To hedge or not to hedge? Evidence via almost stochastic dominance

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Abstract

Under the framework of almost stochastic dominance, we show that portfolio insurance strategies is not preferred for most investors. Several types of portfolio insurance strategies are examined, including stop-loss portfolio insurance, synthetic put portfolio insurance and the constant proportion portfolio insurance. We find that in general portfolio insurance strategies is dominated by a buy-and-hold strategy in terms of almost first- and second-degree stochastic dominance, especially for longer investment horizons. Furthermore, for robustness, we show that the purely buy-and-hold strategy is preferred to the strategy mixed with buy-and-hold and portfolio insurance strategies for most investors.

JEL classification: D80; D81

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1 Introduction

Although the benefits of hedging have been well-promoted by the securities industry, many individual investors still do not adopt portfolio insurance strategies (PI) in the equity market.[1] For example, Ameriks and Zeldes (2004) showed that 44% of investors in their sample did not change their portfolio allocations over a 10-year period. In this paper, we provide a rationale to justify that PI strategy dominated by no hedge, i.e., a buy-and-hold (BH) strategy.

Recently, Annaert et al. (2009) documented that the BH strategy could not dominate the portfolio insurance (PI) strategies in terms of stochastic dominance, and vice versa. Note that stochastic dominance is the necessary and sufficient criterion for all expected-utility maximizers to prefer one portfolio to another.[2] Thus, the results of Annaert et al. (2009) lead us to the conclusion that some investors like the BH strategy whereas some do not. However, failing to recognize that the PI strategies stochastically dominate the BH strategy means that silence is maintained on what types of individuals prefer the BH strategy to the PI strategies. Thus, in this paper, we employ almost stochastic dominance (ASD) to re-examine whether the BH strategy is considered to be a better investment strategy for "most" investors than the hedging portfolio.

ASD first introduced by Leshno and Levy (2002) is the criterion of ranking distributions for "most" individuals rather than all individuals. Leshno and Levy (2002) show that in some cases stochastic dominance fails to provide a determinant suggestion although most decision makers could have a precise call. For example, let lottery X obtain \$100,000 with probability 99.99% and 0 otherwise and Y denote a lottery that gives \$1 for sure. It is obvious that most investors prefer X to Y. However, stochastic dominance rules fail to rank them. Thus, Leshno and Levy (2002) propose employing ASD rather than stochastic dominance to rank distributions.

Since Leshno and Levy (2002), ASD has been applied to several issues in finance. For example, Bali et al. (2009) give empirical support for the popular investment practices which suggest a higher stock to bond ratio for long investment horizons. Bali et al. (2013)

[1]For the infrequent asset re-allocation behavior, please see Lusardi (1999, 2003), Mitchell et al. (2006), Brunnermeier and Nagel (2008), Bilias, Georgarakos, and Haliassos (2009), and Duffie (2010).

[2]In behavioral finance contexts, Dierkes, Erner, and Zeisberger (2010) and Dichtl and Drobetz (2011) justified the popularity of portfolio insurance strategies.

show that hedge funds almost stochastically dominate the U.S. equity market and the U.S. Treasury market. Denuit et al. (2014) extend the concept of ASD to generate almost marginal conditional stochastic dominance. They further show how to construct an efficient frontier under the almost marginal conditional stochastic dominance criterion.

Under the framework of ASD, we demonstrate that most investors in the equity market opt for a buy-and-hold strategy rather than implement PI strategies. We consider different types of PI strategies, including stop-loss portfolio insurance, synthetic put portfolio insurance and constant proportion portfolio insurance. We find that the BH strategy dominates PI strategies in terms of almost first-degree stochastic dominance (AFSD) and almost second-degree stochastic dominance (ASSD) for longer investment horizons. Furthermore, for robustness, we show that the purely BH strategy is preferred to the strategy mixed with BH and PI strategies for most investors.

Our paper contributes to the literature in the following prospects. First, our paper provides an explanation for the common practice that many investors adopt BH strategy in the long run. Second, Bali et al. (2009 and 2013) find that in the long run, some financial products, such as stocks or hedge funds, outperform other products, such as bonds in terms of ASD. Our paper find that in the long run, some investment strategies, such as BH strategy, outperform other strategies, such as PI strategies. Our paper complement the literature by finding a new important application of ASD.

The remainder of the paper is organized as follows. Section 2 introduces the alternative PI strategies. Section 3 reviews ASD. Section 4 describes the data and the simulation setup. Section 5 provides the simulation results, and Section 6 concludes the paper.

2 Alternative portfolio insurance strategies

Three commonly adopted portfolio insurance (PI) trading strategies are examined: stop-loss portfolio insurance (SL), synthetic put portfolio insurance (SP) and the constant proportion portfolio insurance (CPPI). This section provides a description of these strategies.

2.1 Stop-loss portfolio insurance

The SL is a strategy whereby decision makers initially invest their total wealth in the risky asset. The portfolio is maintained as long as the market value of the portfolio at time t (W_t) is above the net present value at time t (NPV_t) of the floor F, where F is set as the minimum target level that the portfolio has to reach at the end of the investment horizon. If the market value of the portfolio drops below the discounted value of the floor, i.e.,

$$W_t < NPV_t(F), \tag{1}$$

then all of the risky asset holdings are sold and the portfolio is re-allocated entirely to the risk-free asset until the end of the investment horizon. If the market value of the portfolio never drops below the discounted floor, then the final wealth will never be lower than the floor. In this strategy, the transaction cost is only incurred once.

2.2 Synthetic put portfolio insurance

The SP strategy is introduced by Rubinstein and Leland (1981). Note that the combination of a risky asset and a put option on this asset is equivalent to purchasing a continuously-adjusted portfolio, which contains the risky asset and the risk-free asset. Using the Black and Scholes (1973) option pricing formula to evaluate the put option, investors could decide the investment weights on the risky asset and the risk-free asset.

Specifically, let S denote the value of a stock, and P be a put option on the stock. Using the Black and Scholes (1973) option pricing formula, the value of the portfolio $S + P$ is equivalent to

$$\begin{aligned} S + P &= S - S \times N(-d_1) + K \times e^{-r_f T} N(-d_2) \\ &= S \times N(d_1) + K \times e^{-r_f T} N(-d_2), \end{aligned} \tag{2}$$

where K is the strike price, r_f is the risk-free rate, T is the time to maturity, $N(\cdot)$ is the standard normal cumulative distribution function and d_1 and d_2 are defined as follows:

$$d_1 = \frac{\ln(S/K) + \left(r_f + \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}} \tag{3}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}, \tag{4}$$

where σ is the standard deviation of risky asset returns.

The delta of the portfolio $S + P$ is

$$\frac{\partial (S + P)}{\partial S} = N(d_1), \tag{5}$$

which indicates how much of the risky asset must be purchased to replicate the portfolio $S + P$. Therefore, the investment weight on the stock is

$$\frac{S \times N(d_1)}{S \times N(d_1) + K \times e^{-r_f T} N(-d_2)} \tag{6}$$

and that on the risk-free asset is

$$\frac{K \times e^{-r_f T} N(-d_2)}{S \times N(d_1) + K \times e^{-r_f T} N(-d_2)}. \tag{7}$$

To maintain a desired protection level, K should be set as the floor F.

The portfolio must be readjusted on a continuous basis to maintain the desired floor. However, while implementing the strategy, transaction costs will be incurred. In order to incorporate this transaction cost effect, we follow Leland (1985) by setting

$$\sigma^2_{adjusted} = \sigma^2 \left(1 + \frac{\sqrt{2/\pi}c}{\sigma\sqrt{\tau}}\right), \tag{8}$$

where c is the relative transaction cost and τ is the rebalancing interval.

2.3 Constant proportion portfolio insurance

Black and Jones (1987) proposed a constant proportion portfolio insurance (CPPI). This strategy also suggests holding both a risky and a risk-free assets in the portfolio but is not based on option pricing theory. The exposure to the risky asset at time t is equal to

$$m \times [W_t - NPV_t(F)] \tag{9}$$

where m is a constant multiplier and $W_t - NPV_t(F)$ represents a cushion. The remainder is invested in the risk-free asset. The setting of m could be arbitrary but it is meaningful. The inverse of m represents the maximum sudden loss from the risky asset such that the cushion is not fully depleted and the portfolio value does not fall below $NPV_t(F)$.

The CPPI strategy can lead to short positions in the risky asset or in the risk-free asset. To follow the common practice in commercial applications, we impose the short-sale and credit constraints as in Benninga (1990), Do (2002), Annaert et al. (2009) and Dichtl and Drobetz (2011). The risky proportion at time t is then given by:

$$max\{min\left(m \times [W_t - NPV_t(F)], W_t\right), 0\}. \tag{10}$$

3 Almost stochastic dominance

This section briefly reviews ASD and introduces how to rank the BH and PI strategies using the ASD rules. Let u denote a decision maker's utility function for wealth x with $u'(x) \geq 0$. To exclude some extreme preferences, Leshno and Levy (2002) proposed considering the set of preferences as

$$U_1(\varepsilon_1) = \left\{ u \middle| u' \geq 0 \text{ and } \sup\{u'(x)\} \leq \inf\{u'(x)\}\left(\frac{1}{\varepsilon_1} - 1\right) \right\}, \tag{11}$$

where ε_1 is a constant and $0 < \varepsilon_1 < \frac{1}{2}$. In the set $U_1(\varepsilon_1)$, the ratio of the maximum to the minimum marginal utility is constrained as

$$\frac{\sup\{u'(z)\}}{\inf\{u'(z)\}} \leq \frac{1}{\varepsilon_1} - 1. \tag{12}$$

If ε_1 approaches zero, then the set $U_1(\varepsilon_1)$ is the same as the set of all decision makers with $u'(x) \geq 0$. If ε_1 approaches $\frac{1}{2}$, then only the utility functions $u'(x) = k$, where k is a positive constant, are in the set, i.e., the set only contains risk-neutral decision makers.

Let $F(x)$ and $G(x)$ represent cumulative probability distributions of a random variable $\tilde{x}$ with support $[a, b]$. Let $S_1(F, G) = \{x \in [a, b] | \; F(x) \geq G(x)\}$. Leshno and Levy (2002) defined almost first-degree stochastic dominance (AFSD) and showed that AFSD is a rule for all decision makers in $U_1(\varepsilon_1)$ to rank F and G as follows:

Proposition 1 *(Leshno and Levy, 2002) For* $0 < \varepsilon_1 < \frac{1}{2}$, $E_F(u) \geq E_G(u)$ *for all* $u \in U_1(\varepsilon_1)$ *if and only if* F *dominates* G *by* ε_1*-AFSD, i.e.,*

$$\int_{S_1(F,G)} [F(x) - G(x)]dx \leq \varepsilon_1 \int_a^b |F(x) - G(x)|\, dx. \tag{13}$$

This Proposition shows that all decision makers in $U_1(\varepsilon_1)$ would prefer F to G as long as the ratio of the area between F and G which violates the rule of first-degree stochastic dominance (FSD) ($\int_{S_1(F,G)}[F(x) - G(x)]dx$) to the total area between F and G ($\int_a^b |F(x) - G(x)|\, dx$) does not exceed ε_1. It is obvious that when ε_1 approaches zero, AFSD collapses to FSD.[3]

Tzeng et al. (2013) further extend AFSD to almost second-degree stochastic dominance (ASSD). Define $F^{(2)}(x) = \int_a^x F(t)dt$, and define $G^{(2)}(x)$ similarly. Let $S_2(F,G) = \{x \in [a,b] | F^{(2)}(x) \geq G^{(2)}(x)\}$ and

$$U_2(\varepsilon_2) = \left\{ u \middle| u' \geq 0,\ u'' \leq 0,\ \text{and}\ \sup\left\{-u''(x)\right\} \leq \inf\left\{-u''(x)\right\} \left(\frac{1}{\varepsilon_2} - 1\right) \right\},$$

where $\varepsilon_2 \in \left(0, \frac{1}{2}\right)$. They defined ASSD and showed that it is a rule for all decision makers in $U_2(\varepsilon_2)$ to rank distributions as follows:

Proposition 2 *(Tzeng et al. 2013) For* $0 < \varepsilon_2 < \frac{1}{2}$, $E_F(u) \geq E_G(u)$ *for all* $u \in U_2(\varepsilon_2)$ *if and only if* F *dominates* G *by* ε_2*-ASSD, i.e.,*

$$\int_{S_2(F,G)} [F^{(2)}(x) - G^{(2)}(x)]dx \leq \varepsilon_2 \int_a^b \left| F^{(2)}(x) - G^{(2)}(x) \right| dx$$

and $E_F(\tilde{x}) \geq E_G(\tilde{x})$*.*

Similarly, the above Proposition shows that all decision makers in $U_2(\varepsilon_2)$ would prefer F to G as long as the ratio of the area between $F^{(2)}$ and $G^{(2)}$ which violates the rule of second-degree stochastic dominance (SSD) ($\int_{S_2(F,G)}[F^{(2)}(x) - G^{(2)}(x)]dx$) to the total area between $F^{(2)}$ and $G^{(2)}$ ($\int_a^b \left|F^{(2)}(x) - G^{(2)}(x)\right| dx$) does not exceed ε_2.

To test whether the BH strategy almost stochastically dominates a PI strategy, we first simulate the empirical distributions of BH and PI as $\hat{F}$ and $\hat{G}$, respectively. We then compute

[3] An example is provided in the Appendix to illustrate Proposition 1.

the empirical $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ as follows:

$$\hat{\varepsilon}_1 = \frac{\int_{S_1(F,G)}[\hat{F}(x) - \hat{G}(x)]dx}{\int_a^b \left|\hat{F}(x) - \hat{G}(x)\right| dx} \tag{14}$$

and

$$\hat{\varepsilon}_2 = \frac{\int_{S_2(F,G)}[\hat{F}^{(2)}(x) - \hat{G}^{(2)}(x)]dx}{\int_a^b \left|\hat{F}^{(2)}(x) - \hat{G}^{(2)}(x)\right| dx}. \tag{15}$$

Let ε_1^* and ε_2^* be the allowed violation ratio for most investors. We adopt the value of ε_1^* and ε_2^* proposed by Levy et al. (2010) and set $\varepsilon_1^* = 5.9\%$ and $\varepsilon_2^* = 3.2\%$. According to the above two Propositions, we will conclude that

$$\text{BH dominates PI in terms of AFSD if } \hat{\varepsilon}_1 \leq \varepsilon_1^*; \tag{16}$$

$$\text{BH dominates PI in terms of ASSD if } \hat{\varepsilon}_2 \leq \varepsilon_2^*. \tag{17}$$

4 Data and simulation setup

Our equity data are collected from Datastream for the period 1 from January 1973 to December 2012. The S&P 500 daily return is constructed by Datastream and referred to as the "S&P 500 COMPOSITE - PRICE INDEX". On the other hand, the data for the risk-free rate in the same time period are downloaded from the US Federal Reserve. We employ the secondary market rate for 3-month Treasury bills with a discount basis (whose code is H15). Non-trading days are deleted. This screen leads to a final sample size of 6,564 daily returns.

The continuously compounded daily returns are calculated. The average annual return of the USA S&P 500 index is 12.1255%, and the standard deviation of the return is 17.8091% per annum. The average return of the 3-month Treasure bills is 5.1192% per year and the standard deviation is 3.5348. These statistics are similar to those in the literature.

To construct the return distributions of the BH and PI strategies, bootstrap simulation is adopted. We first randomly draw a date with replacement. We analyze the performance of each PI strategy for a one- to ten-year investment horizon starting from the randomly drawn date. We assume that there are 252 days per year. If the period from the selected starting date to December 2012 is less than the desired investment horizon, then the data is skipped to avoid missing data problem. The procedure is then repeated 10,000 times. The initial wealth is set as one dollar. We rebalance the portfolio on daily bases.

For the SL strategy, the floor value at time T is set as one dollar as the initial wealth. We use the continuously compounded one-year risk-free rate on the starting date as the discount rate. For the SP strategy, the floor value at time T is also set as one dollar. The continuously compounded one-year risk-free rate on the starting date is used as r_f in Equation (6), and the standard deviation of the 252 continuously compounded daily stock returns prior to the randomly drawn date is used as an estimate of σ in Equation (8). The transaction cost is assumed to be 0.1%. The portfolio is rebalanced daily. For the CPPI strategy, the floor is set as 0.95 starting with 70% invested in the S&P 500 index. We follow Annaert et al. (2009) and set $m = 14$. Thus, in our setting, the risky asset can lose at most 7.1429% ($1/14 = 0.071429$) without violating the floor.

Table 1 reports the descriptive statistics for the above three PI strategies and the BH strategy on the S&P 500 index for different investment time horizons of one year to ten years. Table 1 indicates that the BH strategy obtains the highest average excess return in all investment time horizons. The SL strategy generates the lowest average excess return and standard deviation regardless of the investment horizon. The Sharpe ratio of the BH strategy is greater than those of the SL and SP strategies.

[Insert Table 1 here]

5 Simulation Results

In this section, we demonstrate that most investors prefer BH to PI strategy. In addition, investors might be likely to employ a mixed strategy combining the BH strategy as well as the PI strategies in their portfolios. Thus, in this section, we also check whether the BH strategy is preferred to the strategy mixed with the BH strategy and one of the PI strategies.

5.1 Buy-and-hold vs. portfolio insurance

All PI strategies are compared with the BH strategy respectively under the framework of ASD. The distribution of the BH strategy is treated as $\hat{F}$ and that of the PI strategy is treated as $\hat{G}$ in Equations (14) and (15) to compute $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$. According to Propositions 1 and 2, the dominating strategy has a higher mean. Thus, we do not need to examine whether the PI strategy almost stochastically dominates the BH strategy since the BH strategy has a higher average excess return.

Table 2 shows that the BH strategy outperforms the PI strategies in terms of AFSD for longer investment horizons. Specifically, we find that the BH strategy dominates both SL and SP strategies in terms of AFSD for investment horizons greater than five years. In the sixth and the tenth investment horizons, BH strategy further dominates SP strategy in terms of FSD, i.e., $\hat{\varepsilon}_1 = 0$. In addition, the BH strategy generally dominates CPPI in terms of AFSD for longer investment horizons. However, the results are not significant for 8 and 9 years.

[Insert Table 2 here]

Table 3 demonstrates the results using ASSD criterion. Similar to those in Table 2, we find that, in terms of ASSD, the BH strategy dominates the PI strategies for longer investment horizons. Further, in terms of ASSD, the BH strategy respectively dominates SL, SP and CPPI strategies when the investment horizons are greater than four, four and five years. In Table 2, the corresponding investment horizons are five, five and six years.

[Insert Table 3 here]

Note that our findings are consistent with the findings in Annaert et al. (2009). They showed that for a one-year investment horizon the BH strategy does not stochastically dominate the PI strategies, nor vice versa. We further find that even when using almost stochastic dominance rules, these two strategies cannot be ranked for a one-year investment horizon.

Also note that, as documented by Guo et al. (2013) and Tsetlin et al. (2013), AFSD proposed by Leshno and Levy (2002) may not imply ASSD proposed by Tzeng et al. (2013), and vice versa. Interestingly, our simulation results coincidently find that if the BH strategy dominates the PI strategies in terms of AFSD, then we could observe the dominance in terms of ASSD.

5.2 The performance of a portfolio containing both BH and PI

In this section, we would like to test whether the BH strategy is preferred to the strategy mixed with the BH strategy and one of the PI strategies.

Assume that investors allocate α proportion of their wealth to a portfolio insurance strategy, and $1 - \alpha$ to the BH strategy, where $\alpha \in [0, 1]$. We form the return distributions for 21 portfolios with different settings of α, which starts from 0 with an increment of 0.05. To calculate $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ in Equations (14) and (15), we treat the return distribution of the portfolio with $\alpha = 0$ as F and that of the portfolio with $\alpha > 0$ as G. In other words, we check whether the portfolio with $\alpha = 0$ is preferred by most investors to the portfolio with $\alpha > 0$.

Figure 1 shows the relationship between $\hat{\varepsilon}_1$ and α, whereas Figure 2 shows that between $\hat{\varepsilon}_2$ and α. Both Figures indicate that $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ decreases with α in general. That means that, when the proportion of portfolio insurance is too large, most investors will tend to prefer the BH strategy. It is also likely that the BH strategy will almost stochastically dominate the portfolio with a positive α when the investment horizon is longer.

[Insert Figures 1 and 2 here]

6 Conclusion

In this paper, we have demonstrated that most investors in the equity market prefer BH as their trading strategy. We have found that the BH strategy almost dominates the strategies mixed with the BH and the portfolio insurance strategies in terms of AFSD and ASSD when the proportion of portfolio insurance is large and the investment horizon is sufficiently long.

Annaert et al. (2009) tested whether the portfolio insurance strategy was preferred to the BH strategy for "all" investors. Using SD rules, they found it difficult to show the existence of a stochastic dominance relation. Our paper demonstrates that, although the BH strategy could not be claimed as a preferred strategy for all investors on basis of Annaert et al. (2009), we could assert that the BH strategy could be a dominate strategy for "most" investors in the long run. In addition, our paper complements the literature of ASD by providing empirical evidence to explain a common practice in the market.

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Appendix A An example

Let us consider the example in the Introduction to illustrate Proposition 1. Let

$$F(x) = \begin{cases} 0 \text{ if } x < 0 \\ 0.01\% \text{ if } 0 \leq x < 100,000 \\ 100\% \text{ if } x \geq 100,000 \end{cases}$$

and

$$G(x) = \begin{cases} 0 \text{ if } x < 1 \\ 100\% \text{ if } x \geq 1 \end{cases} .$$

Although most decision makers would prefer $F(x)$ to $G(x)$, it is obvious that FSD cannot rank $F(x)$ better than $G(x)$ since the rule requires that $F(x) \leq G(x)$, $\forall x$. This is because FSD is a rule for all decision makers with $u' \geq 0$. A decision maker with utility function

$$u(x) = \begin{cases} x \text{ if } x \leq 1 \\ 1 \text{ if } x > 1 \end{cases}$$

is in the set of preferences with $u' \geq 0$, and would prefer $G(x)$ to $F(x)$ since

$$E_F(u) = 1 \times 99.99\% < 1 = E_G(u).$$

This drawback is fixed by AFSD by excluding the decision makers with extreme preferences such as the above decision maker. According to the AFSD rule, all decision makers in the set of $U_1(\varepsilon_1)$ with $\varepsilon_1 > 1.00011 \times 10^{-9}$ would prefer $F(x)$ to $G(x)$.

Table 1: Descriptive statistics for 4 trading strategies, including Buy-and-Hold (BH), Constant Portfolio Insurance (CPPI), Synthetic Put (SP), and Stop-loss (SL). Holding period from 1 to 10 years. Daily rebalance.

strategy					BH					
horizon	1	2	3	4	5	6	7	8	9	10
average excess return	9.0%	18.0%	27.2%	35.1%	44.9%	55.5%	66.4%	75.0%	84.0%	93.8%
standard deviation	18.0%	25.0%	31.1%	36.4%	39.8%	41.7%	45.9%	50.1%	54.4%	58.9%
Sharpe ratio	0.50	0.72	0.87	0.96	1.13	1.33	1.45	1.50	1.54	1.59
skewness	-1.16	-1.06	-0.70	-0.38	-0.29	-0.30	-0.46	-0.73	-0.80	-0.81
%<0	23.8%	18.0%	19.3%	23.7%	23.6%	12.0%	7.3%	9.9%	9.7%	13.2%
ave neg excess return	-17.3%	-25.4%	-25.3%	-18.8%	-12.3%	-8.3%	-19.0%	-24.4%	-29.3%	-18.7%
VaR 5%	-24.9%	-37.3%	-37.0%	-29.1%	-19.0%	-8.8%	-8.4%	-15.4%	-29.5%	-20.6%
ES 5%	-39.2%	-49.6%	-42.5%	-32.7%	-22.5%	-13.3%	-25.9%	-40.7%	-42.0%	-29.7%
omega measure	3.20	4.56	4.19	3.22	3.24	7.35	12.62	9.09	9.28	6.59
strategy					SL					
horizon	1	2	3	4	5	6	7	8	9	10
average excess return	0.9%	1.4%	2.1%	3.1%	3.3%	3.9%	4.2%	5.3%	6.5%	7.4%
standard deviation	6.8%	9.6%	12.5%	16.4%	17.8%	20.2%	22.3%	25.5%	28.9%	31.7%
Sharpe ratio	0.14	0.15	0.17	0.19	0.18	0.19	0.19	0.21	0.23	0.23
skewness	3.51	4.19	4.21	4.15	4.29	4.12	4.39	4.11	3.92	3.85
%<0	89.8%	90.8%	90.9%	91.0%	91.6%	91.1%	90.8%	90.7%	89.8%	89.6%
ave neg excess return	-0.9%	-1.0%	-1.0%	-0.9%	-0.9%	-1.0%	-1.0%	-1.0%	-1.0%	-0.9%
VaR 5%	-2.4%	-2.5%	-2.4%	-2.4%	-2.4%	-2.6%	-2.4%	-2.5%	-2.4%	-2.3%
ES 5%	-4.4%	-4.5%	-4.7%	-4.4%	-4.6%	-5.4%	-4.8%	-5.0%	-4.7%	-4.2%
omega measure	0.11	0.10	0.10	0.10	0.09	0.10	0.10	0.10	0.11	0.12

Table 1: (Continued)

strategy					SP					
horizon	1	2	3	4	5	6	7	8	9	10
average excess return	6.6%	12.4%	19.1%	25.4%	30.8%	35.3%	41.7%	50.1%	58.6%	65.8%
standard deviation	13.8%	19.8%	25.0%	32.2%	37.8%	44.4%	49.3%	50.3%	52.6%	57.6%
Sharpe ratio	0.48	0.63	0.77	0.79	0.82	0.79	0.85	1.00	1.11	1.14
skewness	-0.04	0.06	0.12	0.03	-0.08	-0.29	-0.28	-0.42	-0.56	-0.76
%<0	35.1%	30.4%	28.7%	29.6%	29.6%	32.4%	30.5%	25.3%	20.7%	21.2%
ave neg excess return	-8.8%	-11.4%	-11.9%	-14.6%	-17.8%	-20.5%	-22.4%	-22.3%	-26.4%	-32.2%
VaR 5%	-14.3%	-17.1%	-19.3%	-21.7%	-24.2%	-32.4%	-33.5%	-33.9%	-34.2%	-40.6%
ES 5%	-19.8%	-22.9%	-22.7%	-24.6%	-27.9%	-40.7%	-39.0%	-38.3%	-39.0%	-47.0%
omega measure	1.85	2.29	2.48	2.38	2.37	2.08	2.28	2.95	3.82	3.72
strategy					CPPI					
horizon	1	2	3	4	5	6	7	8	9	10
average excess return	6.2%	12.8%	20.6%	28.5%	35.1%	43.1%	50.6%	58.9%	70.5%	78.2%
standard deviation	12.5%	19.6%	26.4%	34.0%	40.6%	46.0%	51.6%	55.0%	58.1%	60.9%
Sharpe ratio	0.49	0.65	0.78	0.84	0.86	0.94	0.98	1.07	1.21	1.28
skewness	0.64	0.51	0.32	0.14	0.11	-0.08	-0.08	-0.19	-0.35	-0.46
%<0	44.5%	39.1%	38.4%	40.4%	41.5%	39.1%	36.6%	33.7%	28.8%	27.1%
ave neg excess return	-5.1%	-6.7%	-7.9%	-8.6%	-8.8%	-9.1%	-9.5%	-10.0%	-10.7%	-11.6%
VaR 5%	-8.0%	-10.6%	-10.8%	-10.9%	-11.1%	-10.5%	-10.2%	-12.6%	-13.1%	-13.2%
ES 5%	-9.8%	-12.4%	-12.8%	-12.9%	-13.1%	-13.1%	-12.8%	-14.5%	-14.5%	-14.6%
omega measure	1.25	1.55	1.60	1.48	1.41	1.56	1.73	1.97	2.47	2.69

Note: This table shows the descriptive statistics for the four trading strategies, including Buy-and-Hold (BH), Constant Portfolio Insurance (CPPI), Synthetic Put (SP), and Stop-loss (SL). The VaR is obtained by first sorting the portfolio returns into ascending order and then looking at the return at the 5% level. ES is the average of VaR exceedences. The Omega measure is defined relative to a zero (excess) return threshold.

Table 2: Performance results of BH strategy vs. PI strategies with different time horizons according to AFSD.

Horizon	1 Year	2 Years	3 Years	4 Years	5 Years
SL	0.237	0. 166	0.13	0.099	0.052*
CPPI	0.324	0.268	0.241	0.2	0.074
SP	0.283	0.209	0.171	0.068	0.000*
Horizon	6 Year	7 Years	8 Years	9 Years	10 Years
SL	0.011*	0.017*	0.028*	0.031*	0.023*
CPPI	0.003*	0.039*	0.071	0.1	0.053*
SP	$0.000^{\#}$	0.000*	0.015*	0.007*	$0.000^{\#}$

Note: * indicates that the empirical ε is smaller than the critical value. The critical value of ε_1 is 5.9% .
# indicates first-degree stochastic dominance.

Table 3: Performance results of BH strategy vs. PI strategies with different time horizons according to ASSD.

Horizon	1 Year	2 Years	3 Years	4 Years	5 Years
SL	0.189	0.101	0.057	0.028*	0.008*
CPPI	0.393	0.251	0.175	0.088	0.010*
SP	0.365	0.200	0.134	0.028*	0. 000*
Horizon	**6 Year**	**7 Years**	**8 Years**	**9 Years**	**10 Years**
SL	0.001*	0.004*	0.007*	0.006*	0.003*
CPPI	0.000*	0.008*	0.021*	0.024*	0.008*
SP	0.000[#]	0.000*	0.003*	0.001*	0.000[#]

Note: * indicates that the empirical ε is smaller than the critical value. The critical value of ε_2 is 3.2% .
# indicates second-degree stochastic dominance.

Figure 1: The relationship between ε_1 and the proportion of the PI strategy with different time horizons.

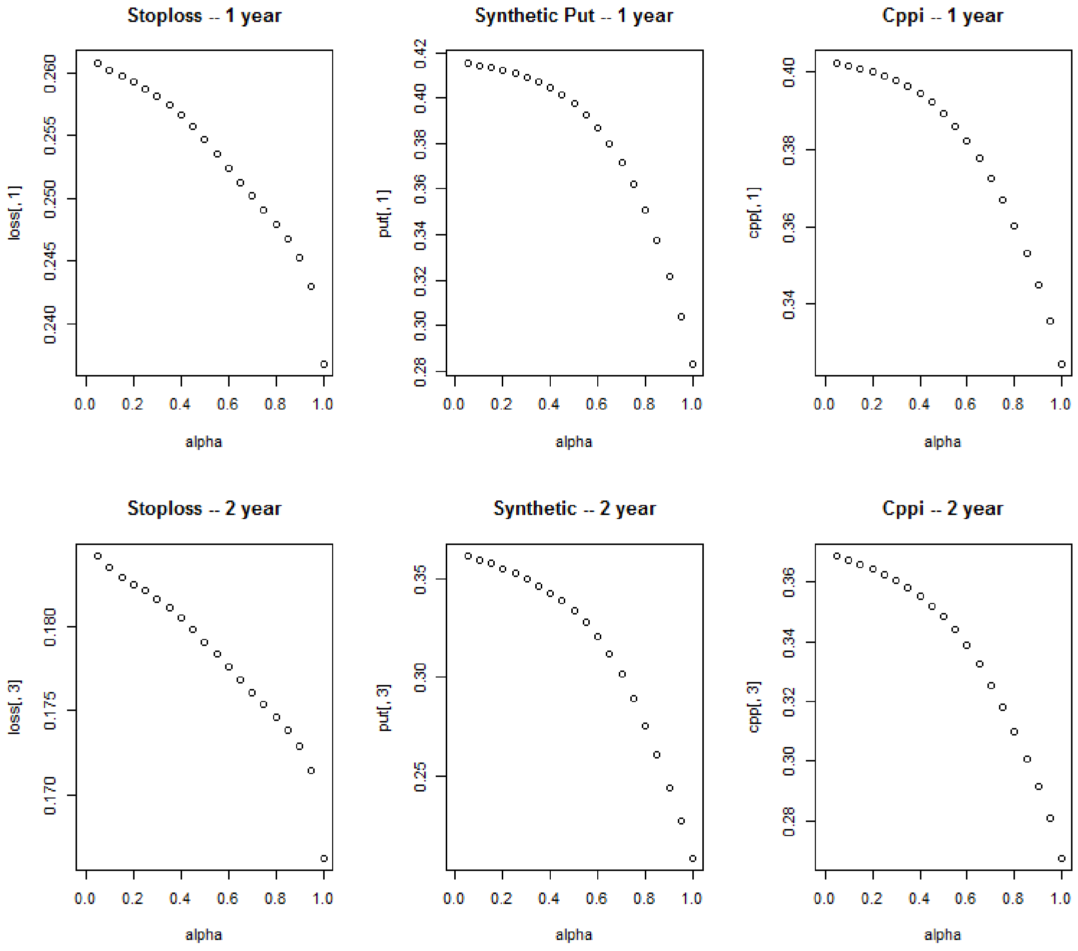

Figure 1: Continued

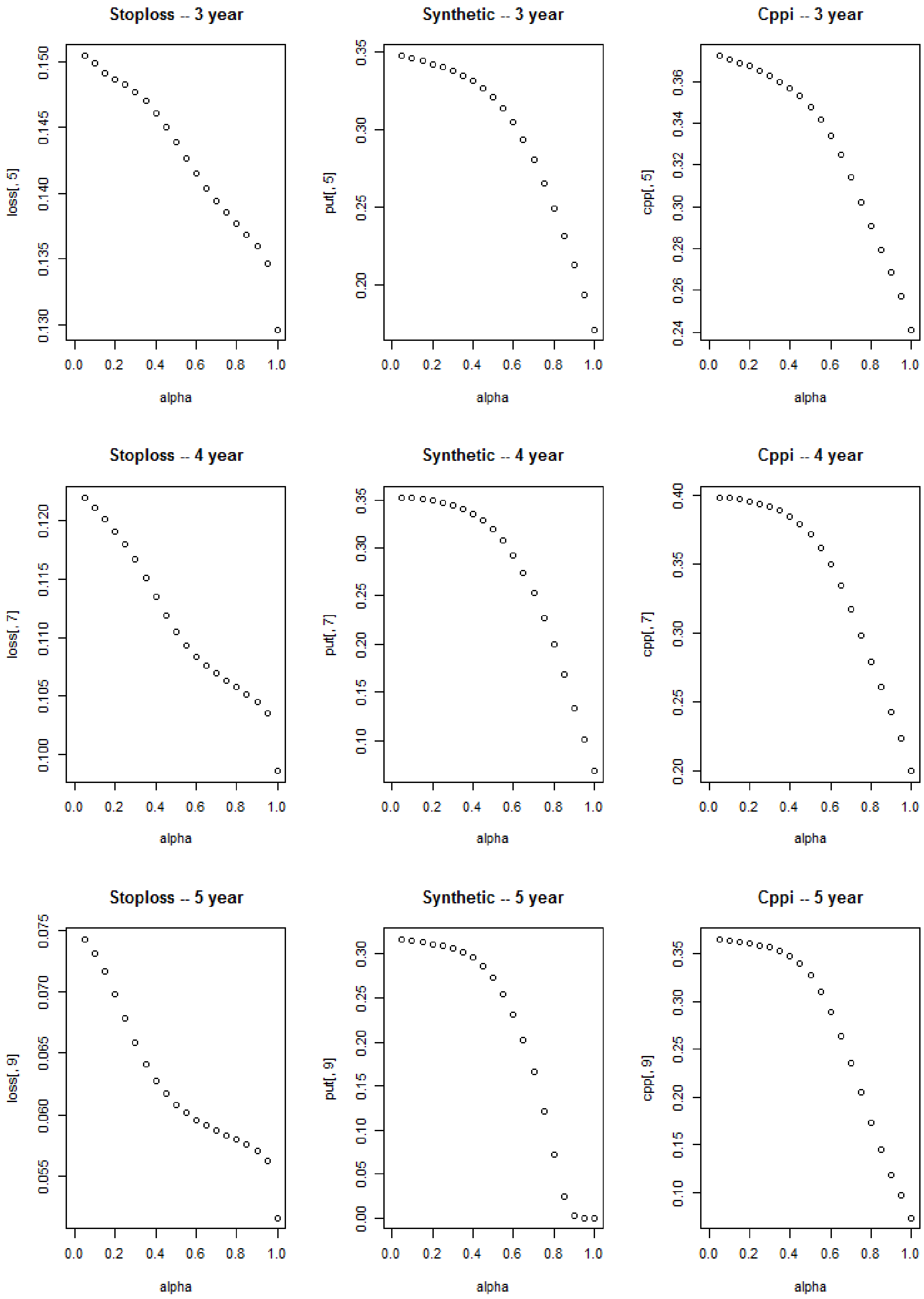

Figure 1: Continued

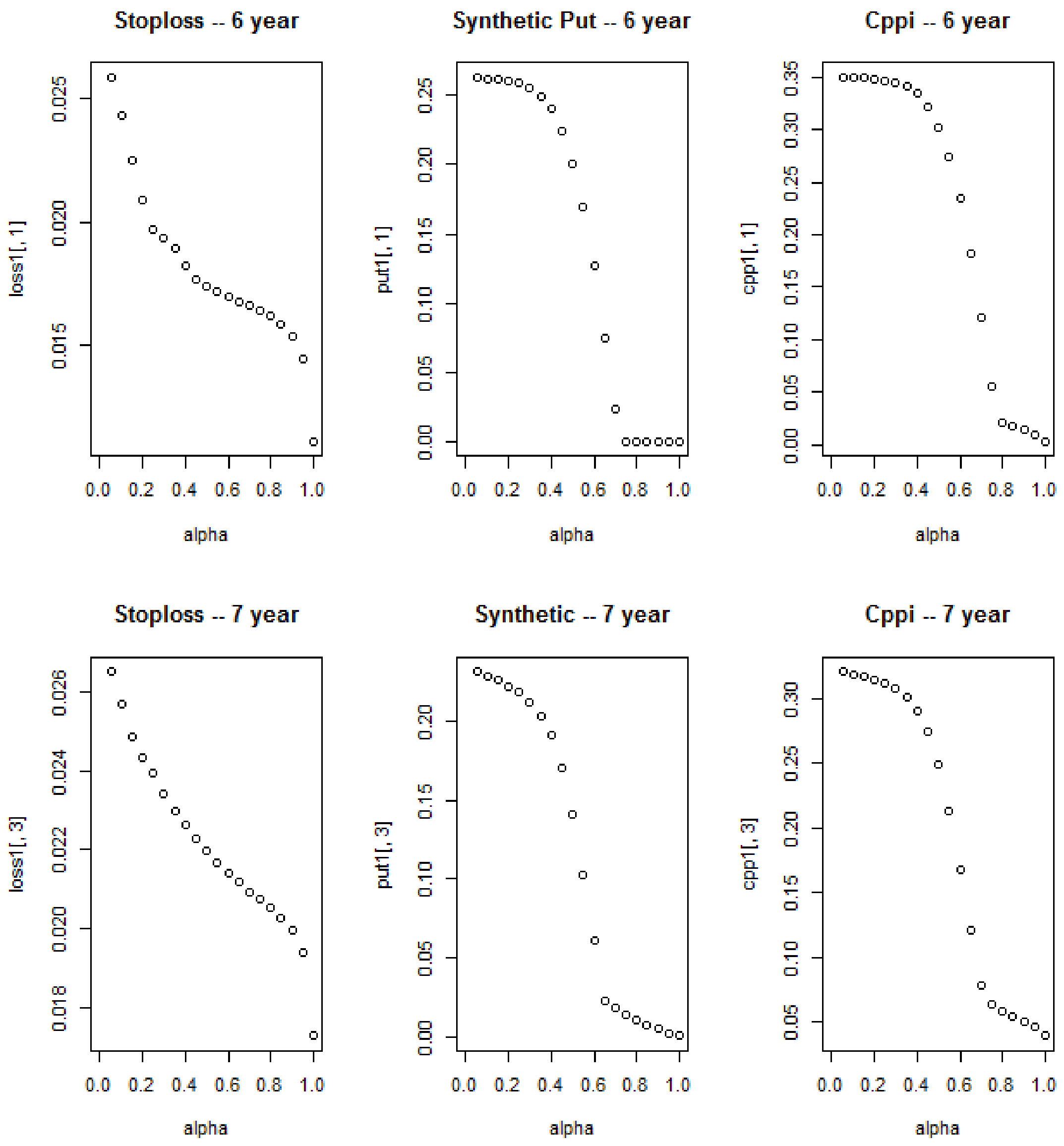

Figure 1: Continued

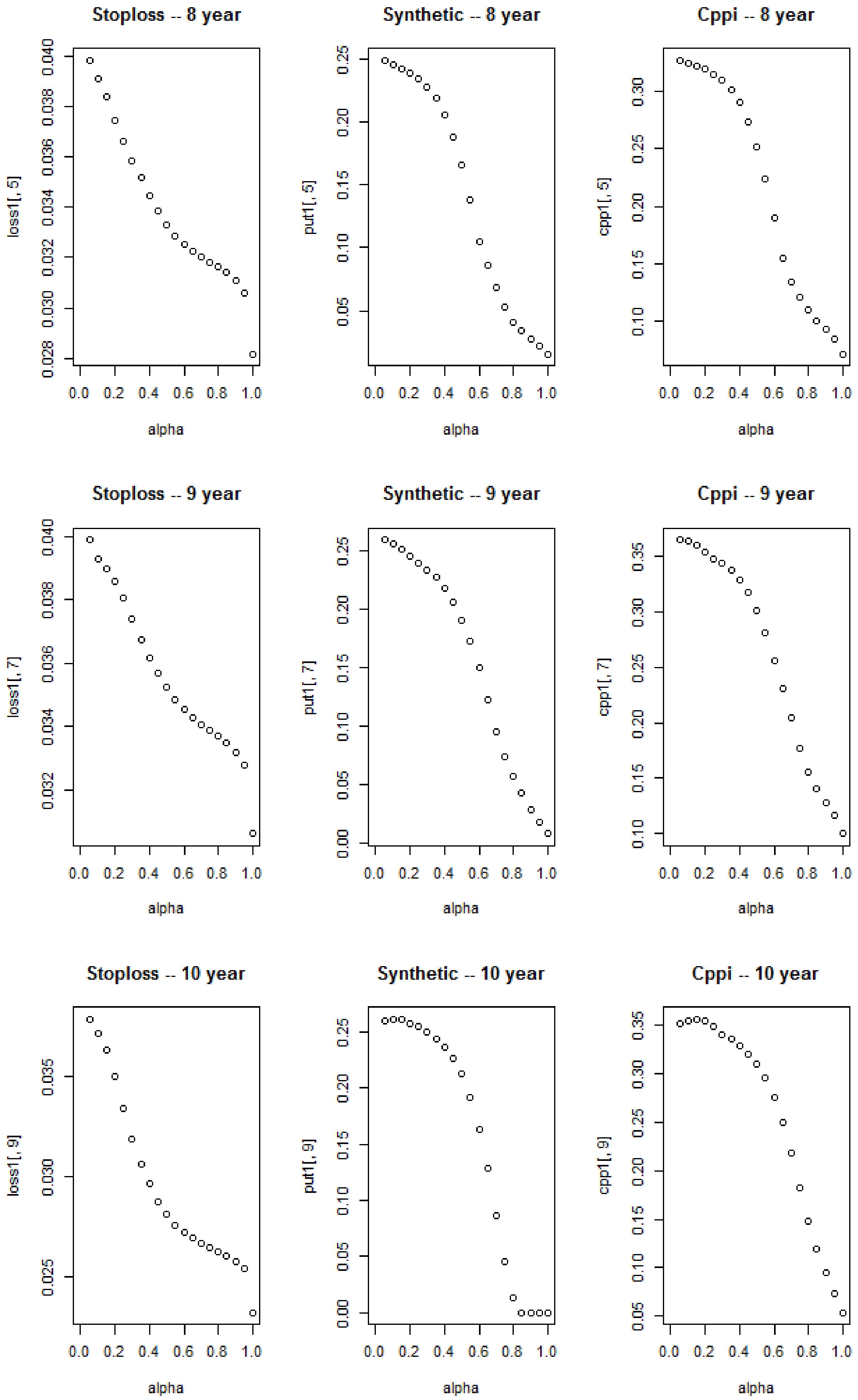

Figure 2: The relationship between ε_2 and the proportion of the PI strategy with different time horizons.

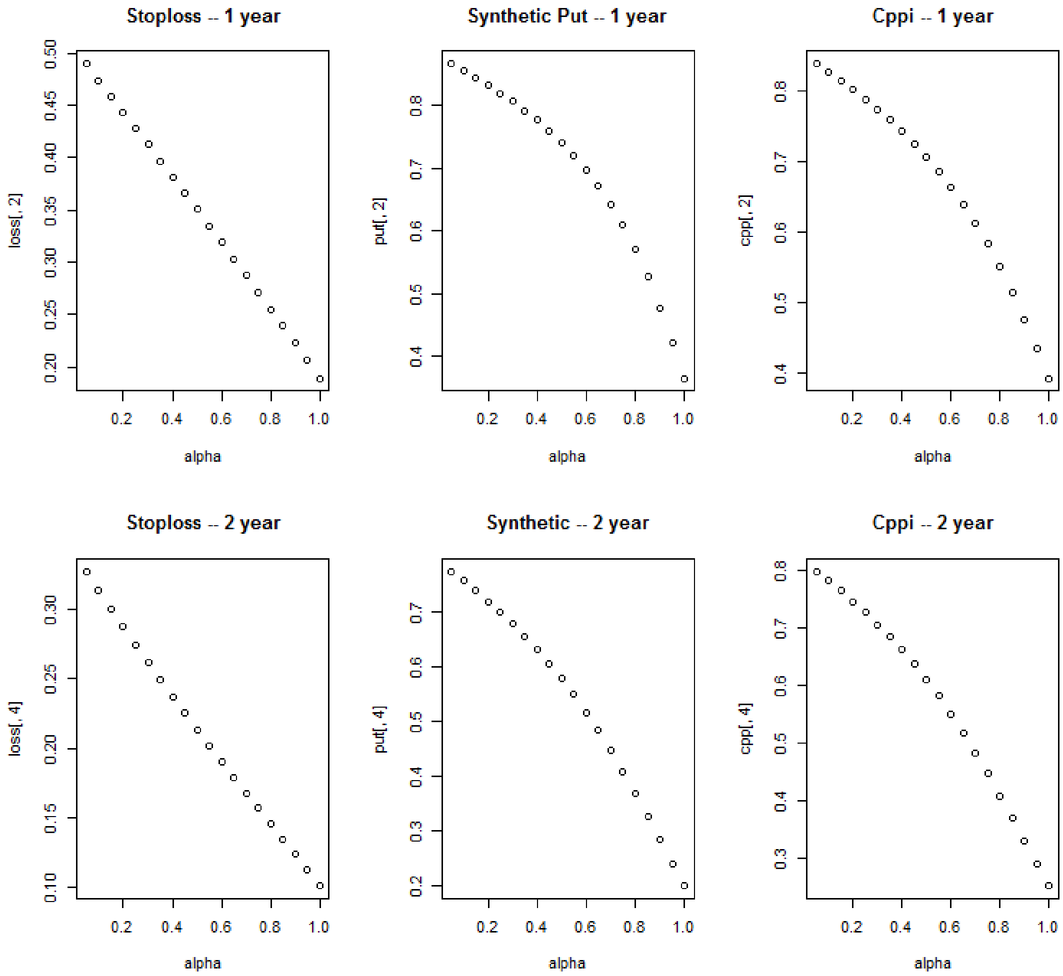

Figure 2: Continued

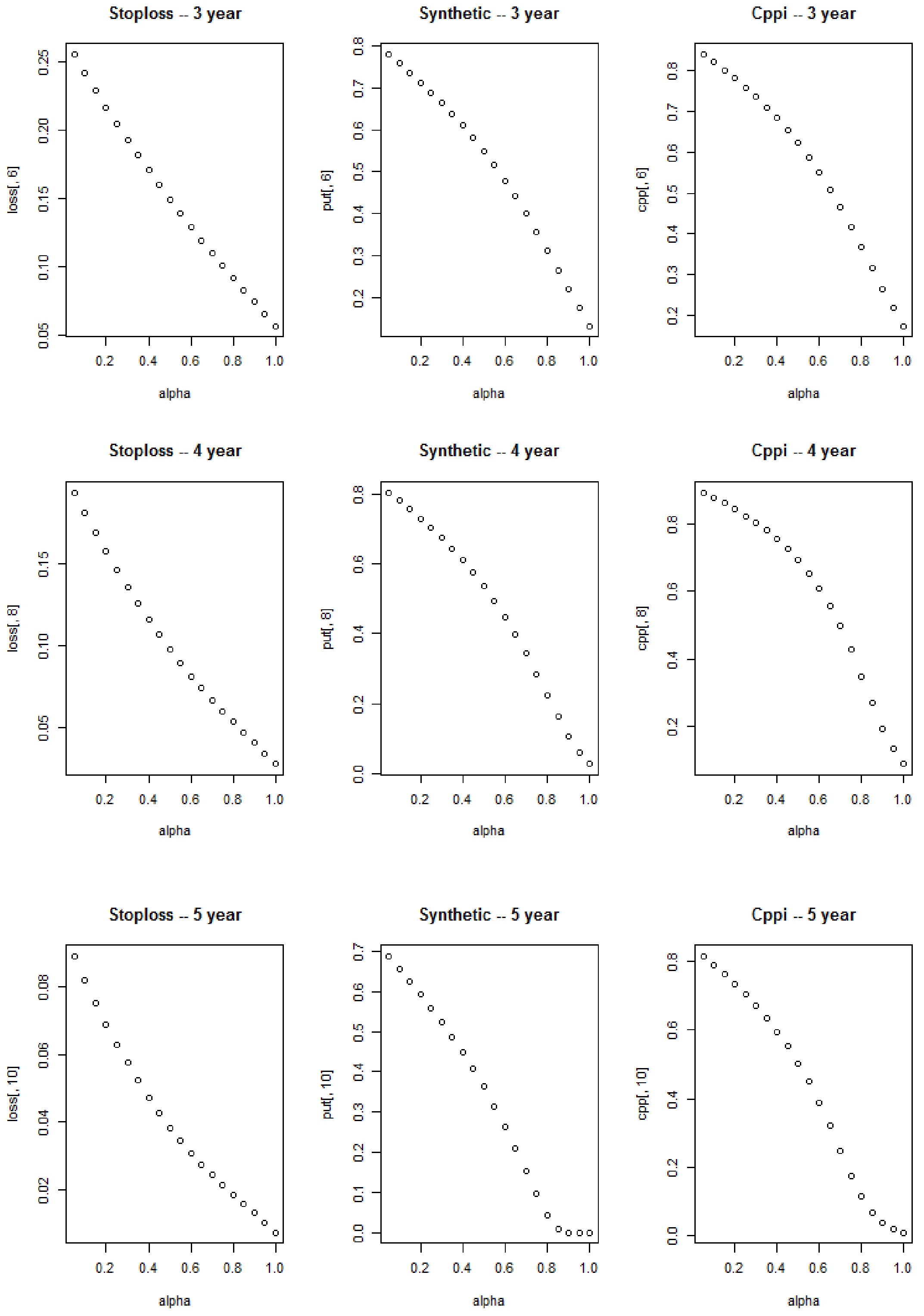

Figure 2: Continued

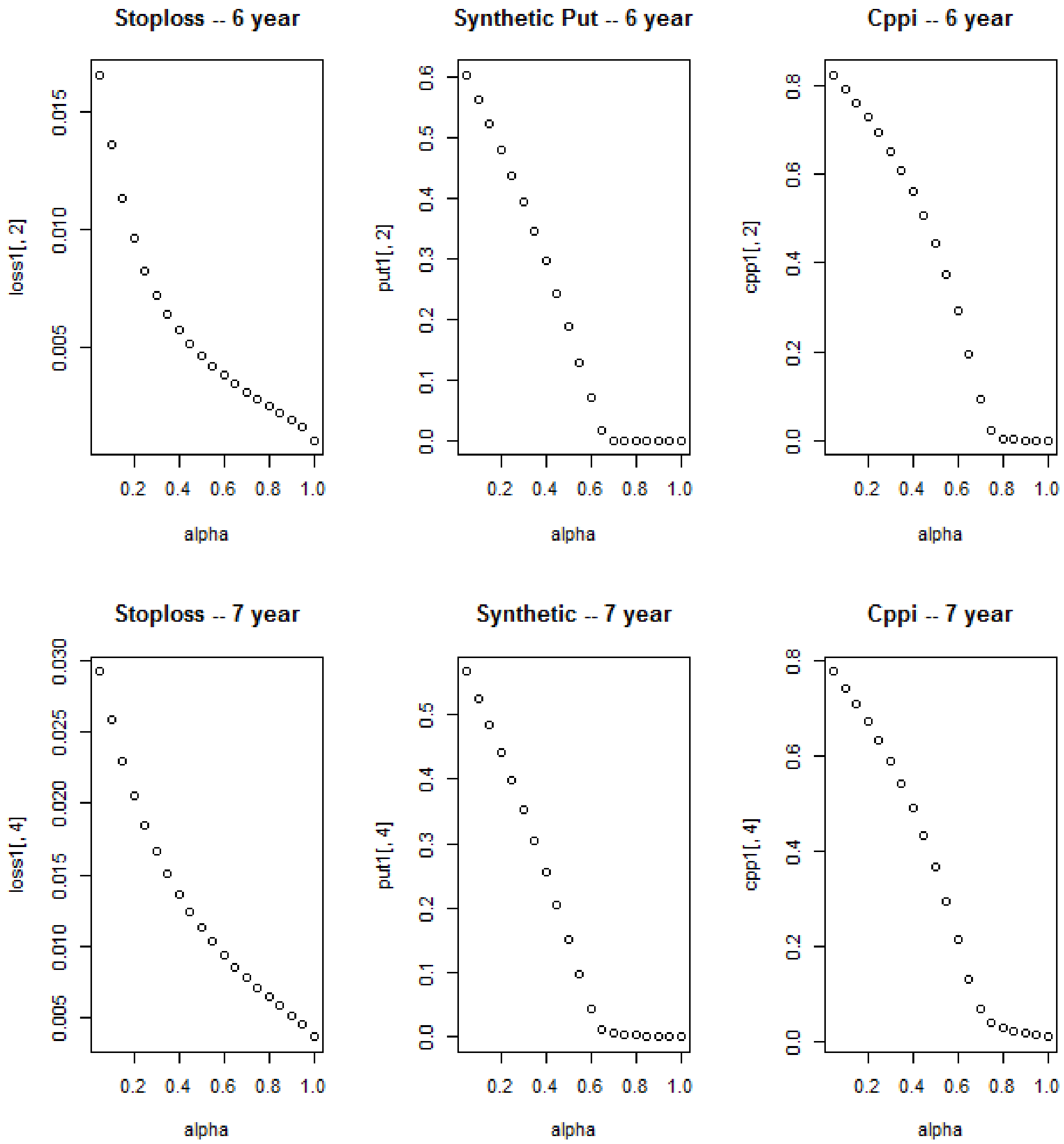

Figure 2: Continued