The Inheritance of the Insurance Purchase Decision: Model and Evidence

Abstract

This study examines the influence in terms of insurance purchase decision-making given from an elder to a younger in a household. The number of in-force policies of individual life insurance per capita increased from 0.82 in 1988 to 1.26 in 2015 in Japan and the dispersion of the figures between prefectures diminished during the periods. However, the minimum is still no more than 51.3% to the maximum in 2015.

In the articles exploring the diffusion of individual insurance, a few studies factor in the behavior of the parents in terms of taking out insurance. A person receiving the insurance benefits likely enlighten her/his children on effects of insurance and the influence from the parent is larger if they live together after becoming adults. Then, the primary variables of this study are the ratio of the adults living together with their parents to the total and the fraction of insureds.

The analysis adopts a two-periods overlapping-generations framework to illustrate the insurance purchase decision-making and to clarify the relationship between variables including those primary ones. Since the results depend on the parameter values, mainly the damages of the accident and its probability, the calibration is carried out for the detail.

1. Introduction

Whether taking out insurance or how much insurance carrying is a personal issue. Some are so cautious that they cannot stop covering for any risks and others do not mind a potential disaster at all. If individuals are homogenously distributed in terms of cautiousness to risks, the fraction of the insureds should be the same in any regions. In the real world, however, the fraction of people taking out insurance is different between regions where they live. In Japan the number of in-force policies of individual life insurance per capita increased from 0.82 in 1988 to 1.26 in 2015. The coefficient of variation of this benchmark between prefectures diminished during the periods due to the rise of the minimum. Though, it is still no more than 51.3% to the maximum in 2015.

Despite numerous causes of the disparity in propensity to purchasing insurance, this study puts the glance to the inheritance of the propensity between generations. Most of people likely gather information as much as possible when they have to estimate risks of disease, injury, accident and disaster in the future. For young individuals without sufficient experiences, their parents may be the most familiar example living under the risks for years. An individual seeing her/his parent received insurance benefits due to
realization of risks more likely takes out insurance when she/he becomes an independent adult. On the other hand, an individual living a life safely but spending a certain amount of money on insurance may recommend saving rather than insurance. In another case, a young person leaving away from home early may not be influenced by her/his parent. In this way, the experience of parents is considered to influence their children in terms of risk assessment or decision-making to take out insurance.

This idea is close to bargaining power in households and habit formation since they are based on the conception that individual decision depends not only on her/his current conditions but also on the opinions of the other family members as well as her/his past utility levels. Manser and Brown (1980) and McElroy and Horney (1981) are the early works placing the household decision problem into a bargaining framework. Subsequent literatures are Browning et al. (1994), Browning and Chiappori (1998), Elder and Rudolph (2003), Friedberg and Anthony (2006), Antman (2014), and Johnston et al. (2016). Whereas these studies treat a bargaining of couple, influence of elders in a household should be taken into consideration in the case of East Asian countries including Japan. Since people in those countries are taught to respect their elders (Bodycott and Lai (2012) and Sohn et al. (2012)), it is worth verifying a hypothesis that children likely make the same decisions as their parents.

Habit formation or habit persistence is represented by the framework in which the current utility depends not only on the current consumption but also the stock of past consumptions. It has been introduced myriad of studies for several decades even if not a few of them denied the existence of it from the empirical data. Most of us, nevertheless, intuitively our consumption decisions are affected by the factors other than current consumption. A person who once enjoyed a luxury good likely feels disappointment if she/he is forced to purchase inferior one which she/he had usually consumed for past years due to decline in income. With habit formation, Dynan (2000) describes “current utility depends not only on current expenditures, but also on a “habit stock” formed by lagged expenditures.” While she uses the food expenditure data for estimation, habit formation framework is applied to the studies of financial economics (Consantinides (1990), Detemple and Zapatero (1991), Heaton (1995), Ben-Arab and Schlesinger (1996))1.

Although myriad of articles investigating the insurance demand are also found, only a few studies introduce the habit forming framework into analysis. Ben-Arab and Schlesinger (1996) apply habit formation to the insurance demand analysis. Their

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1 Besides financial economics, Bover (1991) applies habit formation to the labor supply behavior.
conclusion, which the level of insurance is greater than would exist without habit formation, gives an answer to the question of excess insure lead from many empirical and experimental studies. Cai et al. (2016) test their model introducing habit formation through a two-year study of the adoption of weather insurance by rice farmers in China. In their model with habit formation framework, famers are supposed to evaluate the insurance product from their own experience.

Whereas the habit stock does not affect the utility levels in this study, it is assumed individual's valuation of the insurance can be influenced by her/his parents living together with her/him. The share of such adults living together with their parents to the total gradually declines under the growing population since a certain number of new generations have to find a dwelling house for themselves. In Japan the change of industrial structure fraught with migration of population has attracted people to urban areas for more than half of a century. The younger generations in the local areas leave there at a chance of matriculation and employment.

The share of the adults living together with their parents to the total was already 26.2% in 1990\(^2\). Observing the share by prefecture, however, the maximum and the minimum are 54.7%, and 12.4%, respectively. The difference between prefectures may separate residents' behavior into the one which is inherited from their parents and otherwise. Whichever the consciousness of risks is higher or lower, it is likely maintained in the prefecture where more adults living together with their parents. Indeed, the correlation coefficient between the share of the adults living together with their parents to the total by prefecture in 1990 and the ratio of the change in the amount of inforce life insurance policy per capita from 1990 through 2015 is -0.423. Namely, the higher share a prefecture has, the smaller the difference of the amount of life insurance policy is.

This study assumes the change in evaluation of risks between generations depends on the share of adults living together with their parents and examines the influence of it to the other variables with an overlapping-generations model. The rest of the paper is organized as follows. Section 2 presents the model and the initial conditions. Section 3 discusses the dynamics of the economy. Section 4 shows the results of calibration. Section 5 shows the results of empirical studies. Section 6 concludes.

2. Model and Initial Conditions
A partial equilibrium economy model is used here, omitting the production sector of consumption goods. To depict the decision making for insurance purchase, a

\(^2\) The figure is based on the National Census.
two-periods overlapping generations framework is supposed. For simplicity, let the size of population be unity in every period, that is, population growth is not considered.

The common utility function among individuals living in two periods is given as an ordinary form of equation (1).

\[ u(c_1, c_2), \quad (1) \]

where \( c_1 \) and \( c_2 \) are the consumption in the first and the second periods, respectively. Due to the tractability, the utility function (1) is supposed to be strictly monotonic increasing, strictly concave, and to satisfy the conditions for the inner solution as below.

**Assumption 1.** \( u_1 > 0, \ u_2 > 0, \ u_{11} < 0, \ u_{22} < 0, \ u_{11}u_{22} - (u_{12})^2 > 0, \ u_{12} = u_{21} \geq 0, \) and \( \lim_{c_k \to 0} u_k = \infty, \) where \( u_k \equiv \partial u / \partial c_k \) and \( u_{kl} \equiv \partial^2 u / \partial c_k \partial c_l, \ k, l = 1, 2. \)

To settle the initial conditions of the economy, which has a younger generation only in period \( t = 0, \) the following argument is developed. Each individual inelastically supplies a unit of labor to gain wage income, \( I > 0 \) and spends a fraction of it for purchasing the consumption goods. Simultaneously, she/he allocates the income to insurance premium, \( i \geq 0 \) and saving, \( s \geq 0 \) as well. Like most two-periods overlapping generations frameworks, the individual does not work in the latter half of life time so that she/he pays for consumption in the second period by saving accumulated in the first period. The bequest motives are not considered in this model.

In the beginning of the second period, some of individuals may face an accident which forces them to spend some amount of their savings. Let \( \tilde{\alpha} \in (1, \infty) \) and \( \alpha \equiv 1/\tilde{\alpha} \in [0,1) \) denote the seriousness of the accident and the ratio of the loss to the total saving. Since they cannot observe the true probability value of the accident, they assess the risk by themselves and have their own subjective probability. The insurance provided by the insurer in this model is designed to cover the risk of the accident, that is, a beneficiary receives the insurance benefits as much as the loss of savings only if she/he has the accident.

The utility maximization problems consist of two types; one is for the individuals who

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3 Since this model prohibits an individual from borrowing money, \( s \geq 0. \) While the insurance premium is also non-negative, it will be set at one price by the insurer in the following arguments.

4 E.g., an individual suffers from a serious disease and reduces the savings to pay for the expensive treatment.

5 The amount of savings is common among the individuals so that the insurance benefits received by the recipients are constant. This is because the model assumes the homogenous consumers.
does not purchase the insurance and the other is for the insured. Let $\emptyset_j \in [0,1]$ denote j-th individual’s subjective probability. The utility maximization problem of the individuals not avoiding the risk is expressed as (2).

$$\max_{c_1,c_2} \emptyset_j u(c_1,c_2) + (1 - \emptyset_j)u(c_1,c_2) \quad (2)$$

subject to $c_1 + s = l, c_2 = \alpha(1 + r)s$,

where $r$ is the constant and risk-free interest rate. As the maximization problem of (2) includes the uncertainty based on the risk, the objective function is the form of the expected utility, or the weighted average of utility by risk. On the other hand, the second utility maximization problem corresponding to the insured is

$$\max_{c_1,c_2} u(c_1,c_2) \quad (3)$$

subject to $c_1 + i + s = l, c_2 = (1 + r)s$.

Using the constraints to substitute out for the couple of consumptions in the subjective function, the both maximization problems (2) and (3) can be reduced to the simple form with respect to the single variable, $s$. The first order conditions for problems (2) and (3) are following equations (4) and (5), respectively.

$$\emptyset_j u_1(l - s, \alpha(1 + r)s) = (1 - \emptyset_j)[-u_1(l - s, (1 + r)s) + (1 + r)u_2(l - s, (1 + r)s)] \quad (4)$$

$$u_1(l - i - s, (1 + r)s) = (1 + r)u_2(l - i - s, (1 + r)s) \quad (5)$$

Let $s_j^-$ and $s_j^+$ denote the solution of the equations (4) and (5) for j-th individual, i.e.,

$$s_j^- \equiv \arg\max_s \{\emptyset_j u(l - s, \alpha(1 + r)s) + (1 - \emptyset_j)u(l - s, (1 + r)s)\}, \quad (6)$$

$$s_j^+ \equiv \arg\max_s \{u(l - i - s, (1 + r)s)\}. \quad (7)$$

Using the notations defined by (6) and (7), the indirect utility functions are expressed as (8) and (9).

$$v^-(s_j^-) \equiv \emptyset_j u(l - s_j^-, \alpha(1 + r)s_j^-) + (1 - \emptyset_j)u(l - s_j^-, (1 + r)s_j^-) \quad (8)$$

$$v^+(s_j^+) \equiv u(l - i - s_j^+, (1 + r)s_j^+) \quad (9)$$

Whether an individual purchases the insurance or not depends on the level of utility in the two situations, (8) and (9). Thus, j-th individual buys the insurance if and only if $v^-(s_j^-) < v^+(s_j^+)$. Figure 1 depicts j-th individual’s indifference curves with respect to some different situations. While the curve $\overline{U}^-$ illustrates the largest utility in the situation where she/he neither buys the insurance nor faces to the accident, the indifference curve denoted by $\overline{U}^-$ indicates the utility level of no income in the second period. Depending on individual’s subjective probability, the location of the indifference curve corresponding
to the solution of the maximization problem (8) is determined between the couple of curves, $U^-$ and $U^-$. In Figure 1 it is drawn as the dotted line curve indexed by $U^-$ in the two areas separated by $U^+$ curve. $U^+$ is the indifference curve illustrating the level of utility maximized by the insured individual. Thus, if j-th individual purchases the insurance, we can recognize her/his indifference curve, $U^-$ is in the lower area to the curve $U^+$.

![Indifference Curve](image)

**Figure 1. Indifference Curve**

Following from the above discussion, some definite relations between the variables can be induced. They are described as Lemma 1 and Lemma 2.

**Lemma 1.** For an individual not purchasing the insurance, the optimal saving is decreasing in the subjective probability, i.e., $\frac{\partial s^-_j}{\partial \phi_j} < 0$.

**Proof.** As $s^-_j$ is the solution of the first order condition (4), the total differentiating the equation with respect to $s^-_j$ and $\phi_j$ gives the sign of $\frac{\partial s^-_j}{\partial \phi_j}$. For simplicity, rewrite $u(l-s, \alpha(1+r)s)$, $u(l-s, (1+r)s)$, $s^-_j$, and $\phi_j$ to $u(\alpha)$, $u(\alpha)$, $s$, and $\phi$, respectively. The total differentiation of equation (4) is

$$-\Phi u_{11}(\alpha)ds + u_1(\alpha)d\phi = (1 - \phi)[u_{11}(\alpha) - 2(1+r)u_{12}(\alpha) + (1+r)^2u_{22}(\alpha)]ds - [-u_1(\alpha) + (1+r)u_2(\alpha)]d\phi.$$  

(10)

Using the first order condition (4), equation (10) is briefly expressed as the equation
(11).

\[
\{\phi u_{11}(\alpha) + (1 - \phi)[u_{11}(\cdot) - 2(1 + r)u_{12}(\cdot) + (1 + r)^2u_{22}(\cdot)]\}ds = \frac{1}{1 - \phi} u_1(\cdot, 0)d\phi
\]  

(11)

Since the both sides of (11) have different signs from Assumption 1, \(\partial s_j^- / \partial \phi_j < 0\). 

**Lemma 2.** For an individual not purchasing the insurance, the indirect utility function \(v^- (s_j^-)\) is decreasing in the subjective probability \(\phi_j\).

**Proof.** As the optimal saving \(s_j^-\) is the function of the subjective probability \(\phi_j\), the indirect utility function \(v^- (s_j^-)\) can be treated as the same. By differentiating \(v^- (s_j^-)\) with respect to \(\phi_j\) and omitting the indices,

\[
dv / d\phi = \frac{u(\cdot, \alpha)}{u(\cdot, \cdot)} + \{-u_1(\cdot, \alpha) + (1 - \phi)(-u_1(\cdot) + (1 + r)u_2(\cdot))\} \, (ds / d\phi)
\]

is obtained. As the third term in the right-hand side of the above equation is deleted by the first order condition (4), the derivative is reduced to the subsequent form.

\[
dv / d\phi = \frac{u(\cdot, \alpha)}{u(\cdot, \cdot)} = u(l - s, \alpha) - u(l - s, (1 + r)s).
\]

Due to the monotonicity of the utility function, \(u(l - s, \alpha(1 + r)s) < u(l - s, (1 + r)s)\). Therefore, \(dv / d\phi < 0\), i.e., the indirect utility function \(v^- (s_j^-)\) is decreasing in the subjective probability \(\phi_j\). 

Lemma 2 indicates individuals with a higher subjective probability are likely to sign the insurance contract. Meanwhile, if all of the individuals are so optimistic that estimate the frequency of the accident to be low, then no-one purchases the insurance policy. Since the aim of this study is to investigate the influences given by parent generation to the child generation in the insurance market, a certain number of individuals should purchase the insurance in the model. Therefore, it is assumed that the subjective probability is densely distributed in the interval from zero to one. This requires the subjective probability should be treated as a continuous variable, \(\phi\). Namely, for any value of the subjective probability, a certain number of individuals estimate the risk at that level. Denoting the number of individuals with a subjective probability \(\phi\) as a function \(\Phi(\phi)\), these settings and the integrability of the function are assumed as Assumption 1 below.

**Assumption 2.** \(\Phi(\phi)\) is integrable on the interval of \([0,1]\) and \(\Phi(\phi) > 0\) for any value of the subjective probability \(\phi \in [0,1]\).

To focus on the individuals’ behavior, the insurer is assumed to meet the demand for the insurance. Thus, the insurer has the perfect information about the distribution of the
subjective probability and the true value of the probability of the risk.

**Assumption 3.** The insurer recognizes the preference of individuals and the distribution function of individuals with respect to subjective probabilities, $\Phi(\phi)$, and the true value of probability of accident, $\hat{\phi}$.

Based on Assumption 3, the insurer correctly projects the number of individuals who purchase the insurance and the fraction of them claiming the insurance benefits due to the accident. Whereas the model assumes the insurer meet the demand for the insurance, the insurer’s budget is required to be balanced. Namely, it provides the actuarially fair insurance. The remaining question is whether the price of insurance premium exists under the condition for balancing the budget. The answer to this question is yes under Assumptions 2 and 3.

**Proposition 1.** There exists a price of the insurance under which non-zero number of individuals purchase the policy and the insurer’s budget balances.

**Proof.** Suppose that an insurance premium $i < I$ is given. According to Lemma 2 and Assumption 3, there exists a value of subjective probability, $\bar{\phi} > 0$ such that $v^-(s^-) < v^+(s^+)$ for all $\phi > \bar{\phi}$, i.e., all of the individuals with a subjective probability larger than $\bar{\phi}$ purchase the insurance. Therefore, the remaining of the proof is to show that the insurer can set the insurance premium $i$ to balance the budget of both the insurer and the individuals. Observing the true value of probability of the event, $\hat{\phi}$, the insurer projects the balance. The revenue from the insurance premium is $i \int_{\bar{\phi}}^{1} \Phi(\phi)d\phi$ and the total amount of the benefit payment is $\hat{\phi}(1 - \alpha)(1 + r)s^+ \int_{\bar{\phi}}^{1} \Phi(\phi)d\phi$. Hence, $i = \hat{\phi}(1 - \alpha)(1 + r)s^+$. Substituting $s^+ = i/\hat{\phi}(1 - \alpha)(1 + r)$ for $s$ in the first order condition (5), the equation is rewritten as (12).

$$u_1 \left( I - \left( 1 + \frac{1}{\hat{\phi}(1 - \alpha)(1 + r)} \right) i, \frac{i}{\hat{\phi}(1 - \alpha)} \right) = (1 + r)u_2 \left( I - \left( 1 + \frac{1}{\hat{\phi}(1 - \alpha)(1 + r)} \right) l, \frac{i}{\hat{\phi}(1 - \alpha)} \right)$$

(12)

To examine whether there exists a value of $i$ satisfying equation (12) and the budget constraint of the insurer simultaneously, each side of (12) is treated as a function of $i$ as below.

$$L(i) \equiv u_1 \left( I - \left( 1 + \frac{1}{\hat{\phi}(1 - \alpha)(1 + r)} \right) l, \frac{i}{\hat{\phi}(1 - \alpha)} \right), \quad R(i) \equiv (1 + r)u_2 \left( I - \left( 1 + \frac{1}{\hat{\phi}(1 - \alpha)(1 + r)} \right) l, \frac{i}{\hat{\phi}(1 - \alpha)} \right)$$
\[
\frac{1}{\phi(1-\alpha)(1+r)} i, \frac{i}{\phi(1-\alpha)}.
\]

Then,
\[
\lim_{i \to +0} L(i) = u_1(I, 0) < \lim_{i \to +0} R(i) = \lim_{i \to +0} (1 + \frac{1}{\phi(1-\alpha)(1+r)}) u_2 \left( I - \left( 1 + \frac{1}{\phi(1-\alpha)(1+r)} \right) \frac{i}{\phi(1-\alpha)} \right) = \infty,
\]
\[
L'(i) = -(1 + \frac{1}{\phi(1-\alpha)(1+r)}) u_{11} + \frac{1}{\phi(1-\alpha)} u_{12} > 0, \quad (13)
\]
\[
R'(i) = -(1 + r) \left( 1 + \frac{1}{\phi(1-\alpha)(1+r)} \right) u_{21} + \frac{1}{\phi(1-\alpha)} u_{22} < 0. \quad (15)
\]

Letting \( \bar{i} \) denote the solution of (12), \( \bar{i} \) is a unique and positive from (13), (14) and (15).

Finally, as \( \lim_{i \to \bar{i} - 0} L(i) = \lim_{i \to \bar{i} - 0} u_1 \left( I - \left( 1 + \frac{1}{\phi(1-\alpha)(1+r)} \right) \frac{i}{\phi(1-\alpha)} \right) = \infty \), where \( \bar{i} \equiv \frac{I}{1 + \frac{1}{\phi(1-\alpha)(1+r)}} \), \( \bar{i} < \bar{i} \) holds, i.e., the insurance premium is low enough to remain the consumption of the first period positive. \( \Box \)

Proposition 1 states the insurance premium is always set to be lower than the level at which it depletes the consumption of the first period despite any high true values of probability of the event. Even though it is extremely high, i.e., 1 and then the insurance benefits are smaller than the savings, the individuals with a higher subjective probability than \( \bar{\phi} \) purchase the policy. This is because they predict that they are likely to lose their savings if they do not spend money to the insurance premium. The insurer, on the other hand, garners the premium \((1 + r)\) times as much as individual’s savings to balance the budget. Consequently, demand for the insurance does exist under Assumption 2. The fraction of individuals who purchase the insurance among population is determined and it is denoted by \( \phi \) for the subsequent section. Namely,
\[
\varphi \equiv \int_{\bar{\phi}}^{1} \Phi(\phi) d\phi. \quad (16)
\]

3. Dynamics

As of time \( t = 0 \), every individual has a child and a fraction of population denoted by \( \varphi \) bought the insurance to avoid the risk of losing their assets. In this period, \( t = 1 \), the children grow up to the younger generation with the same preference as the previous generation. Some of them live together with their parent and the others leave from home.
to their own house. The fractions of these two types of younger generation are denoted as below.

\[ \omega \in [0,1] : \text{share of the individuals living with parent} \]
\[ 1 - \omega : \text{otherwise} \]

While the preference of the younger generation is completely inherited from the older one like as the most articles with overlapping-generations economy, our model considers the subjective probability of the risk may be different between generations. This is based on a simple prediction that children of the cautious parents also become cautious adults. Furthermore, the similarity between generations is likely influenced by the dwellings of the younger individuals. The individuals continuously living together with their parents after they become adults may more likely have the same cautiousness as them than the individuals independently live in their own home. Two situations are examined here based on the assumptions concerning the heredity of the subjective probability.

Case-1

**Assumption 4.** The distribution of the subjective probabilities among younger individuals is the same as that of the previous generation except for the individuals living with her/his parent who purchased the insurance in the previous period. They inherit the subjective probability from their parent.

From Assumption 4, the fraction of individuals purchasing the insurance in this period, \( \Psi_1 \), is expressed as the function of \( \varphi \) and \( \omega \).

\[ \Psi_1(\varphi, \omega) = \varphi \omega + \varphi(1 - \omega)\varphi + (1 - \varphi)\varphi \quad (17) \]

The first term of the right-hand side is the fraction of the second generation perfectly inheriting the subjective probability from the parent who owns the insurance policy. The second one is the fraction of the individuals who leave from their parent and decide to purchase the insurance by themselves. The last one is also the fraction of the individuals purchasing the insurance in this period, whose parent is not a policyholder\(^6\). \( \Psi_1 \) is rewritten as (18).

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\(^6\) The individuals whose parent is a policyholder consist of two types; (i) living together with the parent and (ii) leaving from parent’s home. The individuals of type (ii) consist of further two types; (ii-a) purchasing the insurance and (ii-b) not purchasing it. Finally, the individuals whose parent is not a policyholder consist of two types in the same way as (ii-a) and (ii-b). Therefore, three types of individuals purchase the insurance.
\[
\Psi_1(\varphi, \omega) = \varphi (1 - \varphi) \omega + \varphi \quad (18)
\]

The calculation of the fraction of the individuals who own the insurance policy is the same in the subsequent periods. Thus, the general form for period \( t \) can be derived and it converges to a certain value as \( t \to \infty \). Lemma 3 formally describes this fact.

**Lemma 3.** The fraction of the individuals who purchase the insurance in period \( t \geq 0 \) is expressed as the function of \( \varphi \) and \( \omega \).

\[
\Psi_t(\varphi, \omega) = \varphi \left( 1 - \varphi \right)^{t+1} \omega^{t+1} \frac{1}{1 - (1 - \varphi) \omega} \quad (19)
\]

\[
\Psi(\varphi, \omega) = \lim_{t \to \infty} \Psi_t(\varphi, \omega) = \frac{\varphi}{(1 - \varphi)(1 - \omega) + \varphi} \quad (20)
\]

**Proof.** Following from (18), \( \Psi_{s+1} \) is expressed as the following recursion,

\[
\Psi_{s+1} = \Psi_s \omega + \Psi_s (1 - \omega) \varphi + (1 - \Psi_s) \varphi = \Psi_s (1 - \varphi) \omega + \varphi.
\]

Therefore, equation (19) is derived from the above recursion. As \( \varphi, \omega \in [0,1] \), (20) is straightforward. ■

**Case-2**

**Assumption 5.** The younger individuals purchase the insurance as of time \( t = 1 \) if they live together with their parents who faced the accident and received the insurance benefits or lost the savings due to the uninsured. Whether the other individuals leaving their parent’s home purchase the insurance or not depends on each subjective probability.

In this case the individual purchases the insurance if she/he lives together with her/his parent who faced the accident. The fraction of them is the products of the probability of the risk and the fraction of individuals living together with their parent, i.e., \( \hat{\psi} \omega \). Another individual who purchases the insurance leaves from her/his parent’s home and has higher subjective probability. The fraction of them is \( (1 - \omega)\varphi \). Whereas the sum of these two fractions, \( \hat{\psi} \omega + (1 - \omega)\varphi \) is the rate of the individuals purchasing the insurance as of time \( t = 1 \), it does not depend on time. Letting \( \Psi(\varphi, \omega) \) denote it, Lemma 4 corresponding to Lemma 3 can be expressed as below.

**Lemma 4.** The fraction of the individuals who purchase the insurance is constant in time and expressed as the function of \( \varphi \) and \( \omega \).

\[
\Psi(\varphi, \omega) = (\hat{\psi} - \varphi) \omega + \varphi \quad (21)
\]
While in case-1 the fraction of the individuals purchasing the insurance is obviously increasing in the rate of the individuals living together with their parent, it is dependent on the difference between the true value of the probability and the fraction of the insured based on the subjective probabilities in case-2. That is,
\[
\frac{\partial \Psi(\varphi, \omega)}{\partial \omega} = \frac{(1 - \varphi)/((1 - \varphi)(1 - \omega) + \varphi)^2}{0} > 0, \quad (22)
\]
\[
\frac{\partial \Psi(\varphi, \omega)}{\partial \omega} = \frac{\bar{\phi} - \varphi}{\geq 0 \iff \bar{\phi} \geq \varphi}. \quad (23)
\]
Under Assumption 4, a younger individual living together with her/his insured parent perfectly inherits the subjective probability. Simultaneously, \( \varphi \) of individuals leaving from their parent’s home purchase the insurance. Although the increase in \( \omega \) proliferates the former individuals and diminishes the latter, the first effects are superior to the second effects. The total effects are expressed as inequality (22).

On the other hand, Assumption 5 explicitly includes the true value of the probability of the accident. If the distribution of individuals in terms of the subjective probabilities is thick in the lower area, i.e., most of them have comparatively lower subjective probabilities, offspring of the optimistic parent is so surprised by the realized probability that they chose to be a policyholder. In such a situation, the increase in the fraction of the individuals living in their parent’s home enhances the fraction of the insured. Inequalities (23) insist such a behavior can be observed.

4. Calibration

Specifying the functions of the previous sections, the changes of the variables, including the fraction of the insured, can be captured quantitatively. First the utility function is defined as the form analog to the one assumed in the Ramsey model\(^7\):
\[
u(c_1, c_2) = \frac{c_1^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \cdot \frac{c_2^{1-\theta}}{1-\theta}, \quad (24)
\]
where \( \theta > 0, \rho > 0 \). \( \theta \) is the inverse of the intertemporal elasticity of substitution and \( \rho \) is the discount rate. The specified forms corresponding to equations (6) and (7) are as follows.

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\(^7\) If the function should be continuously defined at \( \theta = 1 \), then \( u(c_1, c_2) = \frac{c_1^{1-\theta}-1}{1-\theta} + \frac{1}{1+\rho} \cdot \frac{c_2^{1-\theta}-1}{1-\theta} \) is required. However, the results of calibration depend on the measure of the variables in that case.
\[ s^- = \frac{l}{1+ A (1-(1-\alpha)\theta)\phi^{-1}}, \quad (25) \]

where \( A \equiv (1+\rho)^{\frac{1}{\theta}}(1+r)^{1-\frac{1}{\theta}} \) and

\[ s^+ = \frac{l-i}{1+A}. \quad (26) \]

Whereas (26) implies the relation between individual’s saving and the insurance premium, the insurance payment of the insurer also relates to the amount of saving, i.e.,

\[ i = \hat{\phi}(1-\alpha)(1+r)s^+. \quad (27) \]

Therefore, the insurance premium in equilibrium is determined by equations (26) and (27).

\[ \hat{i} = \frac{\hat{\phi}(1-\alpha)(1+r)}{1+A+\hat{\phi}(1-\alpha)(1+r)} l \quad (28) \]

Whereas insured’s utility level is determined under given values of parameters, \( \theta, \rho, r, \alpha, \) and \( \hat{\phi} \), non-insured’s indirect utility is a decreasing function in the subjective probability, \( \emptyset \). Therefore, the minimum level of the subjective probability of insured individuals, \( \hat{\phi} \) is determined. If the value of the elasticity of substitution is given 2 similar to the many existing studies, then \( \theta = 0.5 \). Setting one period twenty-five years and the subjective discount rate is 0.01, \( \rho = 0.28 \). Since the insurance premium depends on the value of the income in this model, the level itself should not be noticed but the ratio to the income. Setting the value of income unity, \( \hat{i} \) can be treated as the ratio. The percentages of the insured to the total individuals also depends on the distribution of them. It is assumed as the normal distribution with the mean value is 0.5 and the standard deviation is 0.17 based on the six-sigma\(^8\). The other parameters are considered in several combinations as seen in Table 1.

<table>
<thead>
<tr>
<th>Table 1 parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
</tr>
<tr>
<td>( \hat{\phi} )</td>
</tr>
<tr>
<td>( r )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2 calibration results</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0.9, \ r = 0.45 )</td>
</tr>
<tr>
<td>( \hat{\phi} )</td>
</tr>
</tbody>
</table>

\(^8\) Namely, \( \Phi(\phi) \sim N(0.5, 0.17^2) \).
\[
\begin{array}{cccccc}
i/l & 0.007 & 0.013 & 0.020 & 0.027 & 0.033 & 0.039 & 0.045 \\
\bar{\theta} & 0.141 & 0.281 & 0.420 & 0.558 & 0.695 & 0.832 & 0.968 \\
\varphi & 0.981 & 0.900 & 0.679 & 0.365 & 0.124 & 0.024 & 0.001 \\
\end{array}
\]

\[\alpha = 0.9, \ r = 2.39\]

\[
\begin{array}{cccccc}
i/l & 0.011 & 0.022 & 0.044 & 0.064 \\
\bar{\theta} & 0.164 & 0.326 & 0.642 & 0.951 \\
\varphi & 0.974 & 0.845 & 0.200 & 0.002 \\
\end{array}
\]

\[\alpha = 0.5, \ r = 0.45\]

\[
\begin{array}{cccccc}
i/l & 0.033 & 0.064 & 0.093 & 0.120 & 0.145 & 0.170 & 0.192 & 0.214 \\
\bar{\theta} & 0.122 & 0.240 & 0.355 & 0.468 & 0.578 & 0.685 & 0.791 & 0.896 \\
\varphi & 0.985 & 0.935 & 0.802 & 0.573 & 0.322 & 0.137 & 0.042 & 0.008 \\
\end{array}
\]

\[\alpha = 0.5, \ r = 2.39\]

\[
\begin{array}{cccccc}
i/l & 0.103 & 0.186 & 0.255 & 0.314 \\
\bar{\theta} & 0.270 & 0.509 & 0.723 & 0.918 \\
\varphi & 0.910 & 0.477 & 0.093 & 0.005 \\
\end{array}
\]

\[\alpha = 0.01, \ r = 0.45\]

\[
\begin{array}{cccccc}
i/l & 0.063 & 0.119 & 0.168 & 0.212 & 0.252 & 0.288 & 0.321 & 0.350 & 0.378 \\
\bar{\theta} & 0.077 & 0.151 & 0.221 & 0.289 & 0.354 & 0.420 & 0.485 & 0.551 & 0.619 \\
\varphi & 0.992 & 0.978 & 0.948 & 0.891 & 0.803 & 0.679 & 0.534 & 0.380 & 0.240 \\
\end{array}
\]

\[\alpha = 0.01, \ r = 2.39\]

\[
\begin{array}{cccccc}
i/l & 0.185 & 0.312 & 0.404 & 0.475 & 0.531 & 0.576 & 0.613 & 0.644 \\
\bar{\theta} & 0.164 & 0.296 & 0.408 & 0.507 & 0.602 & 0.687 & 0.776 & 0.877 \\
\varphi & 0.974 & 0.883 & 0.704 & 0.482 & 0.273 & 0.134 & 0.051 & 0.012 \\
\end{array}
\]

\(\alpha = 0.9\) implies that the accident damages individual's saving assets by ten percent. If this accident happens with probability 0.1, then the individuals estimating it at higher than 0.141 purchase the insurance. Assuming the standard distribution of the subjective probability with mean value, 0.5 and standard deviation, 0.17, the ratio of the insured to the total population is 98.1% since the insurance premium is no more than 0.7% of the income. Despite low premium of the insurance, the ratio of the insured drops down to zero as the true value of probability of the accident exceeds 0.7. Comparing the two levels of interest rate, it is shown that the larger insurance premium in the case of the higher interest rate reduces the ratio of the insured more drastically. When the interest rate is 2.39 and the true value of the probability is 0.4, no less than 0.2% of individuals purchase the insurance.

Increasing the loss of the saving assets caused by the accident, i.e., the rise of the
insurance premium ensues from the fall of \( \alpha \). This relation seems straightforward from (28) since the fall of \( \alpha \) and the rise of the true value of probability are equivalent with regard to arithmetic. However, the influence of the change in the loss affects the border of the subjective probability through the saving function of the non-insured, (25). While in the cases of the higher interest rate no one purchases the insurance under the true probability larger than 0.4, 1.2% of individuals become insured even when the probability reaches 0.8.

### Table 3 transition of the fraction of the insured

<table>
<thead>
<tr>
<th>( \hat{\phi} )</th>
<th>( \omega )</th>
<th>( \frac{\psi_t}{\bar{\psi}} )</th>
<th>( t = 0 )</th>
<th>( t = 1 )</th>
<th>( t = 2 )</th>
<th>( t = 3 )</th>
<th>( t = \infty )</th>
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</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.547</td>
<td>( \psi_t )</td>
<td>0.9932</td>
<td>0.9933</td>
<td>0.9933</td>
<td>0.9933</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \bar{\psi} )</td>
<td>0.5011</td>
<td>\quad</td>
<td>\quad</td>
<td>\quad</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.329</td>
<td>( \psi_t )</td>
<td>0.9900</td>
<td>0.9901</td>
<td>0.9901</td>
<td>0.9901</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \bar{\psi} )</td>
<td>0.6944</td>
<td>\quad</td>
<td>\quad</td>
<td>\quad</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.124</td>
<td>( \psi_t )</td>
<td>0.9871</td>
<td>0.9871</td>
<td>0.9871</td>
<td>0.9871</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \bar{\psi} )</td>
<td>0.8759</td>
<td>\quad</td>
<td>\quad</td>
<td>\quad</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.547</td>
<td>( \psi_t )</td>
<td>0.4409</td>
<td>0.4851</td>
<td>0.5016</td>
<td>0.5113</td>
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<tr>
<td></td>
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<td>( \bar{\psi} )</td>
<td>0.4191</td>
<td>\quad</td>
<td>\quad</td>
<td>\quad</td>
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</tr>
<tr>
<td></td>
<td>0.329</td>
<td>( \psi_t )</td>
<td>0.3932</td>
<td>0.4092</td>
<td>0.4127</td>
<td>0.4138</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>( \bar{\psi} )</td>
<td>0.3802</td>
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<td>\quad</td>
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<td>0.124</td>
<td>( \psi_t )</td>
<td>0.3485</td>
<td>0.3507</td>
<td>0.3509</td>
<td>0.3510</td>
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<tr>
<td></td>
<td></td>
<td>( \bar{\psi} )</td>
<td>0.3436</td>
<td>\quad</td>
<td>\quad</td>
<td>\quad</td>
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</tr>
<tr>
<td>0.8</td>
<td>0.547</td>
<td>( \psi_t )</td>
<td>0.0128</td>
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<td></td>
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<td>( \bar{\psi} )</td>
<td>0.4960</td>
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<td></td>
<td>0.329</td>
<td>( \psi_t )</td>
<td>0.0110</td>
<td>0.0119</td>
<td>0.0121</td>
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<td></td>
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<td>( \bar{\psi} )</td>
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<td>\quad</td>
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<td>0.124</td>
<td>( \psi_t )</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>( \bar{\psi} )</td>
<td>0.1184</td>
<td>\quad</td>
<td>\quad</td>
<td>\quad</td>
<td></td>
</tr>
</tbody>
</table>

\( r = 0.45, \alpha = 0.5 \)

Table 3 illustrates the transition of the fraction of the individuals purchasing the insurance with respect to each combination of the parameter values. Three true values of the probability of the accident are chosen; 0.1, 0.5, and 0.8. The ratio of the individuals living together with their parent is also applied three different values from the empirical data. The data is extracted from the results of the national census in 1990. As the figures of the ratio of the individuals living together with their parents are available
for forty-seven prefectures, the maximum, the average, and the mean values are chosen\textsuperscript{9}. The interest rate and the ratio of the loss to the total saving asset are 0.45 and 0.5, respectively.

When the true value of probability of the accident is 0.1, the fraction of the insured is 0.985 in period $t = 0$. The fraction in Case-1, $\Psi_t$, grows in the following periods although it approximately reaches the ultimate value in the early period. The lower insurance premium by virtue of the small frequency of the accident entices most of the individuals to the insurance. On the other hand, every fraction in Case-2 declines in the subsequent period since the true value of probability is smaller than the fraction in the initial period as described in Lemma 4. As the accident rarely happens due to the low probability, the younger individuals living together with their parent who did not receive the insurance benefits likely hesitate to buy the insurance.

The fractions in both cases are very close each other when the true value of probability of the accident is 0.5. For the fraction in Case-2 the influence of the changes in the ratio of the individuals living together with their parent is the smallest between three values of the true probability. While the fractions are 0.4191 and 0.3436 for the highest and the lowest values of $\omega$, these figures are 0.5011 and 0.8759 in case that $\phi = 0.1$ and 0.4960 and 0.1184 in case that $\phi = 0.8$. Meanwhile, the difference between convergent values of the fraction in Case-1 is the largest from 0.5113 to 0.3510, whereas they are 0.9933 and 0.9871 in case that $\phi = 0.1$ and 0.0181 and 0.0094 in case that $\phi = 0.8$.

The results of the calibration illustrate the difference in the ratio of the individuals living together with their parent likely influence the tendency to purchase the insurance.

5. Empirical Studies
(Preparing)

6. Conclusions
This study examines the influence in terms of insurance purchase decision-making given from an elder to a younger in a household. In the articles exploring the diffusion of individual insurance, a few studies factor in the behavior of the parents in terms of taking out insurance. A person receiving insurance benefits likely enlightens her/his children on the virtue of insurance and the influence given from the parent is larger if they live together at home after becoming adults. Meanwhile, a miserable elder losing a certain amount of savings due to an accident also promotes the propensity of her/his

\textsuperscript{9} The maximum and the minimum values are of Yamagata and Tokyo, respectively.
successors to purchasing insurance goods as a negative example in a household. Then, this study focuses on the couple of variables, i.e., the ratio of the adults living together with their parents to the total and the fraction of insureds. The analysis adopts a two-periods overlapping-generations framework in which an individual decides to purchase insurance depending on her/his subjective probability of an accident under given conditions.

If the adults living with her/his parent who purchased the insurance are assumed to do the same in the subsequent period, then the higher the fraction of such adults is, the more people take out insurance. On the other hand, if the adults living together with her/his parent who received the benefits or who lost some amount of savings due to an accident are assumed to buy insurance definitely, the relation between primary variables depends on the values of parameters.

The theoretical model includes so many variables that calibration approach is effective to clarify the relations between them. In the case that the accident bereaves an individual of the half of the savings with probability 0.5, the ratio of individuals living together with their parents gives the largest influence to the fraction of the insureds in equilibrium. The other combinations of parameter values will be examined hereafter.

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