

Lapse-and-Reentry in Variable Annuities

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Abstract

Section 1035 of the current US tax code allows policyholders to exchange their variable annuity policy for a similar product while maintaining tax-deferred status. When the variable annuity contains a long-term guarantee, this “lapse-and-reentry” strategy allows the policyholder to potentially increase the value of the embedded guarantee. We show that for a return-of-premium death benefit guarantee this is frequently optimal, which has severe repercussions for pricing. We analyze various policy features that may help mitigate the incentive to lapse, and compare them regarding the insurer’s average expense payments and their post-tax utility to the policyholder.

We find that a ratchet-type guarantee and a state-dependent fee structure best mitigate the lapse-and-reentry problem, outperforming the typical surrender schedule. Further, when accounting for proper tax treatment, the policyholder prefers a variable annuity with either of these three policy features over a comparable stock investment.

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1 Introduction

Investment flexibility, favorable tax treatment, and long-term guarantees have moved variable annuities (VAs) among the most popular long-term savings vehicles in recent years. A typical VA policy may entail an initial lump-sum investment that is placed into a financial stock or mutual fund, according to the policyholder's choosing. To protect the policyholder from adverse market scenarios, the insurance company selling the VA policy enhances the product with a Guaranteed Minimum Death Benefit (GMDB) rider that promises to return the larger of the VA account value and a pre-specified "guaranteed" amount upon the policyholder's death. To cover its expenses and the cost of the GMDB rider, the insurer collects a fee, continuously and in proportion to the concurrent VA account value.¹

However, as recent events have demonstrated, VAs are posing tremendous challenges to life insurers (Reuters, 2009; ING, 2011; Manulife Financial, 2011; Sun Life Financial, 2011). In addition to their exposure to long-term financial risk, insurers are particularly troubled by their poor understanding of policyholder behavior.² For instance, according to Section 1035 of the US tax code, the policyholder typically has the right to surrender his existing VA policy and use the cash value to purchase a new one, without incurring additional tax obligations. In the case of a GMDB, it may be optimal to do so when the VA account value has risen (significantly) above the guaranteed amount. In that case, the GMDB has little value, yet the policyholder is paying a larger amount of fees for it. This so-called *lapse-and-reentry* strategy can have a detrimental effect on the insurer's profit; from the company's perspective, the policyholder's market reentry constitutes the sale of a new VA policy that triggers large payments in the form of commissions and other new-policy expenses. This substantially increases the fee rate that the insurer needs to charge for the policy. As a result, accounting for the policyholder's lapse behavior is critical in determining the proper fee.

In this study we present and contrast the standard return-of-premium GMDB rider with a variety of additional policy features that can help mitigate the policyholder's incentives to lapse: In practice, insurers typically impose a *surrender schedule*, so that for a number of years the policyholder must pay a percentage of the current VA account value when lapsing. A second common feature—though usually offered as an add-on for an additional fee—is a *roll-up guarantee*, whereby the guaranteed minimum death benefit amount increases by a fixed percentage each year

¹For a detailed description of this and other guarantees available in the US VA market we refer to Bauer et al. (2008).

²In a recent study, Moody's Investors Service concludes that policyholder behavior is a "weak spot" for insurers, and that "unpredictable policyholder behavior challenges US life insurers' variable annuity business" (Moody's, 2013). Moreover, in the year 2000, the UK-based mutual life insurer Equitable Life—the world's oldest life insurance company—was closed to new business due to problems arising from a misjudgment of policyholder behavior with respect to exercising guaranteed annuity options within individual pension policies (Boyle and Hardy, 2003).

(Bauer et al., 2008). Hardy (2004) presents a *ratchet-type guarantee*, whereby the guaranteed amount is equal to the largest VA account value at previous policy anniversary dates. More recently, Bernard et al. (2014) propose a *state-dependent fee* structure, under which the policyholder pays the fee only while the guarantee is (close to) in-the-money. Lastly, we consider an *additional* or *enhanced earnings* feature, which provides additional death benefit payouts upon good investment performance.

Implementing the policyholder's dynamic optimization problem under realistic parameter specifications, we find that the policyholder should lapse a VA with a return-of-premium GMDB rider quite frequently (every four years on average). Taking this optimal lapse behavior into account forces the insurer to raise the aggregate fee rate from around 90 to over 320 basis points (bps). This is consistent with the general insight of Kling et al. (2014) that miss-specifying policyholder behavior may lead to considerable deviations in the insurer's expected profit. However, we find that three of the five policy features considered provide effective disincentive to lapse-and-reentry. In particular, we find that the 7-year surrender schedule—which is attached to most current (B-share) VA policies—deters lapses during the surrender fee period. Still, policyholders are likely to lapse immediately after the surrender period ends, which is consistent with anecdotal evidence from actuarial practice. Moreover, under our base-case parameter assumptions the 7-year surrender schedule reduces the fee to around 150 bps, which is in line with typical VA fees in the US market. However, we find that the surrender schedule is not the most appropriate remedy against lapse-and-reentry, as both the ratchet guarantee and the state-dependent fee structure prove more effective. Under these features, lapsing is (almost) never optimal, which leads to a lower fee and a higher overall payout to the policyholder.

Our findings are invariant to parameter specifications and valuation approaches. In particular, the lapse-and-reentry strategy of a policyholder maximizing the expected utility of his post-tax terminal payout is almost identical to that of a policyholder concerned with maximizing the risk-neutral expected value of his investment. The former approach, however, provides additional insight into the product's recent popularity, as (fairly-priced) VAs with GMDB riders that include either a surrender schedule, a ratchet feature, or a state-dependent fee structure yield a greater expected utility at inception than a benchmark stock investment.

Many studies have focused on the valuation of the various guarantees embedded in VAs. Milevsky and Posner (2001) use option pricing techniques to value GMDB riders. More recent contributions specifically account for optimal policyholder behavior as part of pricing guarantees. For instance, Bauer et al. (2008) consider value-maximizing policyholder behavior (in addition to deterministic and probabilistic exercise strategies) in a general VA valuation framework that accounts for various death and living benefit guarantees. Belanger et al. (2009) develop partial differential equations for pricing a GMDB rider and numerically show that the option fee is con-

siderably higher when accounting for optimal partial withdrawals. Bernard et al. (2014) derive the optimal surrender strategy and the resulting fair fee for a Guaranteed Minimum Accumulation Benefit (GMAB) rider.³ Milevsky and Salisbury (2006), among others, derive the optimal withdrawal strategy for the more complex Guaranteed Minimum Withdrawal Benefit (GMWB) rider, while Kling et al. (2014) analyze lifetime withdrawal guarantees. In these studies, the policyholder's decision making is based on financial optimality under arbitrage pricing principles, akin to the early exercise of an American put option (Grosen and Jørgensen, 2000). In contrast, Moenig and Bauer (2015) allow the policyholder's withdrawal decision to be affected by tax considerations, while Steinorth and Mitchell (2015) develop a utility-based framework to examine policyholder behavior for a lifetime withdrawal guarantee.⁴

To our knowledge, very little *empirical* research has been done on policyholder behavior in VAs. Using data from the Japanese VA market, Knoller et al. (2015) confirm the general insight that surrendering should be more likely when the VA account value exceeds the guaranteed amount. In addition, the authors find that policyholders who—due to a larger face value of their VA policy—are suspected to be more financially literate also surrender with greater sensitivity to the “moneyness” of the underlying guarantee. Several studies have been undertaken to contrast lapse rates between ordinary and unit-linked life insurance and annuity products. While in earlier studies unit-linked policies were surrendered more frequently (Renshaw and Haberman, 1986), more recent data from the German market show approximately equal lapse rates (Eling and Kiesenbauer, 2014). A possible explanation is that nearly all modern unit-linked policies have investment guarantees embedded, which could reduce the policyholder's incentive to lapse. This is in contrast to the case of lapse-and-reentry that we are considering, where the policyholder lapses in order to attain an improved guarantee. Here, the presence of the guarantee should *increase* the likelihood of policy lapses. Lastly, a study by Swiss Re (2003) argues that a policyholder should intuitively be more likely to surrender a unit-linked policy such as a VA, compared to a traditional participating life insurance product.

Our research differs from the existing literature on optimal policyholder behavior within VAs in that we not only describe optimal lapse rates and determine the fair guarantee fee, but also analyze and contrast the impact of various product features on lapse rates, fair fee, and the products' respective values to the policyholder. By incorporating the insurer's expenses into our valuation framework, we are able to quantify the latter from both a valuation perspective—targeting a min-

³A GMAB gives the policyholder the right to receive the greater of the terminal VA account value and a pre-specified amount, if he is alive when the policy matures.

⁴The valuation of embedded options has also been extensively studied in other insurance lines, with recent contributions focusing on a numerical analysis in a risk-neutral framework coupled with optimal policyholder behavior. For example, Kling et al. (2006) analyze the fair values of paid-up options in individual pension schemes in Germany. For participating life insurances, Schmeiser and Wagner (2011) develop a joint valuation framework for premium and surrender options when considering optimal exercise strategy.

imal overall expense payment—and by contrasting the policyholder’s after-tax utility. Our study both contributes to the academic research on VAs and provides useful insights for practicing life actuaries.

The remainder of the paper is structured as follows. Section 2 develops a risk-neutral pricing model to determine optimal lapse-and-reentry behavior for a return-of-premium GMDB rider, derives (risk-neutral) break-even fee rates, and assesses the impact of policy lapses. Section 3 extends the model to include various policy features, and tests their effectiveness as “remedies” to the lapse-and-reentry problem. In Section 4 we derive the optimal lapse-and-reentry strategy in a utility-based framework with taxes, and contrast the policyholder’s time-0 expected utility to a benchmark investment. Lastly, Section 5 concludes.

2 Lapse-and-Reentry for a Return-of-Premium GMDB Rider

2.1 Model Setup

At time 0, an individual age x purchases a single-premium VA with face amount A_0 and maturity T years.⁵ The money is placed into a stock (or mutual fund), whereby the time- t stock price S_t evolves according to a Geometric Brownian Motion under the physical probability measure:

$$dS_t = \mu S_t dt + \sigma S_t dZ_t, \quad S_0 > 0, \quad (1)$$

where μ and σ are positive constants, and $(Z_t)_{t>0}$ is a standard Brownian motion.⁶ We denote the time- t account value of the VA by A_t .

The policy includes a GMDB rider that promises to return a guaranteed amount G_t upon the policyholder’s premature death.⁷ In the case of a return-of-premium GMDB this amount is simply the initial investment: $G_t = A_0$ for all t . For simplicity, we assume a fully discrete policy where the death benefit is payable at the end of the year of death. That is, if death occurs in the t -th policy year, the policyholder receives $\max\{A_t, G_t\}$ at time t , with A_t coming from his VA account, while the remainder, $\max\{G_t - A_t, 0\}$, is supplemented by the insurer through the GMDB rider. If the policyholder survives to maturity, he receives the VA account value A_T at that time.

⁵During this accumulation phase of the VA policy lapse-and-reentry is permissible.

⁶We choose the Black-Scholes model due to its simplicity and tractability for numerical implementations. While it is certainly possible to assume more sophisticated financial models to account for (e.g.) stochastic volatility and/or interest rates, we believe this will not qualitatively change our overall findings and insights on optimal lapse behavior and the comparison of the various remedies. For instance, Kling et al. (2014) demonstrate that “the assumption of stochastic equity volatility seems to have only little influence on” pricing results for lifetime withdrawal guarantees, under both optimal and suboptimal policyholder behavior.

⁷Certain living benefit guarantees, such as GMABs, can be implemented in similar fashion.

Furthermore, the insurer faces expenses due to commissions, marketing, and administrative costs. We model them in the form of an initial one-time expense rate ϵ_{ini} and an annually recurring expense rate ϵ_{rec} (charged at the beginning of each year). Both expense rates are assessed in proportion to the VA account value at the time.

The policyholder may lapse his VA contract on policy anniversary dates, that is at times $t = 1, 2, \dots, T - 1$. If he does, he receives the current account value A_t , with which he immediately purchases an identical VA policy with the same insurer and the original year of maturity. In effect, this lapse-and-reentry strategy replaces the policyholder's current guarantee with a new, at-the-money GMDB rider. On the other hand, from the perspective of the insurer, the policyholder's re-entry to the market constitutes the sale of a new VA policy, which again draws the one-time up-front expense at rate ϵ_{ini} . In the case of lapse-and-reentry, the policyholder incurs search costs at rate α , in proportion to the account value at the time.

The insurer aims to recover its costs for expenses and the GMDB rider by charging a recurring fee at annual rate φ^{agg} , assessed continuously and in proportion to the current VA account value A_t . The fee is taken directly out of the VA account so that the policyholder's financial contribution to the VA is restricted to the initial investment A_0 .

To better illustrate the impact of lapse-and-reentry, we divide the aggregate fee rate φ^{agg} into three parts: a base fee rate φ^{base} that covers the insurer's expenses (assuming no lapse-and-reentry); a pure guarantee fee rate φ^{guar} that accounts for the cost of providing the GMDB rider in the absence of lapse-and-reentry; and lastly a lapse-and-reentry fee φ^{LR} that covers the additional guarantee costs and expenses that the insurer faces due to the policyholder's decision to lapse the policy:

$$\varphi^{\text{agg}} = \varphi^{\text{base}} + \varphi^{\text{guar}} + \varphi^{\text{LR}}. \quad (2)$$

Consistent with standard actuarial notation, we denote by ${}_t p_x$ the probability that a person age x survives for t years, by q_x the probability that a person age x dies within the following year, and by ${}_t | q_x = {}_t p_x q_{x+t}$ the probability that a person age x dies exactly t years into the policy, that is between ages $x + t$ and $x + t + 1$.

Following Bauer et al. (2008) we assume independence between financial market risk and biometric risk. We define \mathbb{Q} as the product measure of the risk-neutral measure for financial risk and the real-world measure for (idiosyncratic) mortality risk, and \mathbb{P} as the product measure of the real-world measure for financial risk and the real-world measure for mortality risk.

2.2 Valuation without Lapses

In the absence of (direct) profit considerations, the insurer distributes the initial investment A_0 in the form of either benefits (to the policyholder) or expenses. Therefore, the break-even fee φ^* is

given implicitly by the identity

$$NPV(\varphi^*) = A_0 - EPVB_0(\varphi^*) - EPVE_0(\varphi^*) = 0, \quad (3)$$

where NPV denotes the net present value of the policy to the insurer (for a given fee rate), and $EPVB_0$ and $EPVE_0$ denote the time-0 expected present values of benefits and expenses, respectively. In the spirit of market-consistent valuation, all expected present values are computed under measure \mathbb{Q} .

Finding the Base Fee φ^{base}

Considering first the case without a GMDB rider, we note that the benefits paid out to the policyholder are merely the return of the initial investment, minus fee payments. That is,

$$EPVB_0(\varphi) = \left[\sum_{k=0}^{T-1} {}_kq_x \cdot A_0 \cdot e^{-\varphi(k+1)} \right] + {}_T p_x \cdot A_0 \cdot e^{-\varphi T}.$$

Similarly, the expected present value of expenses is given by

$$EPVE_0(\varphi) = A_0 \cdot \epsilon_{\text{ini}} + \sum_{k=0}^{T-1} {}_k p_x \cdot A_0 \cdot e^{-\varphi k} \cdot \epsilon_{\text{rec}}.$$

The annual base fee φ^{base} that covers the insurer's expenses from the VA (without lapse-and-reentry) is then given as the solution to Equation (3).

Finding the Pure Guarantee Fee φ^{guar}

To determine the insurer's break-even fee without lapses, denoted by φ^1 , we exploit the GMDB's resemblance to a life-contingent put option on the underlying investment, with varying times to maturity. Relying on the classical Black-Scholes formula with dividends, we find that—for a given fee rate φ —the expected present value of all benefits paid to the policyholder from the VA is given by

$$EPVB_0(\varphi) = \sum_{k=0}^{T-1} {}_kq_x \cdot [A_0 \cdot e^{-\varphi(k+1)} + \text{Put}(A_0, G_{k+1}, k+1, \varphi)] + {}_T p_x \cdot A_0 \cdot e^{-\varphi T}, \quad (4)$$

whereby—for a return-of-premium GMDB without lapses— $G_{k+1} = A_0$ for all k , and

$$\begin{aligned} \text{Put}(S_0, K, T, \varphi) &= K e^{-rT} \mathcal{N}(-d_2) - S_0 e^{-\varphi T} \mathcal{N}(-d_1), \text{ with} \\ d_1 &= \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \varphi + \frac{\sigma^2}{2}\right) \cdot T}{\sigma \sqrt{T}} \quad \text{and} \\ d_2 &= d_1 - \sigma \sqrt{T}, \end{aligned}$$

denotes the Black-Scholes price of a put option with current stock price S_0 , strike price K , time to maturity T and dividend yield φ . Note that the latter has the same effect on the stock price as the continuously assessed fee rate has on the VA account value A_t . The first and last term on the right side of Equation (4) denote the return of the initial investment (net of fee payments) upon death or survival to maturity, respectively, while the middle term represents the benefits paid from the GMDB.

Again, the total fee without lapses, φ^1 , is given as the solution to Equation (3). This then implies a pure guarantee fee of $\varphi^{\text{guar}} = \varphi^1 - \varphi^{\text{base}}$.

2.3 Accounting for Lapse-and-Reentry

We now quantify and solve for the optimal lapse-and-reentry decision for a value-maximizing policyholder who—at any given policy anniversary date—aims to maximize the risk-neutral expected value (under \mathbb{Q}) of his investment.⁸

When is Lapsing Financially Optimal?

The policyholder's optimal lapse decision problem bears resemblance to the early exercise of an American option. For a given policy, the implicit value of the VA on the t -th policy anniversary date—prior to making his time- t lapse decision—depends on the current account value A_t and the guaranteed death benefit amount G_t . We denote this value by $V_t(A_t, G_t)$, and in what follows outline how to determine it recursively.

At maturity (time T), the policyholder receives the VA account value:

$$V_T(A_T, G_T) = A_T. \quad (5)$$

For further reference we define:

$$\tilde{V}(t, A_t, G_{t+1}) = q_{x+t} \cdot [A_t e^{-\varphi^{\text{agg}}} + \text{Put}(A_t, G_{t+1}, 1, \varphi^{\text{agg}})] + (1 - q_{x+t}) \cdot e^{-r} \cdot \mathbb{E}^{\mathbb{Q}} [V_{t+1}(A_{t+1}, G_{t+1})], \quad (6)$$

⁸We demonstrate in Section 4 that value maximization is in fact a suitable assumption in this case.

where,

$$\begin{aligned} A_{t+1} &= A_t \cdot \exp \left[r - \varphi^{\text{agg}} - \frac{1}{2} \sigma^2 + \sigma (Z_{t+1} - Z_t) \right], \quad \text{and} \\ Z_{t+1} - Z_t &\sim \mathcal{N}(0, 1). \end{aligned}$$

Then, recursively, for times $t = T-1, T-2, \dots, 1$, and given the value function $V_{t+1}(A_{t+1}, G_{t+1})$, we can compute the continuation value of the VA policy as

$$V_t^{\text{cont}}(A_t, G_t) = \widetilde{V}(t, A_t, G_t), \quad (7)$$

and the VA value upon lapse-and-reentry as

$$V_t^{\text{lapse}}(A_t, G_t) = \widetilde{V}(t, A_t, A_t) - \alpha \cdot A_t, \quad (8)$$

where the guaranteed amount is set equal to the VA account value upon reentry.⁹ Hence, lapse-and-reentry is optimal at time t if and only if $V_t^{\text{lapse}}(A_t, G_t) > V_t^{\text{cont}}(A_t, G_t)$, and we can define

$$V_t(A_t, G_t) = \max\{V_t^{\text{cont}}(A_t, G_t), V_t^{\text{lapse}}(A_t, G_t)\}. \quad (9)$$

Finding the Lapse-and-Reentry Fee φ^{LR}

The aggregate fair fee φ^{agg} is still determined as the solution to Equation (3). However, $EPVB_0$ and $EPVE_0$ now depend on the policyholder's lapse decisions. This requires us to determine these values recursively as well.

To do so, we let $EPVB_t(A_t, G_t)$ and $EPVE_t(A_t, G_t)$ denote the time- t expected present values of all future benefit and expense payouts, respectively. Both are assessed immediately before the policyholder decides whether to lapse the policy at time t (and therefore take his optimal lapse decision—at time t and thereafter—into account). For future reference, we define:

$$\begin{aligned} \widetilde{EPVB}(t, A_t, G_{t+1}) &= q_{x+t} \cdot [A_t e^{-\varphi^{\text{agg}}} + \text{Put}(A_t, G_{t+1}, 1, \varphi^{\text{agg}})] \\ &\quad + (1 - q_{x+t}) \cdot e^{-r} \cdot \mathbb{E}^{\mathbb{Q}}[EPVB_{t+1}(A_{t+1}, G_{t+1})], \end{aligned}$$

and

$$\widetilde{EPVE}(t, A_t, G_{t+1}, \epsilon_{t+1}) = \epsilon_{t+1} \cdot A_t + (1 - q_{x+t}) \cdot e^{-r} \cdot \mathbb{E}^{\mathbb{Q}}[EPVE_{t+1}(A_{t+1}, G_{t+1})].$$

⁹Note that the search cost ($\alpha \cdot A_t$) is external to the VA investment and comes directly out of the policyholder's pocket at time t .

We start the recursion at maturity (time T) with terminal conditions:

$$\begin{aligned} EPVB_T(A_T, G_T) &= A_T \quad \text{and} \\ EPVE_T(A_T, G_T) &= 0. \end{aligned} \tag{10}$$

Proceeding recursively for times $t = T-1, T-2, \dots, 1$, if—for given A_t and G_t —the policyholder chooses to hold on to his current VA policy, we find

$$\begin{aligned} EPVB_t(A_t, G_t) &= \widetilde{EPVB}(t, A_t, G_t) \quad \text{and} \\ EPVE_t(A_t, G_t) &= \widetilde{EPVE}(t, A_t, G_t, \epsilon_{\text{rec}}). \end{aligned} \tag{11}$$

Conversely, upon lapse-and-reentry the time- t expected present values for benefits and expenses are given by

$$\begin{aligned} EPVB_t(A_t, G_t) &= \widetilde{EPVB}(t, A_t, A_t) \quad \text{and} \\ EPVE_t(A_t, G_t) &= \widetilde{EPVE}(t, A_t, A_t, \epsilon_{\text{ini}} + \epsilon_{\text{rec}}). \end{aligned} \tag{12}$$

Finally, for time $t = 0$, we obtain:

$$\begin{aligned} EPVB_0(\varphi^{\text{agg}}) &= \widetilde{EPVB}(0, A_0, A_0) \quad \text{and} \\ EPVE_0(\varphi^{\text{agg}}) &= \widetilde{EPVE}(0, A_0, A_0, \epsilon_{\text{ini}} + \epsilon_{\text{rec}}). \end{aligned} \tag{13}$$

We iterate over φ^{agg} until Equation (3) is satisfied. The lapse-and-reentry portion of the fee rate, φ^{LR} , is then implied by Equation (2) and the fee rates φ^{base} and φ^{guar} derived in Section 2.2.

Numerical Implementation

We implement the policyholder's optimal control problem described above numerically using recursive dynamic programming. For that we discretize the state space consisting of the VA account value A_t and the guaranteed death benefit amount G_t . For each point (A_T, G_T) on this two-dimensional state space grid we determine the terminal values for V_T , $EPVB_T$, and $EPVE_T$, as specified by Equations (5) and (10). Thereafter—again for each point on the (A, G) -grid—we can compute and contrast the continuation and lapse values of the VA policy at time $T - 1$, as per Equations (7) and (8). Thereby we rely on the Gauss-Hermite quadrature method to model the one-year stock returns, and on linear interpolation of our earlier valuation results for $V_T(A_T, G_T)$. This yields the time $T - 1$ VA value at each grid point, as given by Equation (9). Based on the policyholder's lapse decision we can find the time $T - 1$ expected present values of benefits and expenses ($EPVB_{T-1}$ and $EPVE_{T-1}$) at each grid point, using Equation (12) in the case of lapse-and-reentry, and Equation (11) otherwise. We proceed recursively to times $t = T - 2, \dots, 1$, solving the optimal control problem and assigning values to V_t , $EPVB_t$, and $EPVE_t$ —year by

Table 1: Parameter values for our numerical implementation of lapse-and-reentry problem for return-of-premium GMDB rider.

Description	Parameter	Values	
		Base Case	Sensitivity Analysis
Account Value (\$)	A_0	100,000	
Age at Inception (years)	x	55	50, 60
Interest Rate	r	3%	2%, 5%
Volatility	σ	20%	15%, 25%
Initial Expense	ϵ_{ini}	7%	5%, 9%
Recurring Expense	ϵ_{rec}	0.4%	0.2%, 0.6%
Search Cost	α	0	0.2%, 1%

year—for all points on the (A, G) -grid. Lastly, we determine $EPVB_0$ and $EPVE_0$ (using Equation (13)), and iterate to numerically find the fair aggregate fee φ^{agg} that satisfies Equation (3). In similar fashion we can keep track of the average number of lapses that the policyholder expects to make going forward.¹⁰

Lastly, in order to better understand lapse frequencies over the course of the policy life, we use Monte Carlo simulation to generate a large number of paths for stock movements and individual mortality experience. Thereby we embed the optimal lapse decision (at the nearest grid point) from the recursive dynamic programming approach discussed earlier.

2.4 Parameter Specifications

Parameter specifications for our numerical illustration are displayed in Table 1. In particular, for our base case scenario we consider a 55-year old policyholder whose mortality follows the 2012 IAM basic male mortality table. We extensively test our results for their sensitivity to all relevant input parameters. For instance, we consider policyholders age 50 and age 60 at inception of the policy. In each case the policy matures on the policyholder’s 80th birthday.¹¹ The VA policy has face amount \$100,000—since all valuations are assessed in proportion to the face amount, lapse decisions and break-even fees are invariant to this parameter. We assume a risk-free rate of interest of 3% p.a. in the base case (2% and 5% for the sensitivity tests), and a base case volatility of 20% (sensitivity: 15% and 25%). In line with typical values for B-share VAs we assume expense rates

¹⁰However, while we carry out the valuation of the VA policy under the risk-neutral measure \mathbb{Q} , we determine the expected number of lapses over the course of the policy using the real-world measure \mathbb{P} .

¹¹In practice, 80 is typically the maximum age to purchase a new VA policy. As a result, policyholders cannot lapse and reenter after reaching age 80.

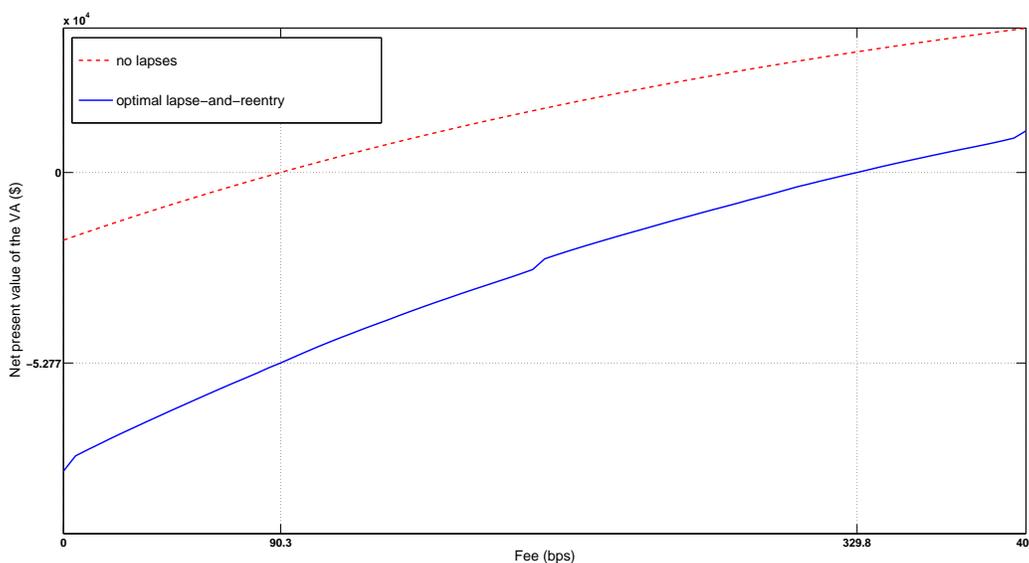


Figure 1: Net present value (NPV) to insurer, under measure \mathbb{Q} , for return-of-premium GMDB rider, as a function of annual fee rate. Parameter values as in base case of Table 1.

of 7% of the face amount at inception (or reentry) of a policy (sensitivity: 5%, 9%) and 0.4% of the account value recurring each year (sensitivity: 0.2%, 0.6%).

We believe that policyholder search costs for VA policies upon lapse-and-reentry are minimal. This is largely due to the fact that these policies can be sold only by licensed insurance agents and brokers, and that the agent/broker receives the full commission of a new VA policy when the policyholder reenters the market. As a result, the agent/broker has strong incentives to offer the policyholder a free consultation and to make the lapse-and-reentry process as easy as possible for him. Nonetheless, in our sensitivity analysis we also consider positive search costs at the amount of 0.2% and 1.0% of the VA account value at the time of lapse-and-reentry, respectively.

2.5 Valuation Results for the Return-of-Premium GMDB

Figure 1 displays the insurer's net present value according to Equation (3) under the base case parameter specifications from Table 1. In the absence of lapses the insurer breaks even at an annual fee rate of approximately 90.3 bps. However, if the policyholder lapses and reenters whenever it is financially optimal to do so, the insurer loses \$52,770 (that is, over half of the contract's face amount). Thus, the opportunity to lapse and reenter appears highly valuable and significantly drives up the fair fee. Table 2 shows that "lapse-and-reentry" makes up the by far largest portion of the aggregate fee rate. In fact, an annual fee rate of 329.8 bps proves necessary for the insurer to break even under financially optimal, unconstrained lapse behavior.

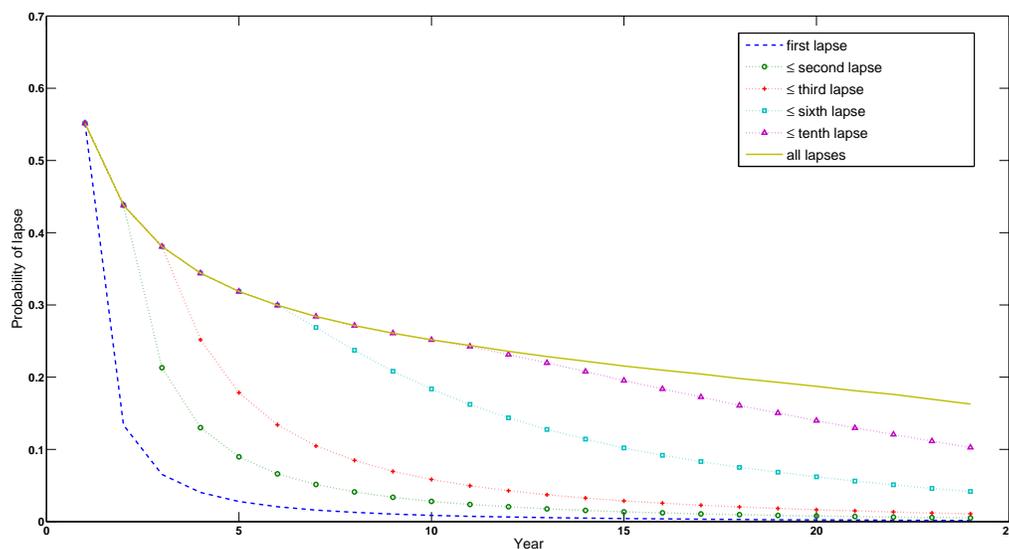


Figure 2: Probability (under measure \mathbb{P}) that the policy is lapsed in a given year, by number of lapses, for a return-of-premium GMDB rider with the base case parameter specifications from Table 1.

Further analysis reveals that the policyholder will lapse and reenter on average 6.11 times prior to maturity (or death, whichever comes first). According to Figure 2, the policyholder lapses after the first year in 55% of all financial market scenarios. The likelihood of a lapse declines over time, but we still expect a 16% lapse rate one year prior to maturity. In fact, there is roughly a 22% chance that a policy will be lapsed at least 10 times over the course of the (at most) 25 policy years.

These frequent lapses significantly increase the value of the death benefit guarantee.¹² However, this makes up only 14% (that is, 33.2 bps)¹³ of the additional fee rate φ^{LR} . The majority of this additional fee (86%, that is 206.3 bps) is used to cover the insurer's new-policy expenses associated with each market reentry. With that, the insurer expects to use more than 40% of the policyholder's investment to cover its own expenses (Column $EPVE_0$ of Table 2), and return less than 60% to the policyholder in the form of death or survival benefits (Column $EPVB_0$). In

¹² Note that in the absence of policyholder search costs (that is, when $\alpha = 0$), the policyholder's optimal control problem can be further simplified: since $\tilde{V}(A_t, G_{t+1})$ is strictly increasing in G_{t+1} (because a put option is more valuable when the strike price is higher, and a VA policy is more valuable when the guaranteed death benefit amount is larger), we see that $V_t^{\text{lapse}}(A_t, G_t) > V_t^{\text{cont}}(A_t, G_t)$ if and only if $A_t > G_t$. That is, the policyholder can improve his financial position through lapse-and-reentry whenever the VA account value exceeds the guaranteed death benefit amount. This optimal lapse-and-reentry strategy makes the return-of-premium GMDB rider essentially identical to a so-called ratchet-type GMDB, which we will discuss further in Section 3.

¹³We obtain the break-even lapse-and-reentry fee of 33.2 bps when $\epsilon_{\text{ini}} = 0$ for all lapse-and-reentry activities (though not for the initial VA purchase).

Table 2: Valuation results and lapse statistics for a return-of-premium GMDB rider. This table lists the aggregate break-even fee φ^{agg} for a return-of-premium GMDB rider, broken down into the base fee φ^{base} , the pure guarantee fee φ^{guar} , and the lapse-and-reentry fee φ^{LR} . Fees are quoted in bps. Furthermore, the table presents the division of the initial investment $A_0 = \$100,000$ into benefits ($EPVB_0$) and expenses ($EPVE_0$), as well as the average number of lapses over the course of the policy (Lapses). Lapse decisions are made by the policyholder from a value-maximizing perspective (under measure \mathbb{Q}). All fees and expected present values are also computed under \mathbb{Q} . Lapse numbers are assessed under the real-world measure \mathbb{P} , whereby the average growth rate of the stock, μ , is calibrated to yield a Sharpe ratio of 0.25 (that is, $\mu = r + 0.25\sigma$). All parameter specifications are provided in Table 1, unless noted in this table.

	φ^{base}	φ^{guar}	φ^{LR}	φ^{agg}	$EPVB_0$	$EPVE_0$	Lapses
Base case	73.7	16.6	239.5	329.8	\$58,150	\$41,850	6.11
$r = 2\%$	73.7	25.0	221.2	319.9	\$61,640	\$38,360	5.55
$r = 5\%$	73.6	7.1	271.2	351.9	\$52,390	\$47,610	7.56
$\sigma = 15\%$	73.7	9.8	216.8	300.3	\$58,390	\$41,610	6.91
$\sigma = 25\%$	73.2	23.5	263.2	359.9	\$58,110	\$41,890	5.71
$x = 50$	68.1	13.4	233.9	315.4	\$53,810	\$46,190	7.20
$x = 60$	81.7	20.3	245.0	347.0	\$62,880	\$37,120	5.03
$\epsilon_{ini} = 5\%$	63.8	15.8	185.4	265.0	\$65,180	\$34,820	6.83
$\epsilon_{ini} = 9\%$	83.8	17.4	292.4	393.6	\$52,230	\$47,770	5.75
$\epsilon_{rec} = 0.2\%$	52.8	15.3	241.1	309.2	\$60,270	\$39,730	6.23
$\epsilon_{rec} = 0.6\%$	94.6	17.9	237.8	350.3	\$56,150	\$43,850	6.00
$\alpha = 0.2\%$	73.7	16.6	206.7	297.0	\$61,500	\$38,500	5.28
$\alpha = 1\%$	73.7	16.6	115.7	206.0	\$71,390	\$28,610	2.84
$\epsilon_{ini} = \epsilon_{rec} = 0$	0	12.0	30.3	42.3	\$100,000	\$0	8.59
Base case, no lapses	73.7	16.6	0	90.3	\$84,770	\$15,230	0

contrast, without lapses only around 15% of the initial investment is used to cover the insurer's expenses (see the last row in Table 2).

As Table 2 demonstrates, our main insight from the base case—that is, the desire to lapse frequently and the enormous impact this has on the break-even fee rate—prevail across parameter specifications. Furthermore, we observe that the (generally minor or moderate) numerical deviations in the break-even fee rate in response to parameter changes conform with our intuition. In a high-interest climate ($r = 5\%$), for instance, the VA account value grows faster on average (under measure \mathbb{Q}). This reduces the likelihood that the guarantee is (deep) “in the money” if and when the policyholder dies, which yields a lower no-lapse pure guarantee fee φ^{guar} (7.1 vs. 16.6 bps). However, based on our earlier discussion, the on average larger account value also increases the policyholder's incentives to lapse and reenter (7.56 vs. 6.11 lapses), and the insurer's resulting

expenses actually drive *up* the aggregate fee rate φ^{agg} (351.9 vs. 329.8 bps).

If the underlying investment is more volatile ($\sigma = 25\%$), the GMDB increases in value—both with and without lapses—as extreme negative investment performances become more likely (23.5 vs. 16.6 bps, and 359.9 vs. 329.8 bps). Moreover, since a younger investor ($x = 50$) faces lower average mortality rates (over a longer policy period), the GMDB rider is less likely to materialize for him; this slightly reduces the break-even fee rate (315.4 vs. 329.8 bps). Naturally, break-even fee rates increase as the insurer’s expense rates ($\epsilon_{\text{ini}}, \epsilon_{\text{rec}}$) increase. Furthermore, we observe that a positive search cost is indeed a deterrent to lapse-and-reentry. However, as stated above, we find that a search cost of approximately \$1,000 (corresponding to $\alpha = 1\%$) far exceeds the experience of a typical VA policyholder. Lastly, note that the inclusion of expenses into our model does not directly impact the policyholder’s incentives to lapse.¹⁴

In conclusion, we find that accounting for the possibility that the policyholder may lapse his VA (and reenter the market at better financial conditions) is crucial for the pricing of a GMDB rider. However, optimal lapse-and-reentry causes the insurer to spend a substantial portion of the policyholder’s investment on commissions and other expenses. This makes the overall product undesirable for most investors. In fact, the policyholder might well be better off under a no-lapse policy due to the resulting lower break-even fee. Marketing and enforcing such a policy might present its own challenges, though. On the other hand, in the following section we introduce and analyze several contract features that indirectly (and in one case directly) strengthen the policyholder’s incentive not to lapse prematurely.

3 Potential Remedies to the Lapse-and-Reentry Problem

Insurers and academics have introduced various policy features that potentially reduce incentives to lapse and reenter. We consider five such features—surrender schedule, roll-up and ratchet-type guarantees, a state-dependent fee structure, and additional earnings riders—and analyze their respective impact on optimal lapses and valuation.

3.1 Surrender Schedule

The insurer can directly counteract the incentive to lapse by imposing a fee schedule upon surrender/lapse. If the policyholder lapses his VA policy in year m , the insurer assesses a surrender fee at rate $s(m)$ applied to the amount withdrawn, that is the current VA account value A_t . The poli-

¹⁴Discrepancies in the lapse rates between the base case and the case without expenses (6.11 vs. 8.59 lapses on average per policy) can be attributed to the higher average growth rate of the VA account value in an environment where the continuously deducted fee rate is much lower due to the absence of expenses (42.3 vs. 329.8 bps).

Table 3: Valuation results and lapse statistics for the contract features presented in Section 3. This table lists the aggregate break-even fee φ^{agg} for each contract feature, broken down into the base fee φ^{base} , the pure guarantee fee φ^{guar} , and the lapse-and-reentry fee φ^{LR} . Fees are quoted in bps. Furthermore, the table presents the division of the initial investment $A_0 = \$100,000$ into benefits ($EPVB_0$) and expenses ($EPVE_0$), as well as the average number of lapses over the course of the policy (Lapses). Lapse decisions are made by the policyholder from a value-maximizing perspective (under measure \mathbb{Q}). All fees and expected present values are also computed under \mathbb{Q} . Lapse numbers are assessed under the real-world measure \mathbb{P} , whereby the average growth rate of the stock $\mu = 8\%$ (hence the Sharpe ratio is 0.25). Parameter values are consistent with the base case specification in Table 1.

	φ^{base}	φ^{guar}	φ^{LR}	φ^{agg}	$EPVB_0$	$EPVE_0$	Lapses
<i>Return-of-premium</i>	73.7	16.6	239.5	329.8	\$58,150	\$41,850	6.11
<i>Surrender Schedule</i>							
5-year	73.7	16.6	88.5	178.8	\$73,980	\$26,020	2.21
7-year	73.7	16.6	60.4	150.7	\$77,340	\$22,660	1.45
10-year	73.7	16.6	34.8	125.1	\$80,450	\$19,550	0.80
<i>Roll-up</i>							
$g = 2\%$	73.7	35.5	210.1	319.3	\$64,190	\$35,810	4.71
$g = 5\%$	73.7	104.5	178.0	356.2	\$73,280	\$26,720	2.94
<i>Ratchet</i>	73.7	49.8	0	123.5	\$85,060	\$14,940	0
<i>State-Dependent^a</i>							
$h = 0$	73.7	58.8	9.3	141.8	\$84,350	\$15,650	0.03
$h = 20\%$	73.7	42.7	48.0	164.4	\$81,530	\$18,470	0.72
<i>Additional Earnings</i>							
$\theta_{AE} = 20\%$	73.7	26.8	230.0	330.5	\$58,180	\$41,820	6.11
$\theta_{AE} = 40\%$	73.7	36.0	220.9	330.6	\$58,270	\$41,730	6.08
$\theta_{AE} = 100\%$	73.7	56.2	181.2	311.1	\$60,720	\$39,280	5.03

^a In any given year the policyholder pays fees at rate φ^{agg} if Inequality (15) holds, and at rate φ^{base} otherwise.

cyholder therefore receives only $A_t \cdot [1 - s(m)]$. For an n -year surrender schedule, the surrender fee is 0 after the policyholder has held the policy for at least n years.

In the absence of lapse-and-reentry, and since adding a surrender schedule does not change the guaranteed amount G_t for any t , the break-even pure guarantee fee φ^{guar} is the same as for the return-of-premium GMDB rider. However, the optimal lapse decision now depends on how long the policyholder has been holding on to his current VA policy. We denote by m_t the policy year in year t , to reflect this added dimension in the state space, and define

$$\begin{aligned} \tilde{V}(t, A_t, G_{t+1}, m_{t+1}) &= q_{x+t} \cdot [A_t e^{-\varphi^{\text{agg}}} + \text{Put}(A_t, G_{t+1}, 1, \varphi^{\text{agg}})] \\ &\quad + (1 - q_{x+t}) \cdot e^{-r} \cdot \mathbb{E}^{\mathbb{Q}} [V_{t+1}(A_{t+1}, G_{t+1}, m_{t+1})] \end{aligned}$$

in lieu of Equation (6). The continuation value of the VA policy is then given by

$$V_t^{\text{cont}}(A_t, G_t, m_t) = \tilde{V}(t, A_t, G_t, \min\{m_t + 1, n\}),$$

while the VA value upon lapse-and-reentry is

$$V_t^{\text{lapse}}(A_t, G_t, m_t) = \tilde{V}(t, A_t \cdot [1 - s(m_t)], A_t \cdot [1 - s(m_t)], 1) - \alpha \cdot A_t.$$

As for the return-of-premium GMDB rider, the policyholder lapses if and only if

$$V_t^{\text{lapse}}(A_t, G_t, m_t) > V_t^{\text{cont}}(A_t, G_t, m_t),$$

and the implicit value of the VA policy is given by the larger of the two values:

$$V_t(A_t, G_t, m_t) = \max\{V_t^{\text{cont}}(A_t, G_t, m_t), V_t^{\text{lapse}}(A_t, G_t, m_t)\}.$$

Following the procedure outlined in Section 2.3—and making similar adjustments to determine the expected present values of benefits and expenses, $EPVB_t(A_t, G_t, m_t)$ and $EPVE_t(A_t, G_t, m_t)$ —we obtain the break-even fee rate φ^{agg} for VA policies with surrender schedules.

Results

A typical B-share VA policy carries a 7-year surrender schedule, whereby the surrender fee rate is 7% in the first year, 6% in the second year, and so on, down to 1% in the seventh year. Therefore, lapsing at time $t = 1$ (that is, at the beginning of the second year) will incur a 6% surrender charge. In addition, we test 5-year and 10-year surrender schedules that are structured in the same way. Table 3 displays lapse and valuation results for the different schedules. We observe that the additionally imposed surrender fees considerably discourage the policyholder from lapsing his VA

policy. In addition, the longer the surrender schedule, the lower the expected number of lapses and the resulting break-even fee.

Figure 3 shows the likelihood of a policy lapse under a 7-year surrender schedule at each policy anniversary date. The spike pattern highlights the presence of the surrender fee and the policyholder's desire to avoid lapses while subject to a positive surrender fee. However, in more than half of all (simulated) financial market scenarios lapsing is optimal at the end of the initial surrender fee period (time $t = 7$). This coincides with anecdotal evidence that a large number of policies are in fact lapsed at that time. Moreover, the resulting aggregate break-even fee of 150.7 bps is around what insurers tend to charge for a standard VA with an embedded return-of-premium GMDB rider and a 7-year surrender schedule.

3.2 Roll-Up Guarantee

Under a roll-up feature, the guaranteed death benefit amount increases at rate g each year. That is, we have

$$G_{k+1} = G_k \cdot (1 + g), \quad G_1 = A_0$$

in lieu of the corresponding declaration in Equation (4), and

$$V_t^{cont}(A_t, G_t) = \tilde{V}(t, A_t, G_t \cdot (1 + g))$$

in lieu of Equation (7). The insurer's valuation equations (11) and (12) can be adjusted accordingly.

Results

This feature is offered by several companies—typically as an enhanced death benefit rider for an additional fee—with roll-up rates of $g = 2\%$ and/or $g = 5\%$. Table 3 and Figure 3 show that the addition of a roll-up feature moderately reduces the incentive to lapse and reenter, and thus also the insurer's overall expenses. On the other hand, the increased death benefit payout drives up the cost of the GMDB rider itself. In fact, for a 5% roll-up rate, the aggregate break-even fee rate exceeds that of the return-of-premium GMDB (356.2 vs. 329.8 bps).

3.3 Ratchet Guarantee

Under a ratchet-type guarantee—also known as an “automatic annual step-up” feature—the death benefit payout is equal to the largest account value at any policy anniversary date:

$$G_t = \max\{A_0, A_1, \dots, A_t\},$$

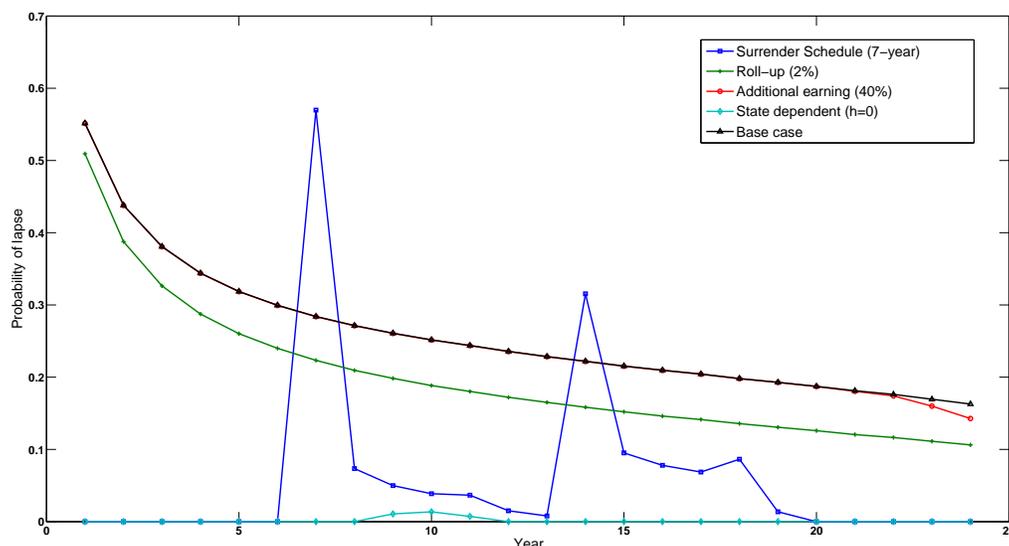


Figure 3: Probability (under measure \mathbb{P}) that the policy is lapsed in a given year, by number of lapses, for select remedies with the base case parameter specifications from Table 1.

or, equivalently,

$$G_{t+1} = \max\{G_t, A_{t+1}\}, \quad G_0 = A_0. \quad (14)$$

This recursive description shows that the optimal lapse decision can be expressed as a function of only A_t and G_t , as for the return-of-premium GMDB rider. Therefore, the numerical implementation of the ratchet guarantee is akin to the return-of-premium case, but with the guaranteed death benefit amount stepped up to the VA account value according to Equation (14).

Results

Based on our discussion from Section 2—and in particular contrasting Equations (7) and (8)—it is apparent that with a ratchet-type guarantee the policyholder has no financial incentive to lapse the VA (see also Footnote 12). In fact, (for $\alpha = 0$) the automatic step-up is equivalent to lapse-and-reentry in the return-of-premium case, albeit with the important benefit that the policyholder does not need to purchase a new VA policy, and the insurer does not incur the corresponding new-policy expenses. This implies that for the ratchet guarantee $\varphi^{\text{LR}} = 0$, and leads to a much lower aggregate break-even fee φ^{agg} (123.5 vs. 329.8 bps, see Table 3).

3.4 State-Dependent Guarantee Fee

Recently, Bernard et al. (2014) introduced the idea of a state-dependent fee structure whereby the policyholder only pays the guarantee fee when the guarantee is in fact “in the money” (or below a specified value). We implement this as follows: the policyholder pays the aggregate fee rate φ^{agg} continuously throughout a given policy year if the account value at the beginning of that policy year is at or below the guaranteed death benefit amount (potentially increased by a factor h), that is, if

$$A_t \leq G_{t+1} \cdot (1 + h). \quad (15)$$

Otherwise he pays only the base fee φ^{base} during that year. The numerical implementation is akin to the return-of-premium case, but includes adjustments to the annual fee rate based on Inequality (15).

Results

Table 3 displays optimization and valuation results for $h = 0$ and $h = 20\%$. In both cases, the state-dependent fee structure reduces the number of lapses considerably. In fact, for $h = 0$ the policyholder has virtually no incentive to lapse (see also Figure 3), which results in a very low aggregate fee rate. This makes the state-dependent fee structure an attractive option for insurers combating the lapse-and-reentry problem.

3.5 Additional Earnings Feature

In recent years some insurers have enhanced their basic GMDB rider with an Additional Earnings (AE) feature, sometimes also called Enhanced Earnings feature. Thereby the insurer increases the death benefit payout by a share θ_{AE} of the policyholder’s VA earnings (defined as the excess of the VA account value at death over the initial investment), capped by a maximum additional payout C_{AE} . In effect, while the basic GMDB rider materializes upon poor investment performance, the AE feature is valuable when the VA account value exceeds the initial investment, represented—even following lapse-and-reentry—by the guaranteed death benefit amount G_t . Therefore, upon death in policy year t , the policyholder receives additional earnings (at time t):

$$P_{AE}(A_t, G_t) = \min \{ \theta_{AE} \cdot \max\{A_t - G_t, 0\}, C_{AE} \}.$$

Since the AE feature acts like a call option on the VA account value (although modified by a

factor of θ_{AE} , and by a cap of C_{AE} , we can adjust our base-case implementation by adding

$$\theta_{AE} \cdot \left(\text{Call}(A_0, A_0, k + 1, \varphi) - \text{Call}\left(A_0, A_0 + \frac{C_{AE}}{\theta_{AE}}, k + 1, \varphi\right) \right)$$

to the right-hand side of Equation (4), and

$$\theta_{AE} \cdot \left(\text{Call}(A_t, G_{t+1}, 1, \varphi^{\text{agg}}) - \text{Call}\left(A_t, G_{t+1} + \frac{C_{AE}}{\theta_{AE}}, 1, \varphi^{\text{agg}}\right) \right)$$

to the right-hand side of Equation (6), as multiples of ${}_k|q_x$ and q_{x+t} , respectively. Thereby,

$$\text{Call}(S_0, K, T, \varphi) = S_0 e^{-\varphi T} \mathcal{N}(d_1) - K e^{-rT} \mathcal{N}(d_2)$$

denotes the Black-Scholes price of a European call option with parameters as defined in Equation (4).

Results

At present, to our knowledge, GMDB riders with an AE feature are offered by only a few insurance companies. They typically use earnings multipliers of $\theta_{AE} = 20\%$ or $\theta_{AE} = 40\%$, and a maximum cap of up to $C_{AE} = 250\%$ of the original investment. Table 3 and Figure 3 show that even a 40% earnings multiplier proves rather ineffective in preventing the policyholder from lapsing and reentering. However, we also observe that this is not purely a consequence of the AE feature itself but rather of the values chosen by the VA providers. For instance, when increasing the multiplier to $\theta_{AE} = 100\%$, the policyholder lapses around one time less on average over the course of the policy (5.03 vs. 6.08), which (moderately) reduces the aggregate break-even fee φ^{agg} (311.1 bps vs. 330.6 bps), despite the increased death benefit payout.

4 Comparing the Remedies

A simple measure of comparison between the various remedies is the portion of the initial investment that is ultimately paid back to the policyholder. From the $EPVB_0$ column in Table 3 we see that the ratchet feature has the highest value in that regard (\$85,060). The state-dependent fee structure is a close second (\$84,350). Both of them clearly outperform the traditional surrender schedule (\$77,340), although the latter also benefits the policyholder significantly. On the other hand, roll-ups and especially AE features prove much less successful in addressing the lapse-and-reentry problem.

However, the above conclusions are based on the assumption that policyholders are purely

interested in maximizing the risk-neutral expected value of their investment. In addition, one might rightfully wonder why the policyholder would purchase a financial product that will return to him at best 85% of his initial investment (in expected present value terms), and if the resulting optimal lapse behavior is indeed viable. To address these two concerns, we account for two sources that help shape the policyholder's purchase and—potentially—lapse decisions in the context of VAs: taxes and individual risk preferences.

4.1 Tax Treatment of Variable Annuities

In the US, earnings from standard financial instruments such as stocks and bonds are taxed when they are realized. Earned interest on bonds is taxed as ordinary income. Dividend payouts from stocks are typically taxed at the long-term capital gains tax rate. In addition, when investors sell their stocks, realized gains are taxed at the short-term or long-term capital gains tax rate, depending on the length of the investment. Similarly, mutual funds are required to pass on interest and dividend earnings as well as realized capital gains to their investors. As a result, a significant portion of earnings from investments in standard financial instruments are taxed each year, which reduces the growth of the investment.

This is not the case for VAs. Their preferential tax treatment has been one of the key drivers that made them such a popular investment vehicle in recent years. Specifically, investments in VAs grow tax-deferred, that is earnings are not taxed until they are withdrawn; upon withdrawal, however, they are treated as ordinary income. In addition, policyholders can take advantage of Section 1035 of US tax law which allows them to change VA policies without having to realize (taxable) earnings at that time. We model taxation of the VA with a constant marginal income tax rate τ on all VA earnings (that is, payout minus initial investment, if positive), assessed upon death or maturity of the policy.

Therefore, if the policyholder survives to maturity (time T), his terminal wealth is his VA account value at the time, net of taxes:

$$X_{T|\text{surv.}} = A_T - \tau \cdot \max\{A_T - A_0, 0\}.$$

On the other hand, if he dies in policy year t , his beneficiaries receive (at time t)

$$Y_t^{\text{gross}} = \max\{A_t, G_t\}$$

before taxes, which equals

$$Y_t^{\text{net}} = Y_t^{\text{gross}} - \tau \cdot \max\{Y_t^{\text{gross}} - A_0, 0\},$$

after taxes on earnings are assessed. To obtain a comparable time- T value, we assume that the death benefit amount is then invested into the risk-free asset until time T . In the process, interest—credited at continuous rate r —is taxed at the end of each policy year at income tax rate τ . Therefore, in the case of death in year t , the VA generates time- T net wealth

$$X_{T|t} = Y_t^{\text{net}} \cdot [e^r - \tau (e^r - 1)]^{T-t} .$$

4.2 Lapse-and-Reentry in a Utility-Based Framework

To account for the policyholder's risk preferences in a potentially incomplete market, we study the optimal lapse decisions in a utility-based framework. Thereby the policyholder maximizes his discounted expected utility of terminal (net) wealth. Let $u(\cdot)$ denote the policyholder's utility function, β his subjective (annual) discount rate, and B his bequest motive.

For a return-of premium GMDB rider, the policyholder's optimal control problem in the utility framework with taxes can be described and implemented similar to the value-maximization case in Section 2.3, by replacing Equation (5) with the terminal utility

$$V_T(A_T, G_T) = u(X_{T|\text{surv}}) ,$$

and using

$$\tilde{V}(t, A_t, G_{t+1}) = q_{x+t} \cdot [e^{-\beta(T-t+1)} \cdot B \cdot u(X_{T|t+1})] + (1 - q_{x+t}) \cdot e^{-\beta} \cdot \mathbb{E}^{\mathbb{P}} [V_{t+1}(A_{t+1}, G_{t+1})] ,$$

with

$$\begin{aligned} A_{t+1} &= A_t \cdot \exp \left[\mu - \varphi^{\text{agg}} - \frac{1}{2} \sigma^2 + \sigma (Z_{t+1} - Z_t) \right] , \quad \text{and} \\ Z_{t+1} - Z_t &\sim \mathcal{N}(0, 1) , \end{aligned}$$

in place of Equation (6). Proceeding recursively, we ultimately obtain the policyholder's time-0 discounted expected utility, denoted by U_0 , of the VA+GMDB investment:

$$U_0 = \tilde{V}(0, A_0, A_0) .$$

The insurer's valuation, in particular the calculations of the expected present values $EPVB(\cdot)$ and $EPVE(\cdot)$, continues to be taken out under the risk-neutral measure \mathbb{Q} , as outlined in Section 2.3. The modifications in the policyholder's valuation extend naturally to the various policy features under consideration, based on our descriptions in Section 3.

4.3 Parameter Specifications

For our numerical illustration we rely on the base case parameter specifications from Table 1 as well as the utility and tax parameter values summarized in Table 4. In addition, we assume $\mu = 8\%$ as mean growth rate of the stock.¹⁵

We assume that the policyholder exhibits constant relative risk aversion (CRRA) with utility function

$$u(X) = \frac{X^{1-\gamma}}{1-\gamma},$$

where γ is known as the coefficient of relative risk aversion. A number of experimental studies have been conducted to elicit this coefficient for various populations. Estimates vary—among other things—by geographic region, age, gender, education, and socio-economic standing. Based on lab experiments with undergraduate students, MBA students, and business school faculty, Holt and Laury (2002) find that the level of risk aversion is “centered around the 0.3–0.5 range, which is roughly consistent with estimates implied by behavior in games, auctions, and other decision tasks.” Using Hey and Orme (1994) data from lab experiments with US students, Harrison and Rustrom (2008) estimate a CRRA coefficient between 0.66 and 0.80. Harrison et al. (2007) conduct a field experiment on a representative sample of the Danish population and find an average relative risk aversion of 0.39 for individuals age 50 and above.¹⁶ Field experiments in developing countries find similar levels of risk aversion (Binswanger, 1981; Harrison et al., 2010; Tanaka et al., 2010, among others). Therefore we choose $\gamma = 0.5$ as our base case risk aversion coefficient. However, we also compare optimal lapse behavior to the case of a very risk averse policyholder ($\gamma = 5$).

Furthermore, we assume that the policyholder has subjective discount rate $\beta = \mu = 0.08$ and bequest motive $B = 1$ —though we separately test $\beta = 0$ and $B = 0.1$ as well. The marginal income tax rate varies by individual. Following Moenig and Bauer (2015) we assume $\tau = 30\%$ in the base case, but for comparison also consider a much lower tax rate of $\tau = 20\%$.

4.4 Results: Optimal Lapse Behavior with Risk Preferences and Taxes

Table 5 displays valuation results and lapse statistics for the utility-based model with taxes. Results are shown for the base case and sensitivity parameters stated in Table 4; however we omit them for the cases $B = 0.1$ and $\beta = 0$ since these are almost identical to the “base case” numbers.

Contrasting the values in Table 3 and Table 5 for the base case parameter values, we notice that

¹⁵This implies a Sharpe Ratio of 0.25, consistent with our analysis in Sections 2 and 3.

¹⁶Harrison et al. (2007) find an average risk aversion coefficient of 0.67, “weighted to reflect the Danish population”. Considering the *total* effect of key demographic variables, the authors find that being age 50 and above decreases the CRRA coefficient by 0.28 (see Table 4 in Harrison et al. (2007)). Both values are statistically significantly different from zero. In conclusion, the authors find that individuals age 50 and above have an average coefficient of relative risk aversion of 0.39.

Table 4: Parameter values for our numerical implementation of policyholder’s optimization problem in utility framework with taxes.

Description	Parameter	Values	
		Base Case	Sensitivity Analysis
Coefficient of relative risk aversion	γ	0.5	5
Subjective discount rate	β	0.08	0
Bequest motive	B	1	0.1
Income tax rate	τ	30%	20%

the policyholder’s risk preferences and the deferred tax treatment of VAs do not have a significant impact on his optimal lapse behavior. In fact, the break-even fees φ^{agg} are all within a few bps of the fees in the value-maximizing case. Moreover, comparing the discounted expected utility (U_0) for the various remedies leads us to the same conclusion as earlier: ratchet, state-dependent, and (to a lesser degree) surrender schedule are the policyholder’s preferred contract features.

Table 5 further shows that results are robust across utility and tax parameter specifications. Even a policyholder who is significantly more risk averse ($\gamma = 5$) or who faces a much lower income tax rate ($\tau = 20\%$) makes very similar lapse-and-reentry decisions. The latter is in stark contrast to findings from Moenig and Bauer (2015) in the case of withdrawal guarantees; there the amount withdrawn each year is taxed at the time of the withdrawal. In our study, however, the tax benefits of the VA investment are carried through the lapse-and-reentry process and materialize only when the policy “matures” (at age 80 or at death, whichever occurs first). Therefore the lapse-and-reentry decision is not directly impacted by tax considerations.

Moreover, the impact of β on the policyholder’s decision making is minimal because utilities are only assessed at time T , and then discounted back for the same amount of time. The same can be said for the effect of the bequest motive B since the policyholder’s lapse decision only impacts payouts in the death state.¹⁷ In effect, the policyholder aims to maximize his expected payout upon death, which is a scalar multiple of B and therefore the optimal lapse decision is invariant in B .

4.5 Comparison to a Benchmark Investment

In order to assess the subjective value of the VA investment to the policyholder, we consider a comparable outside investment. Thereby, at time $t = 0$ the initial amount A_0 is invested in the

¹⁷The exception to that is the state-dependent fee structure where lapses might result in a higher guarantee fee, which lowers the VA account value going forward. Even so, a substantial reduction in the value of B has only a very minor impact on lapse behavior and the resulting break-even (state-dependent) fee.

Table 5: Valuation results, lapse statistics, and discounted expected utility for select contract features. This table lists the aggregate break-even fee φ^{agg} for select contract features, broken down into the base fee φ^{base} , the pure guarantee fee φ^{guar} , and the lapse-and-reentry fee φ^{LR} . In addition to the return-of-premium GMDB rider, we consider the 7-year surrender schedule, the 2% roll-up, the ratchet, the state-dependent fee structure (with $h = 0$), and the AE feature with 40% earnings multiplier. Fees are quoted in bps. Furthermore, the table presents the division of the initial investment $A_0 = \$100,000$ into benefits ($EPVB_0$) and expenses ($EPVE_0$), as well as the average number of lapses over the course of the policy (Lapses). The last column (U_0) denotes the discounted expected utility that the policyholder receives from the respective VA investment (under tax considerations). Lapse decisions are made by a policyholder maximizing the expected utility of his terminal payout (under measure \mathbb{P} with $\mu = 8\%$). All fees and expected present values are computed under \mathbb{Q} . Lapse numbers are assessed under the real-world measure \mathbb{P} . Parameter values are consistent with the base case specifications in Table 1 and Table 4.

	φ^{base}	φ^{guar}	φ^{LR}	φ^{agg}	$EPVB_0$	$EPVE_0$	Lapses	U_0
<i>Base case</i>								
Return-of-premium	73.7	16.6	236.7	327.0	\$58,440	\$41,560	6.06	126.24
Surrender schedule	73.7	16.6	57.8	148.0	\$77,700	\$22,300	1.39	148.59
Roll-up	73.7	35.5	210.4	319.6	\$64,170	\$35,830	4.79	128.66
Ratchet	73.7	49.8	0	123.5	\$85,060	\$14,940	0	153.87
State-dependent	73.7	58.8	0.2	132.7	\$84,810	\$15,190	0.00	157.56
AE	73.7	36.0	215.0	324.7	\$58,850	\$41,150	5.86	135.03
$\gamma = 5^{\text{a}}$								
Return-of-premium	73.7	16.6	236.7	327.0	\$58,440	\$41,560	6.06	-40.17
Surrender schedule	73.7	16.6	61.3	151.6	\$77,190	\$22,810	1.47	-9.20
Roll-up	73.7	35.5	210.4	319.6	\$64,170	\$35,830	4.79	-37.83
Ratchet	73.7	49.8	0	123.5	\$85,060	\$14,940	0	-7.17
State-dependent	73.7	58.8	0	132.5	\$84,820	\$15,180	0	-7.61
AE	73.7	36.0	218.5	328.2	\$58,510	\$41,490	6.05	-35.85
$\tau = 20\%$								
Return-of-premium	73.7	16.6	236.7	327.0	\$58,440	\$41,560	6.06	131.15
Surrender schedule	73.7	16.6	57.8	148.1	\$77,700	\$22,300	1.39	155.38
Roll-up	73.7	35.5	210.4	319.6	\$64,170	\$35,830	4.79	133.82
Ratchet	73.7	49.8	0	123.5	\$85,060	\$14,940	0	161.17
State-dependent	73.7	58.8	0.2	132.7	\$84,810	\$15,190	0	165.11
AE	73.7	36.0	215.1	324.8	\$58,830	\$41,170	5.86	140.15

^a In the case of $\gamma = 5$, U_0 is equal to the stated value, times 10^{-19} .

Table 6: Utility for the benchmark investment. Discounted expected utility (U_0) is displayed for various capital gains tax parameters κ . All other parameter values are in line with base case assumptions of Tables 1 and 4.

	$\kappa = 20\%$	$\kappa = 23\%$	$\kappa = 25\%$
U_0	150.97	145.13	141.34

stock described by Equation (1).¹⁸ For simplicity, we assume that at the end of each policy year a constant marginal (“capital gains”) tax rate κ is applied to all earnings within that year. As before, the investment is liquidated when the policyholder reaches age 80. In the event that the policyholder dies prior to age 80, and consistent with our treatment of the death benefit payout from the VA, the investment is liquidated at the end of the year of death and invested entirely in a bond that matures at the original date of maturity of the VA. In either case, the investment provides utility to the policyholder (potentially through his beneficiaries) at that time, consistent with the utility framework for the VA policy.

Table 6 displays the time-0 utility to the policyholder from the benchmark investment vehicle, for various marginal capital gains tax rates (κ). In the US, state tax rates on long-term gains vary greatly. Following Moenig and Bauer (2015) we use $\kappa = 23\%$ as our base case capital gains tax rate (reflecting federal and state taxes), but also consider $\kappa = 20\%$ and $\kappa = 25\%$.

Contrasting Table 6 with the last column of Table 5, we find that for our base case tax parameters ($\tau = 30\%$, $\kappa = 23\%$) the VA plus GMDB rider is preferable (that is, has a higher initial utility U_0) to the benchmark investment for the state-dependent fee structure, for the ratchet feature, and with a surrender schedule.¹⁹ In particular, the fact that a VA with a plain GMDB rider and a 7-year surrender schedule yields a higher expected after-tax utility than a traditional stock investment may help justify the popularity of VAs as long-term investment vehicles.²⁰

Even for a lower capital gains tax rate ($\kappa = 20\%$), the ratchet and state-dependent cases are preferable to the benchmark investment. In contrast, the return-of-premium GMDB rider, the roll-up, and the AE feature have a lower utility than the benchmark investment even if returns on the latter are taxable at high rates ($\kappa = 25\%$).

¹⁸Alternative investment strategies can be implemented in similar fashion. However, for proper comparison with the VA, we assume that the benchmark investment is placed in the same stock as the VA.

¹⁹Our finding is in line with Milevsky and Panyagometh (2001) who show that in the long run the tax benefits of the VA outweigh the annual fees.

²⁰This relates to our earlier result (cf. Section 3.1) that the break-even fee we find for this rider ($\varphi^{\text{agg}} = 150.7$ bps) is in line with current market rates.

5 Conclusion

Policyholder behavior constitutes an important risk factor impacting the profitability of life insurance companies offering VA products with long-term guarantees. We show that in the presence of a GMDB rider the policyholder frequently takes advantage of the opportunity to lapse his VA policy and reenter the market at more favorable financial conditions. This increases the insurer's expenses and drives up the cost of the guarantee. However, we also show that the insurer can embed specific policy features that mitigate the lapse-and-reentry problem: the commonly employed surrender schedule works reasonably well, but can be approved upon by a ratchet-type guarantee or a state-dependent fee structure. Our findings are consistent across parameter specifications and invariant to the inclusion of risk preferences and taxes. Furthermore, we show that (largely due to its favorable tax treatment) a VA with a lapse-reducing GMDB feature gives the policyholder a greater utility than a comparable stock investment. While our analysis focuses on death benefit guarantees, its insights extend to simple living benefit guarantees (e.g. GMABs).

Since lapse-and-reentry is costly, and since that cost is ultimately borne by the policyholder, he has a clear preference for policy features that provide little incentive to lapse. Even though our pricing model assumes that the insurer breaks even in expectation, the insurer likely also prefers these features for reasons not captured in our model. For one, it is arguably easier to market a product that is beneficial to the customer. Moreover, the insurer favors these policy features from a risk-management perspective: Kling et al. (2014) show that frequent lapsing makes it more difficult for an insurer to hedge its risk exposure from a VA guarantee.

There are a number of interesting issues beyond the scope of the current paper. For instance, in line with our theoretical findings, Knoller et al. (2015) empirically identify “moneyness” as the main driver behind lapse decisions (for GMABs). However, the authors also find (weak) empirical evidence supporting the so-called *emergency fund hypothesis*, where policyholders lapse due to liquidity constraints (and do not reenter the market). If such behavior is indeed systematic, it may impact not only the break-even fee rate, but also the popularity of particular policy features: anticipating the possibility of a liquidity constraint, an investor might look unfavorably at policy features that make lapsing relatively costly. Lastly, some policy features provide a (on average) much larger payout to the policyholder upon death than upon survival to maturity. This may impact the product choice of policyholders with private information (or particular subjective beliefs) about their own mortality risk, and lead to adverse selection. We leave these topics for future research.

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