Can Life Settlement Assets be Marked to Market?

February 1, 2019

Abstract

Life settlement prices are commonly determined by present value calculus. Yet, the asset class lacks an established approach for the determination of adequate discount rates. We estimate historical yield spreads used for pricing based on a large data set of 2,863 transactions that occurred between 2011 and 2016. Subsequently, we explain the cross section of the former based on hedonic regression methodology and a comprehensive set of attributes motivated by industry know-how as well as earlier studies. Out-of-sample results indicate that market-consistent life settlement prices can be conclusively predicted by employing market-consistent discount rates generated with our model.

Keywords: Life Settlements, Valuation, Mark-to-Market, Hedonic Regression

JEL classification: G11; G22; G28; G32; G38
1 Introduction

Life settlements are traded life insurance policies. They originated in the 1980s, when terminally ill policyholders monetized their life insurance contracts to fund medical expenses (see, e.g., Braun et al., 2012). Today, the main market comprises policies of senior citizens with preexisting health impairments. The cash flow pattern of life settlement assets resembles that of swaps. One side, the investor, pays regular premiums to keep the policy in force, while the other side, the insurance carrier, is obliged to disburse the death benefit when the insured passes away.\(^1\) Hence, unsurprisingly, the price of a life settlement, i.e., the amount that the investor hands over to the insured in exchange for his policy, is commonly determined by present value calculus. However, there are two main differences to swap pricing. Firstly, risk-adjusted discount rates are not readily observable and secondly, risk-neutral valuation is not applicable. Instead, actuarial methods are needed to estimate the cash flow probabilities. The latter are governed by the insureds life expectancy (LE). Higher LE-values are associated with lower prices, since the investor will likely have to pay insurance premiums over a longer time horizon and expects to receive the death benefit later (see, e.g., Bayston et al., 2010).

The total face value sold into the life settlement market amounted to $2.8 billion in 2017 (Horowitz, 2018). Given more than $100 billion worth of face value terminated by senior insureds each year (Braun et al., 2018), the market has a huge growing potential. However, despite a recent uptick, scholarly research on life settlements remains sparse. Earlier work has mainly considered the impact on policyholder surrender behavior (see Gatzert et al., 2009), price determinants (see Brockett et al., 2013; Zhu and Bauer, 2013), the performance of the asset class (see Braun et al., 2012; Giaccotto et al., 2015), the impact of adverse selection on expected returns (see Januário and Naik, 2014), and risk management aspects for investors (see MacMinn and Zhu, 2017). A detailed empirical analysis of yields spreads used for pricing, however, is missing to date. It turns out that is a severe problem for the market’s further development, since a consistent valuation of life settlement portfolios requires the selection of adequate discount rates (see, e.g., Braun et al., 2015). We address this gap by means of hedonic regression methodology and a comprehensive data set of historical transactions. More specifically, we econometrically explain the yield spread used for pricing through life settlement attributes motivated by industry know-how and relevant research. This approach is common for other illiquid markets with heterogeneous assets, such as real estate (see, e.g., Shiller, 1993; Lin and Vandell, 2007). As in the case of property, the immediate trading of life settlements is impossible and each asset exhibits different characteristics.

Our results indicate that longevity risk and premium risk are the most important drivers of life settlement yield spreads and that market-consistent prices can be conclusively predicted by employing risk-adjusted discount rates generated with the proposed model. When calibration is kept up to date, both the in-sample and out-of-sample accuracy of this approach turns out to be encouragingly high. This is also of substantial practical relevance. Although the International Financial Reporting Standard (IFRS) 13 and the Alternative Investment Fund Managers Directive (AIFMD) require assets to be held at fair value, some fund managers still maintain that fair valuation for life settlements is unattainable.\(^2\) In line with this claim, an empirical study conducted by Braun et al. (2015) revealed differences between the portfolio valuations reported by certain fund managers and the prices for which similar policies traded in the market. We equip investors with a straightforward toolkit to reveal such practices.

The remainder of the manuscript is structured as follows. In Section 2, we provide a brief introduction to the current practice in life settlement pricing. Testable hypotheses with regard to the key drivers of yield spreads are then developed in Section 3. Section 4 is the empirical part. Here we discuss the data and the procedure for the extraction of yield spreads, show descriptive statistics for our sample, run hedonic regressions to explain the estimated yield spreads, and provide a number of robustness tests. We also assess the in-sample

\(^1\)This is virtually identical to the cash flow pattern of a credit default swap (CDS). Under such a contract, the protection buyer makes continuous payments of the CDS spread in return for a compensation from the protection seller in case a default of the underlying reference entity occurs (see, e.g., ?).

\(^2\)Based on conversations with industry practitioners.
and out-of-sample suitability of our framework for the derivation of risk-adequate discount rates. Finally, in
Section 4.6, we discuss the limitations of our results and in Section 5, we draw our conclusion.

2 Life Settlement Pricing: A Primer

The following is a formal expression for the price of a life settlement asset:

\[ \text{TP} = \sum_{t=0}^{\infty} -tp_x \cdot \pi_t \cdot \frac{1}{(1+r)^t} + \sum_{t=1}^{\infty} tp_x \cdot q_{x+1:t} \cdot DB \cdot \frac{1}{(1+r)^t} \]  

where

- \( TP \): transaction price for the life insurance policy,
- \( DB \): death benefit,
- \( tp_x \): the probability that the \( x \)-year-old insured will survive to time \( t \),
- \( q_{x+1:t} \): the probability that the \( x \)-year-old insured will die between time \( t-1 \) and \( t \),
- \( \pi_t \): premium to be paid at time \( t \),
- \( r \): risk-adjusted discount rate,

Analogous to swap pricing terminology, we dub the first summand premium leg and the second death benefit
leg. The yield spread \( YS \) is embedded in the discount rate. Formally,

\[ YS = r - r_f, \]  

where \( r_f \) denotes the risk-free rate. Risk-adequate discount rates are chosen by life settlement investors so as
to match their specific return targets. Riskier policies should be cheaper than less risky ones. Thus, observed
market prices reflect investors’ perceptions of the riskiness of a deal and can be employed to extract market-
consistent yield spreads. The sizes of \( DB \) and the \( \pi_t \) are known from the policy terms and conditions. Investors
that employ Equation (1) to calculate prices or value portfolios need to additionally enter survival rates and
discount rates. Both are typically determined subjectively. Mortality profiles, including life expectancy \( LE \)
and the corresponding actuarial probabilities \( (tp_x) \) are estimated by specialized medical underwriters. There is
a direct link between both magnitudes in that \( LE \) consists of the \( tp_x \) (see Appendix A for details). Medical
underwriters draw on standard actuarial life tables (e.g., VBT 2015) and modify them with so-called mortality
multipliers that reflect an insured’s health impairment relative to the average individual in his age bracket (see,
e.g., Xu, 2018). Investors may use a single estimate for \( LE \), and hence the survival probabilities, or blend those
of several medical underwriters (see Figure 1).
This figure summarizes the various methods used to derive the LE for the closing of a senior life settlement transaction and indicates their relative degree of implementation. In most cases, investors use an LE that lies within the scope of the estimates issued by medical underwriters. In rare cases, the blended LE falls below the lowest (min(LE)), or exceeds the highest (max(LE)), estimate.

Source: Authors illustration on data from AA-Partners.

Figure 2 illustrates the pricing relationship based on a hypothetical life insurance policy with a death benefit of 3,645 kUSD from a 75-year old male non-smoker. The death benefit and the respective premium streams are averages of all universal life policies in our data set (details see empirical section) with policyholders that fit the aforementioned gender, smoking status and age at the transaction date. The shapes of the graphs in Figure 2 are intuitive. As the price of the policy is equal to the sum of the discounted expected cashflows, it decreases nonlinearly in \( r \). For an \( r \) of 0%, the present value equals the sum of expected death benefits less the sum of expected premium payments. In contrast, the price converges to zero as \( r \) goes to infinity.

Figure 2: Transaction price (\( TP \)) against discount rate (\( r \)) by mortality multiplier (\( k \))

This figure illustrates the life settlement pricing relationship in Equation (1) based on a hypothetical policy of a 75-year old male non-smoker. The respective premium streams are averages of all universal life policies in our data set (see empirical section) with insured that exhibit the same gender, smoking status and age. Prices are negatively related to \( LE \) (survival rates) and \( r \). The mortality multiplier \( k \) represents the degree of the health impairment of the insured. The higher the value of \( k \), the shorter the respective \( LE \) for a given age. Put differently, for a constant \( r \), a higher \( k \) is associated with a shorter \( LE \) and therefore higher \( TP \).

\( ^3 \)Note that the difference in the present values of the two cash flow streams is positive, since the sum of expected death benefits exceeds the sum of probability-weighted premiums.
3 Testable Hypotheses

In line with the work of Braun et al. (2012), we identify a total of five important risk types associated with senior life settlements: longevity risk, premium risk, default risk, rescission risk and liquidity risk. Below we formulate hypotheses regarding their impact on \( YS \) and introduce variables that are employed to measure them.

**Longevity Risk**

Longevity risk means that an insured may live longer than expected. It is the most prominent risk in life settlements and emanates primarily from the possibility of inaccurate (too short) \( LE \) estimates. Below, we discuss a number of independent variables that are linked to this type of risk.

*LE*: *Life expectancy used to close the deal.* As discussed in the previous section, \( LE \) estimates for life settlement transactions are issued by specialized firms called medical underwriters. Based on these estimates, policy buyers and sellers agree on the \( LE \) value that they use to close the deal. Xu (2018) provides empirical evidence that shorter \( LE \)s are more likely to be underestimated than longer ones, meaning that they exhibit a higher degree of longevity risk. This should be reflected in higher values of \( YS \):

\[
H1: YS \text{ decreases in } LE.
\]

*DB*: *Death benefit.* Policyholders whose death benefit \( DB \) is high tend to be wealthy people who have access to advanced healthcare and therefore exhibit a greater longevity compared to their less well-off peers (see, e.g., Verdon, 2010). \( DB \), however, is regularly not disclosed to medical underwriters and consequently not captured in their \( LE \) estimates. Hence, policies with higher \( DB \) values can be expected to exhibit a higher degree of longevity risk, leading us to postulate:

\[
H2: YS \text{ increases in } DB.
\]

*DI*: *Difference in \( LE \) estimates.* Conventionally, a senior life settlement requires \( LE \) certificates from at least two medical underwriters. As \( LE \) estimation involves different quantitative models and subjective judgment, results concerning the same life can notably differ (see Xu, 2018). The variable \( DI \) denotes the gap between the longest and the shortest available \( LE \) estimate for a given policy. The larger this deviation, the higher the assumed uncertainty surrounding the accuracy of the \( LE \) and, in turn, the longevity risk. Therefore, we expect to find the following effect:

\[
H3: YS \text{ increases in } DI.
\]

*MK*: *Market.* \( MK \) is a binary variable that indicates whether the transaction occurred in the secondary (\( MK = 0 \)) or the tertiary market (\( MK = 1 \)) for life insurance policies. Ceteris paribus, a policy in the former should carry a higher longevity risk than one in the latter. The reason is an adverse selection effect (see Bauer et al., 2014). Insureds who are inclined to sell their policies usually think they are healthy, and most likely this feeling reflects their real health condition regardless of what their medical records imply. Based on this notion, we postulate:

\[
H4: YS \text{ is negatively related to } MK.
\]

*NO*: *Number of \( LE \) estimates.* \( NO \) is a binary variable denoting the number of \( LE \) estimates from the four biggest medical underwriters (ITM TwentyFirst, AVS, Fasano and LSI) considered in a transaction. If multiple \( LE \) estimates are available, \( NO = 1 \). Otherwise \( NO = 0 \). In line with Januário and Naik (2014), we suppose that
buyers associate less longevity risk with a policy for which multiple LE estimates are available. Consequently, they are willing to pay a higher price or accept a lower yield spread:

\[ H5: \text{YS is negatively related to NO}. \]

AGE: Insured’s age. The life expectancies of older people are more difficult to forecast due to paucity of historical data (see, e.g., Bahna-Nolan, 2014). Consequently, policies of more senior insureds are likely to include a higher degree of longevity risk, and should therefore be associated with a higher yield spread:

\[ H6: \text{YS increases with AGE}. \]

CO: Premium convexity. We adopt the notion of premium convexity from Januário and Naik (2014) and define the variable CO as the sum of time-weighted premium fractions:

\[
CO = \sum_{t=0}^{\infty} \frac{t^2 \pi_t}{\sum_{t=0}^{\infty} \pi_t},
\]

where \( \pi_t \) is the dollar amount of premium to be paid at time \( t \). CO captures the latent longevity risk associated with insureds’ outliving their LE estimates: ceteris paribus, the more convex a premium stream, the heavier the loss a policy buyer would suffer should the insured live longer than expected. This leads us to assume the following:

\[ H7: \text{YS increases in CO}. \]

Premium Risk

Premium risk pertains to a hike in the premiums of an in-force policy, which means higher cash outflows for investors (see, e.g., Hong and Seog, 2018). We measure this risk type by the following variable:

PM/DB: Sum of expected premiums as a fraction of the death benefit. The current premium level is known to be an indicator for the likelihood of increases. Sheridan (2017), e.g., suggests that low-premium policies are more likely to experience premium rises. We use the sum of expected premiums \( PM \) until \( LE \) normalized to \( DB \) as a measure for the premium level:

\[
PM = \sum_{t=0}^{LE} \pi_t,
\]

Other things equal (especially \( LE \)), the lower \( PM/DB \), the higher a policy’s premium risk. Life settlements with lower premium levels should therefore be priced more conservatively, namely with a higher \( YS \). Accordingly, we formulate the following hypothesis:

\[ H8: \text{YS decreases in PM/DB}. \]

Default Risk

Despite their relatively high financial strength, insurance carriers may become unable to pay death benefits should financial distress occur. An insurer’s A.M. Best credit rating is a good proxy to gauge this risk.

RT: Credit rating. For each policy, RT is a binary variable that denotes the insurer’s credit rating assigned by A.M. Best, a U.S.-based rating agency with a focus on the insurance industry. For policies issued by A-rated
(A-, A, A+, AA-, AA, AA+, AAA) insurers, $RT = 1$, otherwise $RT = 0$. Higher ratings imply a lower default risk associated with the payout of death benefit. Therefore, we expect to find the following relationship:

$$H9: YS \text{ is negatively related to } RT.$$  

Rescission Risk

Rescission is the revocation of a contract. In the life settlements market, it means the insurance carriers’ refusal to pay the death benefit. This could happen due to a lack of insurable interest or other fraudulent behavior at issuance (Chancy et al., 2010). We proxy this risk by the tenure of a policy (Sadowsky and Browndorf, 2009).

$TE$: Tenure. The tenure of a policy is represented by the time elapsed between the issuance and the settlement date. The sooner a life insurance policy is available for sale after its issuance, the more likely it is that investors would believe the policy was originated with the intention to be life-settled. Such contracts are called stranger-originated life insurance (STOLI). As a carrier can contest a claim in the absence of insurable interest, policies with shorter tenure carry higher rescission risk, and should therefore achieve lower prices than policies with longer tenure. Hence:

$$H10: YS \text{ has a positive relation with } TE.$$  

Liquidity Risk

Liquidity risk describes the ability of investors to liquidate their assets in a crisis. The nature of the life settlements market per se implies high liquidity risk. We assume the same level of liquidity risk for all policies, and let the constant term $c$ capture this risk factor.

Control Variables

We add a control variable $PT$ for the policy type. $PT = 1$ designates universal life policies and $PT = 0$ other types. We have also considered variables such as cash surrender value,\footnote{Cash surrender value (CSV) is the money that sits in a policy’s cash account. If CSV is large, the policyowner can enjoy a “premium holiday”, meaning that premiums are funded from the account so that no out-of-pocket payment to the insurer is needed. Since CSV is immediately obtainable upon lapse, it forms the floor of the policy price. We do not deduct CSV from TP, because the premiums in our sample are optimized. Therefore, any CSV effect is already captured through the premium stream.} total premiums until terminal age, transaction date, smoking status, (implied) mortality multiplier and a premium financing dummy. Furthermore, potential interactions between the variables have been taken into account. We refrain from reporting these results, because there is either a lack of sound theory for these variables, or a conceptual overlap with the factors already discussed in this section (e.g. between mortality multiplier $k$ and life expectancy $LE$). In addition, the inclusion of these regressors does not markedly change our results.

4 Empirical Analysis

4.1 Data and Sample Selection

We obtained our data from AA-Partners Ltd (AAP), a Zurich-based consulting firm specialized in life settlements. AAP is one of the few reliable providers of data on this asset class. It maintains a comprehensive network in the industry, through which it collects audited transaction information from over a dozen of life
settlement providers on a monthly basis (see Braun et al., 2015). Based on AAP’s own estimation, their data cover approximately 20% of the total deal flow before 2014, and 60% after. Our sample consists of 2,863 life settlement transactions from both the secondary and tertiary markets and spans the time period from July 2011 to December 2016. It allows us to measure all effects discussed in the previous section. We take the values for \(LE\), \(DB\), and \(AGE\) as given. The remaining variables (\(DI\), \(MK\), \(NO\), \(CO\), \(PM/DB\), \(RT\), \(TE\)) are coded/computed from the information in the sample. The majority of the recorded policies (56%) were traded in the two most recent years. Hence, we are able to analyze a substantial part of the overall market.

4.2 Estimation of Yield Spreads

While the AAP dataset comprises all salient deal characteristics mentioned above (including transaction price \(TP\)), the mortality and discount rates that enter Equation (1) are not shared by investors. We therefore need to estimate these magnitudes from our market data. To this end, we rewrite the pricing relation in terms of the yield to maturity \(ytm\):

\[
TP = \sum_{t=0}^{\infty} -tP_x \cdot \pi_t \cdot (1 + ytm)^t + \sum_{t=1}^{\infty} tP_x \cdot q_{x+t} \cdot DB \cdot (1 + ytm)^t
\]

\(YS\) is now defined as the yield spread, i.e., it equals the difference between \(ytm\) and the risk-free rate \(r_f\). For each life settlement transaction, the risk-free rate \(r_f\) can be measured as the probability-weighted average yield of zero-coupon treasury bonds, formally

\[
r_f = \sum_{t=1}^{\infty} (t-1)p_x \cdot q_{x+t} \cdot y_t,
\]

where \(y_t\) represents the \(t\)-year spot rate of the U.S. treasury zero bond yield curve at the time of the life settlement transaction. The respective data has been downloaded from from FRED Economic Data (https://fred.stlouisfed.org/). Linear interpolation is applied to synthesize rates at maturities where no bonds exist. For example, averaging interest rates of a 3-year bond and a 5-year bond forms the interest rate of a 4-year bond.

Before we can back out \(ytm\), we first need to determine the survival probabilities (mortality rates) implied by the \(LE\) that has been used to close the deal. This is done in three steps:

1. We employ VBT Table 2015 as the basis mortality curve for each insured, given his gender, smoking status, and age.
2. Using the relationship shown in Appendix A, we estimate the mortality multiplier \(k\) implied by the \(LE\)-value reported for each transaction.\(^5\)
3. We combine the estimated \(k\) with our basis mortality curves to generate individual mortality curves implied by the market data.

Finally, based on the price, premium stream and individual mortality curve of each transaction, we can calculate \(ytm\) and, in turn, \(YS\ (ytm - r_f)\).

\(^5\)Recall from the second section that \(k\) represents the insured’s health impairment relative to an average individual.
4.3 Descriptive Statistics

In this section, we provide a variety of descriptive statistics. Table 1 contains the number of observations \((n)\), mean, median, minimum (Min.), maximum (Max.), and standard deviation (StDev.) for the major variables in the sample. To get a sense of the typical transaction characteristics, consider the following values: the average \(TP\) amounts to USD 368.11 thousand at an average \(LE\) of 6.64 years, an average \(PM/DB\) of 26.35\%, an average \(DB\) of USD 1.8 million, and an average \(YS\) of 21.89\%. Both \(TP\) and \(YS\) vary considerably across policies, which is indicated by the respective standard deviations as well as the minimum and maximum values. The extreme \(YS\) values of -1.95\% and 247.48\% indicate the presence of outliers. The same is true for \(k\). A high standard deviation is also observed for the death benefit (\(DB\)), for which the minimum and maximum values differ by almost USD 30 million.

Further descriptive statistics for different categories of policies are presented in Table 2. When focusing on gender, we observe that the majority (71\%) of the insureds in our sample are male. Even though the average male is two years younger than the average female, their respective \(LEs\) differ only slightly. Apart from that, we notice that nonsmokers dominate the sample (97\%). The average age of nonsmokers (78 years) is significantly higher than that of smokers (72 years). Despite being that much older than the average smoker, the \(LE\) of the average nonsmoker is one year longer. In line with the lower \(LEs\), both mean \(TP/DB\) and \(YS\) are higher for smoker than for nonsmoker policies. Furthermore, secondary market transactions make up nearly 70\% of the sample. While the mean \(LE\) is roughly the same for secondary and tertiary market deals, the average insured in the latter is 7 years older. This implies that health impairments are substantially larger in the secondary market, as also reflected by the higher mortality multiplier \(k\). Accordingly, secondary market transactions, on average, exhibit a higher \(TP/DB\). Concerning policy type, we notice that the sample comprises mainly universal life contracts (84\%). The average \(TP/DB\), \(PM\) and \(DB\) are considerably lower for term life and whole life than for universal life policies. It is also evident that the average insured’s age differs greatly among these product categories. In terms of credit ratings, nearly the whole sample (96\%) consists of policies from A-rated carriers, which seem to be associated with a lower average \(YS\) than those of B-rated or unrated insurers. Finally, about 17\% of the policies originate from California, and 56\% were sold in the years 2015 and 2016. Interestingly, \(TE\) increases with the transaction year, indicating that most issuance dates lie between 2002 and 2004. In this period, the number of manufactured policies, including stranger originated life insurance (STOLI), was relatively high.

<table>
<thead>
<tr>
<th>DB (kUSD)</th>
<th>2,863</th>
<th>1,322.78</th>
<th>1,000.00</th>
<th>20.00</th>
<th>30,000.00</th>
<th>2,583.92</th>
</tr>
</thead>
<tbody>
<tr>
<td>TP (kUSD)</td>
<td>2,863</td>
<td>368.11</td>
<td>178.03</td>
<td>0.30</td>
<td>16,191.00</td>
<td>739.69</td>
</tr>
<tr>
<td>TP/DB (%)</td>
<td>2,863</td>
<td>26.87</td>
<td>20.84</td>
<td>0.25</td>
<td>85.38</td>
<td>20.66</td>
</tr>
<tr>
<td>PM/DB (%)</td>
<td>2,863</td>
<td>26.35</td>
<td>26.62</td>
<td>0.00</td>
<td>96.50</td>
<td>17.28</td>
</tr>
<tr>
<td>CSV/DB (%)</td>
<td>2,838</td>
<td>1.64</td>
<td>0.00</td>
<td>-4.02</td>
<td>44.42</td>
<td>4.15</td>
</tr>
<tr>
<td>YS (%)</td>
<td>2,863</td>
<td>21.89</td>
<td>16.60</td>
<td>-1.95</td>
<td>247.48</td>
<td>21.59</td>
</tr>
<tr>
<td>LE (years)</td>
<td>2,863</td>
<td>6.64</td>
<td>6.26</td>
<td>0.43</td>
<td>28.50</td>
<td>3.76</td>
</tr>
<tr>
<td>AGE (years)</td>
<td>2,863</td>
<td>77.89</td>
<td>80.31</td>
<td>20.22</td>
<td>97.80</td>
<td>11.32</td>
</tr>
<tr>
<td>TE (years)</td>
<td>2,697</td>
<td>11.99</td>
<td>10.34</td>
<td>1.14</td>
<td>30.92</td>
<td>7.07</td>
</tr>
<tr>
<td>k (—)</td>
<td>2,863</td>
<td>67.92</td>
<td>3.31</td>
<td>0.39</td>
<td>4,625.67</td>
<td>273.45</td>
</tr>
</tbody>
</table>

This table shows the number of observations \((n)\), mean, median, minimum (Min.), maximum (Max.), and standard deviation (StDev.) of the major variables in the sample. The binary variables introduced in the previous section have been omitted.
Table 2: Descriptive statistics for different categories

<table>
<thead>
<tr>
<th>Gender</th>
<th>n</th>
<th>Percent (%)</th>
<th>$DB$ (kUSD)</th>
<th>$TP$ (kUSD)</th>
<th>$TP/DB$ (%)</th>
<th>$PM/DB$ (%)</th>
<th>$CSV/DB$ (%)</th>
<th>$YS$ (%)</th>
<th>$LE$ (years)</th>
<th>$AGE$ (years)</th>
<th>$TE$ (years)</th>
<th>$k$ (—)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>2,025</td>
<td>70.73</td>
<td>1,765.45</td>
<td>26.17</td>
<td>26.72</td>
<td>1.63</td>
<td>21.30</td>
<td>6.77</td>
<td>77.22</td>
<td>12.42</td>
<td>46.14</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>838</td>
<td>29.27</td>
<td>1,995.49</td>
<td>28.57</td>
<td>25.45</td>
<td>1.68</td>
<td>23.32</td>
<td>6.32</td>
<td>79.51</td>
<td>11.00</td>
<td>120.53</td>
<td></td>
</tr>
<tr>
<td>Smoker</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Non smoker</td>
<td>2,784</td>
<td>97.24</td>
<td>1,853.58</td>
<td>26.77</td>
<td>26.43</td>
<td>1.65</td>
<td>21.74</td>
<td>6.66</td>
<td>78.05</td>
<td>11.98</td>
<td>68.21</td>
<td></td>
</tr>
<tr>
<td>Smoker</td>
<td>79</td>
<td>2.76</td>
<td>1,099.97</td>
<td>30.51</td>
<td>23.38</td>
<td>1.65</td>
<td>27.14</td>
<td>5.87</td>
<td>72.32</td>
<td>12.58</td>
<td>57.61</td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary</td>
<td>1,991</td>
<td>69.54</td>
<td>1,772.16</td>
<td>27.58</td>
<td>24.01</td>
<td>1.77</td>
<td>23.28</td>
<td>6.68</td>
<td>75.71</td>
<td>11.72</td>
<td>92.26</td>
<td></td>
</tr>
<tr>
<td>Tertiary</td>
<td>872</td>
<td>30.46</td>
<td>1,971.20</td>
<td>25.27</td>
<td>31.69</td>
<td>1.36</td>
<td>18.71</td>
<td>6.54</td>
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<td>28.59</td>
<td>4.35</td>
<td>62.75</td>
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<td>1.68</td>
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<td>62.75</td>
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<td>1.63</td>
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<td>6.91</td>
<td>77.80</td>
<td>10.73</td>
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<td>25.51</td>
<td>1.67</td>
<td>22.84</td>
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<td>30.98</td>
<td>21.74</td>
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<td>24.24</td>
<td>6.27</td>
<td>73.94</td>
<td>12.43</td>
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<td>1,728.73</td>
<td>25.75</td>
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<td>1.88</td>
<td>24.81</td>
<td>6.77</td>
<td>78.08</td>
<td>14.14</td>
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<td>23.98</td>
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<td>11.96</td>
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<td>1,596.87</td>
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<td>29.63</td>
<td>1.34</td>
<td>19.60</td>
<td>7.12</td>
<td>78.17</td>
<td>10.25</td>
<td>64.89</td>
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<td>27.17</td>
<td>1.72</td>
<td>22.20</td>
<td>6.38</td>
<td>78.17</td>
<td>12.57</td>
<td>70.72</td>
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</tr>
</tbody>
</table>

In this table, the sample is classified according to gender, smoking status, market, policy type, insurer’s credit rating, originating state and transaction year. For each category, presented are the number of observations (n), mean death benefit ($DB$), transaction price ($TP$), price as a fraction of death benefit ($TP/DB$), sum of premiums as a fraction of death benefit ($PM/DB$), cash surrender value as a fraction of death benefit ($CSV/DB$), yield spread ($YS$), life expectancy ($LE$), insured’s age ($AGE$), tenure ($TE$) and mortality multiplier ($k$).
4.4 Hedonic Regressions

Historical composition of the yield spread

Consistent with the extant literature on asset pricing in illiquid markets with heterogeneous assets, we explain the market-implied yield spread $YS$ with an econometric model of the form:

$$YS = c + X\beta + \epsilon,$$

where the vectors $X$ and $\beta$ denote the predictors (including control variables) as well as their coefficients, and $\epsilon$ is the error term. $LE$ and $DB$ enter in logarithmic form.

In a first step, we want to test our hypotheses and develop an understanding of the general composition of the life settlement yield spread over the historical time period covered by our sample (July 2011 to December 2016). To this end, we run a preliminary regression with all predictors on the full sample. Table 3 shows the results. The majority of effects is statistically significant and the respective signs were correctly anticipated. More specifically, we find evidence for all hypotheses presented above, apart from $H7$ and $H10$. Therefore, the life settlement yield spread seems to predominantly consist of loadings for longevity risk (measured by the six factors $\ln LE$, $\ln DB$, $DI$, $MK$, $NO$ and $AGE$). Premium risk and default risk, in contrast, play a much lesser role, whereas rescission risk is likely not priced at all. Figure 3 is a graphical illustration of the weights of all the significant components as reflected by their standardized regression coefficients.

Table 3: Estimating the general composition of $YS$ (preliminary regression)

<table>
<thead>
<tr>
<th></th>
<th>Coef.</th>
<th>StCoef.</th>
<th>(StErr.)</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
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<td>(0.279)</td>
<td></td>
<td></td>
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<tr>
<td>$\ln LE$</td>
<td>-0.087</td>
<td>-0.261</td>
<td>(0.012)</td>
<td>***</td>
</tr>
<tr>
<td>$\ln DB$</td>
<td>0.032</td>
<td>0.176</td>
<td>(0.005)</td>
<td>***</td>
</tr>
<tr>
<td>$DI$</td>
<td>0.013</td>
<td>0.088</td>
<td>(0.004)</td>
<td>***</td>
</tr>
<tr>
<td>$MK$</td>
<td>-0.025</td>
<td>-0.054</td>
<td>(0.010)</td>
<td>***</td>
</tr>
<tr>
<td>$NO$</td>
<td>-0.051</td>
<td>-0.109</td>
<td>(0.011)</td>
<td>***</td>
</tr>
<tr>
<td>$AGE$</td>
<td>0.002</td>
<td>0.091</td>
<td>(0.001)</td>
<td>***</td>
</tr>
<tr>
<td>$CO$</td>
<td>0.001</td>
<td>0.040</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>$PM$</td>
<td>-0.247</td>
<td>-0.195</td>
<td>(0.031)</td>
<td>***</td>
</tr>
<tr>
<td>$DB$</td>
<td>-0.073</td>
<td>-0.063</td>
<td>(0.035)</td>
<td>**</td>
</tr>
<tr>
<td>$RT$</td>
<td>0.001</td>
<td>0.022</td>
<td>(0.001)</td>
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<tr>
<td>$PT$</td>
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<td>$df$</td>
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<tr>
<td>$SEE$</td>
<td>$R^2_{adj}$</td>
<td></td>
<td>0.160</td>
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</table>

This figure shows the results for a regression of $YS$ on all potential explanatory variables introduced in the third section. For each considered variable, we present least squares estimates of unstandardized (Coef.) and standardized (StCoef.) regression coefficients, as well as Newey-West standard errors (StErr.) and their significance levels (Sig.). Significance levels of 0.1, 0.05 and 0.01 are marked with "+", "++" and "+++" respectively. $df$ represents the degree of freedom of the regression model. Standard error of the estimate (SEE) and $R^2_{adj}$ explain variance and goodness of fit for the model.

6Recall that we extracted $YS$ from the empirically-estimated $ytm$, since the true underlying values were unobservable. Hence, we work with a market-implied $YS$.
7The existing literature on life settlements does not yet include a model that is well suited to explain $YS$. A promising attempt was made by Januário and Naik (2014), who have used $ytm$ as the dependent variable in a regression with various specifications. Their model, however, focuses on the identification of potential adverse selection effects and only explains a minor part of the variance of $ytm$. In addition, they do not assess the suitability of the predicted yields for the market-consistent valuation of life settlement assets.
8Note that the VIF (variance inflation factor), which has not been reported, is below 5 for each variable, indicating absence of collinearity.
This figure illustrates the composition of the historical life settlement yield spread over the time period from July 2011 to December 2016. The weight (in %) of each component is based on its standardized regression coefficient (StCoef.) shown in Table 3. Only variables with statistically significant coefficients are considered in this break-down.

Identification of abnormal cases

The descriptive statistics (Tables 1 and 2) indicated the existence of extreme values. We now aim to exclude abnormal cases, since they distort the estimation of the least squares coefficients. To identify outliers, we compute the externally studentized residuals as well as Cook’s D of the preliminary regression shown above. The results are shown in Figure 4(a). A Bonferroni-corrected confidence level at 0.95 is set for externally studentized residuals (dashed vertical line), whereas a critical value of $\frac{4}{df}$ is set for Cook’s D (dashed horizontal line). Data points beyond those thresholds (red dots) are deemed outliers. The latter account for 3.5% of the full sample (101 transactions). In Figure 4(b) we highlight the previously identified outliers in a TP-YS plot. Most of those transactions are characterized by very low TP values (mostly below USD 1 million) and therefore excessively high yield spreads (YS mostly above 50%). This might be due to the fact that low-value policies are sometimes priced in an ad-hoc fashion instead of strictly following an actuarial pricing formula. Hence, for those policies, the market-implied YS values extracted from the ytm appear abnormal. After all outliers have been removed, we are left with 2,762 transactions.
Figure 4: Identification of outliers

(a) Cook’s D vs. externally studentized residuals

(b) Transaction price $TP$ vs. yield spread $YS$

Figure 5(a) serves to identify outliers in the data set. Cook’s D has been plotted against the externally studentized residuals for each case. The critical values for both measures are represented by dotted lines, which form a box in the bottom left. Observations outside this box are regarded outliers. The majority of the selected cases exhibit low transaction price and abnormally high yield spread as shown in Figure 5(b).

Comparison with fixed-income yield spreads

Figure 5 is a graphical illustration of the evolution of the average $YS$ between July 2011 and December 2016. For comparison purposes, it also includes the historical spreads on high yield fixed-income securities, which we downloaded from FRED Economic Data. The $YS$ time series has been calculated based on a centered 120-day moving average of the truncated sample, without the abnormal cases identified above. We notice that the yield spread for life settlements was historically substantially higher than for corporate default risk. The high volatility of $YS$ is likely attributable to the riskiness, heterogeneity and illiquidity of the asset class.

Predicting the life settlement yield spread

We now turn to our main research goal of developing a model for the derivation of risk-adjusted discount rates, which allows for an accurate market-consistent pricing of life settlement assets. Before we continue, we divide the overall data set into three equally-sized subsamples. Sorted chronologically, the first third of the transactions (training sample) is used for model development, in-sample fitting and the pre-selection of models. We then draw on the second subsample (validation sample) for an assessment of the out-of-sample pricing accuracy of the short-listed models. Based on the respective results, we chose our final model and test its out-of-sample performance on the third subsample (test sample).

Methodologically, we rely on forward selection in a series of eleven ordinary least squares (OLS) regressions with robust standard errors based on the Newey-West heteroskedasticity and autocorrelation consistent (HAC) covariance matrix. In each round, we add the regressor with a statistically significant coefficient that delivers the largest improvement in the model fit. Standard error of the estimate (SEE), adjusted $R^2$ ($R^2_{adj}$), and the

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9 Backward deletion and exhaustive selection deliver virtually the same results and are thus not reported here.
The life settlement yield spread is based on the $YS$ values in our sample, excluding outliers. The time series has been calculated as a centered 120-day moving average. The U.S. High Yield Option-Adjusted Spread is downloaded from FRED Economic Data (https://fred.stlouisfed.org/).

Bayesian information criterion (BIC) are employed as performance indicators to assess how well a combination of coefficients and variables explains the sample data. For a formal definition of these measures refer to Appendix B. A complementary analysis for cross-validation using the Least Absolute Shrinkage and Selection Operator (LASSO) method can be found in Appendix C.

The estimation results are presented in Table 4 and mostly confirm our earlier findings. The significance of the intercept in most model specifications indicates the existence of a base-line yield spread, possibly resulting from factors that are not captured by the independent variables, such as liquidity risk. Model performance can generally be enhanced by adding more independent variables. However, this effect becomes smaller from Model 3 onwards. Following Model 6, we even detect a deterioration in the BIC, although SEE and $R^2_{adj}$ show further improvements. Consequently, we decide to avoid in-sample overfitting by sticking to the most prevalent factors. Of all examined alternatives, we eliminate Models 1, 2, 7, 8, 9, 10 and 11 and continue our analysis with Models 3-6.

To assess the pricing accuracy associated with Models 3-6, we use their versions fitted to the training sample (as shown in Table 4) to generate predictions for the yield spread $\hat{YS}$ and subsequently insert them into Equation (1) to compute model prices $\hat{TP}$ for all transactions in the training and the validation sample. Based on the differences between observed and fitted values for the transaction price, we calculate four common performance indicators: mean error (ME), mean absolute error (MAE), root mean square error (RMSE) and (out-of-sample) $R^2$ (see Appendix B for formal definitions). Figure 6 is a graphical illustration of the results in both the training sample (left hand side) and the validation sample (right hand side). The 45-degree line (red, dashed) in each plot implies equality between model prediction $\hat{TP}$ (horizontal axis) and empirical observation $TP$ (vertical axis). The points above (below) it imply underestimation (overestimation). Deviations from the line correspond to pricing errors. Overall, the prices generated by using the model-predicted yield spreads in combination with the present value relationship in Equation (1) are relatively well aligned along the 45-degree line. A solid precision is also reflected by the $R^2$-measures, implying that our framework is capable of describing a large portion of the price variance both in sample and out of sample.\(^{10}\)

\(^{10}\)The ME and MAE values, however, are relatively high compared to other asset pricing models for insurance risk (see, e.g., Braun, 2016). This observation warrants a discussion of key limitations associated with our approach.
Table 4: Regression model development for $YS$ with training sample

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<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
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<td>(0.044)</td>
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<td>(0.009)</td>
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<td>(0.009)</td>
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<td>(0.009)</td>
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<td>0.015</td>
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<td>(0.002)</td>
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<td>(0.008)</td>
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<td>-0.061</td>
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<td>(0.010)</td>
<td>**</td>
<td>(0.010)</td>
<td>**</td>
<td>(0.010)</td>
<td>**</td>
<td>(0.010)</td>
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<td>(0.010)</td>
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<td>0.000</td>
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<td>0.000</td>
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<tr>
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<td>(0.001)</td>
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<td>(0.001)</td>
<td>***</td>
<td>(0.001)</td>
<td>***</td>
<td>(0.001)</td>
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<td>(0.001)</td>
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<td>0.001</td>
<td>0.021</td>
<td>0.001</td>
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<tr>
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<td>(0.001)</td>
<td>***</td>
<td>(0.001)</td>
<td>***</td>
<td>(0.001)</td>
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<td>(0.001)</td>
<td>***</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$PM$</td>
<td>-0.189</td>
<td>-0.269</td>
<td>-0.169</td>
<td>-0.240</td>
<td>-0.168</td>
<td>-0.239</td>
<td>-0.158</td>
<td>-0.225</td>
<td>-0.157</td>
<td>-0.214</td>
<td>-0.152</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>***</td>
<td>(0.024)</td>
<td>***</td>
<td>(0.023)</td>
<td>***</td>
<td>(0.023)</td>
<td>***</td>
<td>(0.023)</td>
<td>***</td>
<td>(0.030)</td>
</tr>
<tr>
<td>$PT$</td>
<td>-0.043</td>
<td>-0.061</td>
<td>-0.043</td>
<td>-0.061</td>
<td>-0.031</td>
<td>-0.047</td>
<td>-0.032</td>
<td>-0.048</td>
<td>-0.031</td>
<td>-0.047</td>
<td>-0.032</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>*</td>
<td>(0.025)</td>
<td>*</td>
<td>(0.026)</td>
<td>*</td>
<td>(0.026)</td>
<td>*</td>
<td>(0.026)</td>
<td>*</td>
<td>(0.026)</td>
</tr>
<tr>
<td>$df$</td>
<td>919</td>
<td>918</td>
<td>917</td>
<td>916</td>
<td>915</td>
<td>914</td>
<td>913</td>
<td>912</td>
<td>911</td>
<td>827</td>
<td>788</td>
</tr>
<tr>
<td>$SEE$</td>
<td>0.099</td>
<td>0.095</td>
<td>0.093</td>
<td>0.092</td>
<td>0.091</td>
<td>0.091</td>
<td>0.091</td>
<td>0.091</td>
<td>0.091</td>
<td>0.087</td>
<td>0.088</td>
</tr>
<tr>
<td>$R^2_{adj}$</td>
<td>0.226</td>
<td>0.283</td>
<td>0.316</td>
<td>0.334</td>
<td>0.340</td>
<td>0.344</td>
<td>0.347</td>
<td>0.350</td>
<td>0.349</td>
<td>0.345</td>
<td>0.344</td>
</tr>
<tr>
<td>$BIC$</td>
<td>-1.627</td>
<td>-1.692</td>
<td>-1.729</td>
<td>-1.748</td>
<td>-1.750</td>
<td>-1.750</td>
<td>-1.748</td>
<td>-1.747</td>
<td>-1.740</td>
<td>-1.642</td>
<td>-1.544</td>
</tr>
</tbody>
</table>

This figure shows the results for a series of eleven regressions of $YS$ on different combinations of the potential explanatory variables introduced in the third section. For each model, we present variables’ least squares estimates of unstandardized (Coef.) and standardized (StCoef.) regression coefficients, as well as Newey-West standard errors (StErr.) and their significance levels (Sig.). Significance levels of 0.1, 0.05 and 0.01 are marked with "*", "**" and "***" respectively. $df$ represents the degree of freedom of the regression model. Standard error of the estimate (SEE) and $R^2_{adj}$ explain variance and goodness of fit for the model. Bayesian information criterion (BIC) allows us to compare models that differ in number of variables.
Figure 6: Observed versus fitted values prices based on yield spreads predicted by models 3, 4, 5, and 6

(a) Model 3: \( \widehat{YS} = 0.541 - 0.065 \ln LE - 0.044MK - 0.169\frac{FM}{FTP} \)

(b) Model 4: \( \widehat{YS} = 0.376 - 0.073 \ln LE + 0.014 \ln DB - 0.040MK - 0.168\frac{FM}{FTP} \)

(c) Model 5: \( \widehat{YS} = 0.374 - 0.069 \ln LE + 0.015 \ln DB - 0.038MK - 0.158\frac{FM}{FTP} - 0.035PT \)

(d) Model 6: \( \widehat{YS} = 0.379 - 0.073 \ln LE + 0.014 \ln DB + 0.005DI - 0.038MK - 0.157\frac{FM}{FTP} - 0.034PT \)

This figure is a comparison of predicted transaction prices (\( \widehat{TP} \), horizontal axis) and observed transaction prices (\( TP \), vertical axis) in both the training (left hand side) and validation samples (right hand side). The \( \widehat{TP} \) have been calculated with the present value relationship shown in Equation (1) and the yield spreads \( \widehat{YS} \) generated by our regression models estimated on the training sample. The 45-degree line (red, dashed) in each plot implies equality between \( \widehat{TP} \) and \( TP \). The points above (below) it imply underestimation (overestimation). Deviations from the line correspond to pricing errors. Based on the differences between observed and fitted values for the transaction price, we calculate four common performance indicators: mean error (ME), mean absolute error (MAE), root mean square error (RMSE) and (out-of-sample) \( R^2 \).
Although all four short-listed models perform similarly, the quantitative performance measures indicate a superiority of Model 3: it delivers the lowest ME, MAE and RMSE, and the highest $R^2$. Thus, a linear model for the yield spread, comprising the factors $\ln LE$, $MK$ and $PM/DB$, in combination with standard present value calculus seems to be a very effective way to determine market-consistent life settlement prices. The variable selection is confirmed by the LASSO regression presented in Figure 8 of Appendix C. Based on these insights, we move on to the final analysis, in which we run Model 3 on the test sample. The results are illustrated in Figure 7. Evidently, the out-of-sample price predictions based on the predicted yield spreads $\hat{YS}$ continue to be precise. Hence, Model 3 performs consistently well over all three subsamples, covering different time periods.

Figure 7: Observed versus fitted values prices based on yield spreads predicted by model 3

$$\hat{YS} = 0.541 - 0.065 \ln LE - 0.044 MK - 0.169 \frac{PM}{DB}$$

Test Sample (Oct. 09, 2015 – Dec. 31, 2016)

This figure is a comparison of predicted transaction prices ($\hat{TP}$, horizontal axis) and observed transaction prices ($TP$, vertical axis). The former have been calculated with the present value relationship shown in Equation (1) and the yield spreads $\hat{YS}$ generated by Model 3 estimated on the training sample. The 45-degree line (red, dashed) in each plot implies equality between $\hat{TP}$ and $TP$. The points above (below) it imply underestimation (overestimation). Deviations from the line correspond to pricing errors. Based on the differences between observed and fitted values for the transaction price, we calculate four common performance indicators: mean error (ME), mean absolute error (MAE), root mean square error (RMSE) and (out-of-sample) $R^2$.

4.5 Robustness Tests

In the following, we want to evaluate the robustness of Model 3 across policy types, carrier ratings, and medical underwriters. We tranche our data accordingly and recalibrate the model separately for each subcategory. Furthermore, we use two thirds of our overall sample (previously training sample and validation sample) for fitting, and one third (previously test sample) for testing. To calculate $\hat{TP}$, we again plug the model-predicted yield spreads $\hat{YS}$ into the pricing relationship shown in Equation (1). We then measure the model performance based on the deviation of $\hat{TP}$ from $TP$, using the four performance indicators ME, MAE, RMSE and (out-of-sample) $R^2$.

Policy type

The regression results and performance measures for the subsamples of different policy types are shown in Table 5. Both the in-sample and out-of-sample figures for universal life and term life contracts are strong. Yet for the whole life category, we observe insignificant coefficients and a poor out-of-sample performance. This

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11The date that separates the two subsamples is October 9, 2015.
is likely due to the paucity of whole life data, which comprises only 46 cases and thus constitutes less than 2% of the full sample. Until this problem can be resolved, it is advisable to exclusively apply the model to universal life and term life policies. For the latter two categories, model estimation on the specific subsamples does generally not improve pricing accuracy, which can be seen by comparing the results in Table 5 with those for the calibration on the untranched data in Figure 7. This confirms our earlier finding that policy type PT has a negligible impact on YS. Hence, its exclusion from the model was warranted.

Table 5: Robustness test for model 3 on policy-type subsamples

<table>
<thead>
<tr>
<th>Policy type</th>
<th>Universal life</th>
<th>Term life</th>
<th>Whole life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.407 (0.028)***</td>
<td>0.489 (0.059)***</td>
<td>0.271 (0.197)</td>
</tr>
<tr>
<td>ln LE</td>
<td>-0.039 (0.006)***</td>
<td>-0.070 (0.016)***</td>
<td>-0.020 (0.075)</td>
</tr>
<tr>
<td>MK</td>
<td>-0.031 (0.006)***</td>
<td>-0.039 (0.024)</td>
<td>-0.073 (0.036)</td>
</tr>
<tr>
<td>PM/DB</td>
<td>-0.173 (0.018)***</td>
<td>-0.047 (0.119)</td>
<td>0.325 (0.523)</td>
</tr>
</tbody>
</table>

Panel A: separate model calibration for policy-type subsamples

Panel B: model performance for calibration on policy-type subsamples

Rating

The results for the subsamples of issuing-insurer rating classes are shown in Table 6. Just as the whole life subsample, the no rating subsample exhibits a very small size. Therefore, its out-of-sample performance is no reliable indication of the model accuracy. Comparing the categories A-rated and B-rated, we observe that the sensitivity of YS with regard to all predictors, as represented by the absolute values of the regression coefficients, is higher in the latter. Moreover, we find a larger base-line yield spread (intercept) for policies issued by B-rated insurers. In both cases, the model performs similarly well as on the untranched sample.

Medical Underwriters

The LE estimates in our data set come from the four major U.S. medical underwriters (ITM TwentyFirst, AVS, Fasano and LSI). Most life settlements exhibit at least two of them. In addition, for each transaction there is an LE figure which has been used to close the deal. Recall from the second section that the market lacks a universal rule based on which the LE for pricing purposes is determined. The buy and sell side can agree on the estimate of a single medical underwriter, or average the LEs from several. Despite the industrial jargon “blended LE”, the LE used to close a life settlement deal is not always a “blend” in the conventional sense:
Table 6: Robustness test for model 3 on rating-based subsamples

### Panel A: separate model calibration for rating-based subsamples

<table>
<thead>
<tr>
<th>Rating</th>
<th>A-rated</th>
<th></th>
<th>B-rated</th>
<th></th>
<th>No rating</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coeff.</td>
<td>(StErr.)</td>
<td>Sig.</td>
<td>Coeff.</td>
<td>(StErr.)</td>
<td>Sig.</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.410</td>
<td>(0.025)</td>
<td>***</td>
<td>0.806</td>
<td>(0.123)</td>
<td>***</td>
</tr>
<tr>
<td>ln LE</td>
<td>-0.040</td>
<td>(0.006)</td>
<td>***</td>
<td>-0.111</td>
<td>(0.039)</td>
<td>***</td>
</tr>
<tr>
<td>MK</td>
<td>-0.033</td>
<td>(0.006)</td>
<td>***</td>
<td>-0.056</td>
<td>(0.031)</td>
<td></td>
</tr>
<tr>
<td>PM/DB</td>
<td>-0.166</td>
<td>(0.017)</td>
<td>***</td>
<td>-0.247</td>
<td>(0.141)</td>
<td></td>
</tr>
</tbody>
</table>

### Panel B: model performance for calibration on rating-based subsamples

<table>
<thead>
<tr>
<th></th>
<th>InS.</th>
<th>OutS.</th>
<th>InS.</th>
<th>OutS.</th>
<th>InS.</th>
<th>OutS.</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>1,803</td>
<td>873</td>
<td>27</td>
<td>29</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>ME</td>
<td>9</td>
<td>-11</td>
<td>10</td>
<td>-8</td>
<td>-5</td>
<td>-33</td>
</tr>
<tr>
<td>MAE</td>
<td>109</td>
<td>100</td>
<td>119</td>
<td>71</td>
<td>14</td>
<td>121</td>
</tr>
<tr>
<td>RMSE</td>
<td>255</td>
<td>208</td>
<td>232</td>
<td>125</td>
<td>18</td>
<td>201</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.913</td>
<td>0.777</td>
<td>0.813</td>
<td>0.775</td>
<td>0.984</td>
<td>0.218</td>
</tr>
</tbody>
</table>

**Panel A:** model 3 calibrated on subsamples for different rating categories. We present the unstandardized (Coeff.) and standardized (StCoef.) regression coefficients, as well as Newey-West standard errors (StErr.) and their significance levels (Sig.). Significance levels of 0.1, 0.05 and 0.01 are marked with '*', '**' and '***' respectively. **Panel B:** we plug $\hat{YS}$, generated with the calibrated model, into the pricing relationship shown in Equation (1) to calculate $\hat{TP}$. Based on the differences between observed values ($TP$) and fitted values ($\hat{TP}$) for the transaction price, we calculate four common performance indicators: mean error (ME), mean absolute error (MAE), root mean square error (RMSE) and $R^2$. InS.: in-sample estimation (sample period: Jan. 07, 2011 — Oct. 09, 2015). OutS.: out-of-sample prediction (sample period: Oct. 09, 2015 — Dec. 31, 2016).

It can exceed (undercut) the highest (lowest) underwriter estimate (Figure 1). In the absence of reports from medical underwriters, a “home-brewed” $LE$ may be generated. This is common for the pricing of small-face policies where the cost of obtaining an $LE$ report is prohibitive. So far, our analyses relied on the blended $LE$. To assess the model’s robustness with regard to a change in this input, we will now exclusively employ $LE$ estimates of the same underwriter for the recalibration. Consistent with the latter, $PM$ is also recalculated according to Equation (4). Table 7 shows that both the in-sample and out-of-sample performance remain solid when LEs from different underwriters are employed for parameter estimation.
Table 7: Robustness test for model 3 on subsamples of LEs from different medical underwriters

Panel A: separate model calibration for medical underwriter subsamples

<table>
<thead>
<tr>
<th>Underwriter</th>
<th>ITM</th>
<th>AVS</th>
<th>Fasaro</th>
<th>LSI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>(StErr.)</td>
<td>Sig.</td>
<td>Coef.</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.378</td>
<td>(0.027)</td>
<td>***</td>
<td>0.384</td>
</tr>
<tr>
<td>ln LE</td>
<td>-0.034</td>
<td>(0.007)</td>
<td>***</td>
<td>-0.036</td>
</tr>
<tr>
<td>MK</td>
<td>-0.045</td>
<td>(0.007)</td>
<td>***</td>
<td>-0.037</td>
</tr>
<tr>
<td>PM/DB</td>
<td>-0.139</td>
<td>(0.019)</td>
<td>***</td>
<td>-0.112</td>
</tr>
</tbody>
</table>

Panel B: model performance for calibration on medical underwriter subsamples

<table>
<thead>
<tr>
<th>InS.</th>
<th>OutS.</th>
<th>InS.</th>
<th>OutS.</th>
<th>InS.</th>
<th>OutS.</th>
<th>InS.</th>
<th>OutS.</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>1,256</td>
<td>507</td>
<td>1,641</td>
<td>786</td>
<td>293</td>
<td>112</td>
<td>91</td>
</tr>
<tr>
<td>ME</td>
<td>0</td>
<td>-47</td>
<td>10</td>
<td>-15</td>
<td>61</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>MAE</td>
<td>108</td>
<td>126</td>
<td>113</td>
<td>108</td>
<td>167</td>
<td>80</td>
<td>114</td>
</tr>
<tr>
<td>RMSE</td>
<td>205</td>
<td>292</td>
<td>253</td>
<td>229</td>
<td>458</td>
<td>136</td>
<td>175</td>
</tr>
<tr>
<td>R²</td>
<td>0.884</td>
<td>0.594</td>
<td>0.918</td>
<td>0.747</td>
<td>0.934</td>
<td>0.913</td>
<td>0.780</td>
</tr>
</tbody>
</table>

Panel A: model 3 calibrated on subsamples of LEs from different medical underwriters. We present the unstandardized (Coef.) and standardized (StCoef.) regression coefficients, as well as Newey-West standard errors (StErr.) and their significance levels (Sig.). Significance levels of 0.1, 0.05 and 0.01 are marked with “*”, “**” and “***” respectively. Panel B: we plug $\hat{Y}S$, generated with the calibrated model, into the pricing relationship shown in Equation (1) to calculate $\hat{TP}$. Based on the differences between observed values ($TP$) and fitted values ($\hat{TP}$) for the transaction price, we calculate four common performance indicators: mean error (ME), mean absolute error (MAE), root mean square error (RMSE) and $R^2$. InS.: in-sample estimation (sample period: Jan. 07, 2011 — Oct. 09, 2015). OutS.: out-of-sample prediction (sample period: Oct. 09, 2015 — Dec. 31, 2016).

4.6 Limitations

As noted earlier, the discount rates associated with the transaction prices in our sample are not disclosed by investors. Therefore, we extracted implied yield spreads ($Y\ S$) from yields ($ytm$) that we computed based on Equation (5), given prices, premiums and mortality rates. However, the mortality rates used for pricing were also unobservable and had to be inferred from the reported LE-values (via Equation (8) shown in the Appendix). This proceeding relies on two assumptions: (1) LE is the mean of an insured’s survival distribution; (2) an insured’s mortality rates exhibit a constant ratio (the mortality multiplier $k$) with regard to the base mortality rates of his cohort. Differences between our estimates and the true, unobserved mortality rates used by investors would cause discrepancies between actual and implied yield spreads. Practitioners sometimes use median LE instead of mean LE and/or a different mortality table than the VBT15-ANB, which we applied in this study, to extract standard mortality rates.\textsuperscript{12} Additionally, if clinical judgment replaces a full medical underwriting, mortality rates are determined ad hoc and not by multiplying $k$ with standard rates.

The economic and formal characteristics of $ytm$ as an internal rate of return also contribute to the disjunction between actual and implied yield spreads. First, the $ytm$ used for our analysis are expected returns that incorporate investors’ perceptions at the time of the transaction. As such, they will deviate from the realized returns, which can only be assessed at maturity of the life insurance policies. Therefore, the yield spreads derived by means of our model are helpful for pricing but not for performance measurement purposes. Second, if the sign of the probabilistic cash flow changes more than once (such as $-1, +1, +1, +1, -...$), the function $TP(ytm)$ can be non-monotonic\textsuperscript{14} and Equation (5) may have two positive roots. The algorithm that we applied in this

\textsuperscript{12}The crudeness of cohorting in different mortality tables varies. Valuation Basic Tables (VBT), e.g., are gender-smoker-distinct and age-specific. More granular tables also consider primary health impairments.

\textsuperscript{13}This barbelled cash flow pattern can be generated by very high probabilistic tail premiums ($i\ pi \cdot \pi_i$) greater than the probabilistic death benefit receipts ($i\ (1-q_i) \cdot DB$). See Sheridan (2013) for an example.

\textsuperscript{14}The non-monotonicity can also be observed from Figure 2 where $x = 65$, $k = 1$. 

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study to determine the implied yield spread (YS) systematically searches an interval from lower to upper limit for the root (zero) of Equation (5). If multiple roots exist, the smallest is selected. One may consider the usage of the modified internal rate of return, which can resolve the aforementioned problems associated with ytm. However, measurement of the former requires assumptions on reinvestment and finance rates which vary between investors and is thus difficult to implement in this study.

5 Conclusion

In deriving risk-adequate discount rates for senior life settlements, one faces similar problems to other illiquid markets with heterogeneous assets, such as real estate or fine arts. The extant literature has not come up with a reliable solution to the problem yet. To fill the gap, we estimate historical yield spreads for life settlement assets based on a large data set of 2,863 transactions that occurred between 2011 and 2016. Subsequently, we explain the cross section of the former based on hedonic regression methodology and a comprehensive set of attributes motivated by industry know-how as well as earlier studies. Based on the aforementioned findings, we propose a parsimonious model for the prediction of risk-adjusted discount rates in the life settlements market.

We find evidence for the majority of our hypotheses. More specifically, the life settlement yield spread seems to predominantly consist of loadings for longevity risk. Premium risk and default risk, in contrast, play a much lesser role, whereas rescission risk is likely not priced at all. A comprehensive battery of in-sample and out-of-sample tests indicates that market-consistent life settlement prices can be conclusively predicted by employing risk-adjusted discount rates generated with our model. Once its parameters have been estimated, the approach can be used to price new transactions or to run portfolio valuations. Accordingly, our findings rebut the claim of certain fund managers that life settlement assets cannot be marked to market.

The composition of the yield spreads merits further research. Although we were able to provide initial evidence on their sizes and constituent parts, our findings are restricted by the fact that we needed to rely on implied instead of observed discount rates. The former were derived using a specifically chosen mortality table and may thus deviate from the values actually applied by investors. Consequently, a confirmation of our results based on the true yield spreads would be desirable. Furthermore, our study revealed high expected returns on life settlement investments. However, there is anecdotal evidence of historical underperformance, which calls the realizability of these return figures into question and raises concerns regarding mispricing. This problem could be addressed by a cash-flow-based performance analysis for open-end life settlement funds. Finally, it remains an open question, to which extent the observed yield spreads actually reflect risk premiums. After all, risks that are uncorrelated with economic fundamentals such as capital markets or consumption are fully diversifiable by the global investor and should therefore carry no risk premium. Against this background, much if not all of the excess returns on life settlements, may be attributable to frictions.
Appendices

A Calculation of LE

In actuarial science, life expectancy $LE$ is defined as follows:

$$LE = \sum_{i=0}^{\infty} (i+1)p_x = \sum_{i=0}^{\infty} (i \cdot |p_x|)$$  \hspace{1cm} (8)

where

$$i \cdot |p_x| = \begin{cases} 1, & i \leq 0 \\ \prod_{j=0}^{i-1} |p_x|, & i \geq 1 \end{cases}  \hspace{1cm} (9)$$

$$i \cdot |p_x| = \sqrt{\max(0, 1 - k \cdot (\frac{i}{n}Q_x))}  \hspace{1cm} (10)$$

- $iQ_x$: standard mortality rate (probability that the $x$-year-old insured’s age cohort will live $i$ periods).
- $iP_x$: individual mortality rate (probability that the $x$-year-old insured will live $i$ periods)
- $k$: customized mortality multiplier (describes the relationship between the individual mortality rate and the standard mortality rate).
- $i \cdot |p_x|$: the $x$-year-old insured’s one-period conditional survival probability at time $i$ (probability that the insured will be alive at the end of the $(i+1)^{th}$ period, given that the person is alive at the end of the $i^{th}$ period).
- $n$: number of periods in a year (e.g., months: $n = 12$).

B Model performance measures

Let $\{y_i\}_{i=1,2,\ldots,n}$ denote the observed values of a variable and $\{\hat{y}_i\}_{i=1,2,\ldots,n}$ the estimated values using multiple linear regression $M$ that contains $q$ slope parameters.

In-sample performance measurements of model $M$ include:

Standard error of the estimate (SEE):

$$\text{SEE} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n - q - 1}}  \hspace{1cm} (11)$$

Adjusted $R^2$ ($R^2_{adj}$):

$$R^2_{adj} = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2} - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2} \frac{q}{n - q - 1}  \hspace{1cm} (12)$$

Bayesian information criterion (BIC):
\[ \text{BIC} = (q + 2) \ln n - 2 \ln p(y|\hat{\theta}, M) \]  
(13)

where \( \hat{\theta} \) denotes the estimated model parameters.

Out-of-sample performance measurements of model \( M \) include (Braun, 2016, pp. 837-840):

Mean error (ME):

\[ \text{ME} = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)}{n} \]  
(14)

Mean absolute error (MAE):

\[ \text{MAE} = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{n} \]  
(15)

Root mean square error (RMSE):

\[ \text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}} \]  
(16)

Out-of-sample coefficient of determination (\( R^2 \)):

\[ R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \]  
(17)

C  LASSO regression

Consider the regression:

\[ y_i = \beta_0 + \beta_1 x_{i1} + ... + \beta_p x_{ip} + \epsilon_i, i = 1, ..., n \]  
(18)

The OLS estimator \( (\beta_0, \beta_1, ..., \beta_p)^{\text{OLS}} \) and the LASSO estimator \( (\beta_0, \beta_1, ..., \beta_p)^{\text{LASSO}} \) are solved as below (Tibshirani, 1996):

\[ (\beta_0, \beta_1, ..., \beta_p)^{\text{OLS}} = \arg \min \left\{ \sum_{i=1}^{n} (\epsilon_i)^2 \right\} \]  
(19)

\[ (\beta_0, \beta_1, ..., \beta_p)^{\text{LASSO}} = \arg \min \left\{ \sum_{i=1}^{n} (\epsilon_i)^2 + \sum_{j=1}^{p} \lambda |\beta_j| \right\} \]  
(20)

The LASSO regression presented in Figure 8 corroborates the variable selection conducted with OLS regression in Section 4.4. For \( YS \) modeling, three variables are selected (\( \ln LE, MK \) and \( PM_{DB} \)) at \( \lambda = 0.014 \). A decrease of \( \lambda \), which increases number of variables selected, does not markedly improve the regression performance with regard to both \( R^2 \) and RMSE.
Figure 8: LASSO regression modeling for $YS$

In-sample data are used for the LASSO regression. Variables selected by LASSO coincide with those by OLS as shown in Section 4.4. At $\lambda = 0.014$, three variables are selected (ln $LE$, MK and $PM_DB$). A decrease of $\lambda$, which increases number of variables selected, does not markedly improve the regression performance with regard to both $R^2$ and RMSE.
References


