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Life insurance is a large yet poorly understood industry. A final death benefit is not paid for a majority of policies. Insurers make money on customers that lapse their policies and lose money on customers that keep their coverage. Policy loads are inverted relative to the dynamic pattern consistent with reclassification risk insurance. As an industry, insurers lobby to ban secondary markets despite the liquidity provided. These (and other) stylized facts cannot easily be explained by information problems alone. We demonstrate that a simple model of narrow framing, where consumers do not fully account for their need for future liquidity when purchasing insurance, offers a simple and unified explanation.
Narrow Framing and Life Insurance

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November 30, 2012

Abstract

Life insurance is a large yet poorly understood industry. A final death benefit is not paid for a majority of policies. Insurers make money on customers that lapse their policies and lose money on customers that keep their coverage. Policy loads are inverted relative to the dynamic pattern consistent with reclassification risk insurance. As an industry, insurers lobby to ban secondary markets despite the liquidity provided. These (and other) stylized facts cannot easily be explained by information problems alone. We demonstrate that a simple model of narrow framing, where consumers do not fully account for their need for future liquidity when purchasing insurance, offers a simple and unified explanation.

JEL No. D03, G22, G02

“I don’t have to be an insurance salesman!” — Tom Brady, NFL quarterback, describing the relief that he felt after finally being selected in Round 6, pick No. 199, of the 2000 NFL draft.¹

1 Introduction

Narrow framing states that when an individual evaluates a gamble “she does not fully merge it with her preexisting risk but, rather, thinks about it in isolation, to some extent; in other words, she frames the gamble narrowly” (Barberis, Huang, and Thaler 2006). There is a large experimental literature documenting narrow framing when making decisions under risk. Barberis et al. show that narrow framing can solve the Rabin calibration result and argue that it can potentially explain some paradoxes in finance, such as insufficient stock market participation, home bias, and the prevalence of households who allocate a large fraction of their wealth to very

¹See http://profootballtalk.nbcsports.com/2011/04/13/bradys-perceived-slap-against-insurance-salesmen-makes-waves/
A death benefit is not paid on most policies. For “term policies” that offer coverage over a fixed number of years, most are “lapsed” prior to the end of the term; a majority of permanent (e.g., “whole life”) policies are “surrendered” (i.e., lapsed and a cash value is paid) before death.

- Insurers make substantial amounts of money on clients that lapse their policies and lose money on those that do not. Insurers, however, do not earn extra-ordinary profits. Rather, lapsing policyholders cross subsidize households who keep their coverage.

- Real premiums decrease over time (i.e., policies are “front loaded”) rather than increasing with age in a manner more consistent with either actuarily fair pricing or optimal insurance in the presence of reclassification risk where new information about mortality risk is revealed.

- As an industry, insurers lobby intensely to restrict the operations of secondary markets. In other markets (e.g., initial public offerings or certificates of deposit), the ability to resell helps support the demand for the primary offering.

These stylized facts would not emerge from a standard insurance model with information asymmetries or insurance against reclassification risk. We show that narrow framing, however, can reconcile them in a fairly parsimonious manner.

In our model, consumers face two sources of risk: standard mortality risk that motivates the purchase of life insurance as well as other non-mortality “background” shocks that produce a subsequent demand for liquidity prior to death. Examples of background risks include unemployment, medical expenditure, stock market fluctuations, real estate prices, new consumption opportunities, and the needs of dependents. Consistent with narrow framing, while consumers correctly account for mortality risk when buying life insurance, they fail to sufficiently weight the importance of background risks. Risk neutral insurance firms, however, are fully aware of consumers’ narrow framing and price accordingly.

Relative to a rational expectations benchmark, where background shocks are properly incorporated into the insurance purchase decision, narrow framing produces more lapsing during the policy period after unanticipated shocks increase the need for liquidity. Sophisticated insurers, therefore, charge consumers a premium above the actuarily-fair rate early into the policy term, producing a subsequent surrender fee to the insurer upon lapsation. In exchange, consumers receive a smaller perceived present value of payments over the life of the policy assuming that it will be held to maturity. Since consumers do not anticipate the need to lapse, this front-loaded policy appears to be cheaper than a policy that is actuarily fair each period. (The presence

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2The term was introduced by Kahneman and Lovallo (1993), although the more general idea of “decision framing” was introduced earlier by Tversky and Kahneman (1981). See, for example, Redelmeier and Tversky (1992), Langer and Weber (2001), Rabin and Weiszäcker (2009), Eyster and Weizsacker (2011), and references therein for experimental work on narrow framing and Barberis and Huang (2008) for a survey of narrow framing applications in Finance.

3See ACLI 2011, Table 7.9. An additional $7.8 trillion is provided as group coverage through employers.
of paternalistic not-for-profit firms does not affect this result since consumers do not anticipate their future liquidity needs anyway.) Competition, however, forces insurers to earn zero economic profit, inclusive of surrender income. Hence, insurers do not earn any economic rents; rather, lapsing policyholders subsidize policyholders that hold to term. The introduction of a secondary market undermines this cross-subsidy by offering lapsing households better terms relative to surrendering. In the short run, insurers, therefore, lose money on existing contracts written to those households that do not experience background shocks.

Our formulation of narrow framing is consistent with several other lines of work in behavioral economics. Perhaps the closest is Gennaioli and Shleifer’s (2010) notion of “local thinking,” according to which inferences are drawn based on selected and limited samples. Individuals who think locally may underestimate the probability of residual hypotheses (“disjunction fallacy”). As in our model, someone who exhibits the disjunction fallacy overweights mortality risk at the expense of other shocks when purchasing life insurance. The introduction of a secondary market undermines this cross-subsidy by offering lapsing households better terms relative to surrendering. In the short run, insurers, therefore, lose money on existing contracts written to those households that do not experience background shocks.

The rest of the paper is organized as follows. The key motivating stylized facts summarized above are discussed in more detail in Section 2. Section 3 presents a model of a competitive life insurance market where consumers exhibit narrow framing. Section 4 further discusses how our model explains the key stylized facts. Section 5 concludes. Proofs as well as an extension of the “rational model” (without framing problems) are presented in various appendices.

2 Key Stylized Facts

This section reviews the key stylized facts of the life insurance industry.

2.1 Substantial Lapsation

For the most part, the life insurance industry sells a product that never pays. To be sure, by design, most property and casualty insurance policies do not make a payment either, but that is because the insured interest usually does not suffer an actual loss. In contrast, human death is a certain event. The majority of life insurance policies, however, “lapse” before a death benefit is paid.

The Society of Actuaries and LIMRA, a large trade association representing major life insurers, define an insurance policy lapse as “termination for nonpayment of premium, insufficient cash value or full surrender of a policy, transfer to reduced paid-up or extended term status, and in most cases, terminations for unknown reason” (LIMRA 2011A, P. 7). About 4.2% of all life insurance policies lapse each year, representing about 5.2% of the value actually insured (“face amount”). The degree of lapsation varies a bit, though, by the type of policy. For “term” policies,
which contractually expire after a fixed number of years if death does not occur, about 6.4% lapse each year. For “permanent policies,” which do not expire prior to death, the lapsation rate varies from 3.0% per year (3.7% on a face amount-weighted basis) for “traditional whole life” policies to 4.6% for “universal life” policies. So-called “variable life” and “variable universal life” permanent policies lapse at an even higher rate, equal to around 5.0% per year (LIMRA 2011A). While the majority of policies issued are permanent, the majority of face value now takes the term form (LIMRA 2011A, P. 10; ACLI 2011, P. 64).

These annualized rates of lapsation lead to substantial lapsing over the multi-year life of the policies. Indeed, $29.7 trillion of new individual life insurance coverage was issued in the United States between 1991 and 2010. However, almost $24 trillion of coverage also lapsed during this same period. According to Milliam USA (2004), almost 85% of term policies fail to end with a death claim; nearly 88% of universal life policies ultimately do not terminate with a death benefit claim. In fact, 74% of term policies and 76% of universal life policies sold to seniors at age 65 never pay a claim. (With some recent changes in regulatory oversight, it is likely that these lapse rates will actually increase in the future.) As we show below, substantial lapse is consistent with the narrow framing hypothesis, according to which background risk is not fully appreciated at the time of the insurance purchase.

## 2.2 Lapse-Supported Pricing

Insurers profit from policyholders who lapse. In the case of permanent insurance, policyholders build up a “cash value” that allows them to pay a fairly level premium over the life of the policy instead of a premium that rises with age (and, hence, mortality risk). Upon surrendering these contracts prior to death, the cash value paid to the policyholder is much smaller in present value than the premiums paid to date (correcting for the risk). Insurers keep the difference. For term contracts, no cash value is accumulated. But because mortality risk rises with age, the typical level nature of premiums over the fixed number of years means that the insurer still saves money if the policy is dropped.

It is widely recognized that insurers anticipate the subsequent profits from lapses when setting their premiums. For example, in explaining the rise in secondary markets (discussed below), the National Underwriter Company writes: “Policy lapse arbitrage results because of assumptions made by life insurance companies. Policies were priced lower by insurance companies on the assumption that a given number of policies would lapse.” (NUC 2008, P.88) Similarly, Dominique LeBel, actuary at Towers Perrin Tillinghast, defines a “lapse-supported product” as a “product where there would be a material decrease in profitability if, in the pricing calculation, the ultimate lapse rates where set to zero (assuming all other pricing parameters remain the same).” (Society of Actuaries 2006)

Precisely measuring the effects of lapses on initial premium pricing is challenging since insurers do not report the underlying numbers. One reason is regulatory: for determining the insurer’s reserve requirements, the historic NAIC “Model Regulation XXX” discouraged reliance on significant income from lapses for those policies surviving a certain threshold of time. A second

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8 Of course, some of the lapses represent policies issued prior to 1991, which explains why there is still almost $10 trillion of policies in force today.

9 While term policies have a larger annual lapse rate, permanent policies are usually more likely to lapse over the actual life of the policy due to their longer duration.

10 Most recently, principles-based regulations (PBR) have emerged, which are widely regarded to allow for more consideration of policy lapsation for purposes of reserve calculations.
motivation is competitive: insurers are naturally tight-lipped about their pricing strategies.

Nonetheless, there is significant evidence of lapse-supported pricing from various sources. First, like economists, actuaries employed by major insurers give seminars to their peers. The Society of Actuaries 2006 Annual Meetings held a session on lapse-supported pricing that included presentations from actuaries employed by several leading insurance companies and consultants. Kevin Howard, Vice President of Protective Life Insurance Company, for example, demonstrated the impact of lapsation on profit margins for a representative male client who bought a level premium secondary guarantee universal life policy, with the premium set equal to the average amount paid by such males in August 2006 in the company’s sample. Assuming a zero lapse rate, the insurer projected a substantial negative profit margin, equal to -12.8%. However, at a typical 4.0% lapse rate, the insurer’s projected profit margin was +13.6%, or a 26.4% increase relative to no lapsation.11

Similarly, at the 1998 Society of Actuaries meeting, Mark Mahony, marketing actuary at Transamerica Reinsurance, presented calculations for a large 30-year term insurance policy often sold by the company. The insurer stood to gain $103,000 in present value using historical standard lapse rate patterns over time. But, with lapsation turned off and no change in other parameter assumptions, the insurer was projected to lose $942,000 in present value. He noted: “I would highly recommend that in pricing this type of product, you do a lot of sensitivity testing.” (Society of Actuaries 1998, p. 11)

In order to provide a clear picture on lapse-supported pricing, we also gathered data on life insurance policies offered by nine large U.S. insurers. In projecting their profits, we used the most recent “ultimate” mortality tables that are based on actual experience. These tables are used by insurers for regulatory reporting purposes and are intended to correct for adverse selection relative to the general population. Our calculations are discussed in more detail in Appendix A. The results confirm a considerable reliance on lapse income.

Consider, for example, a standard 30-year term policy with $750,000 in coverage that is purchased by a 50-year male non-smoker under a projected annual nominal interest rate of seven percent (Figure 1). These major life insurers are projected to earn between $3,250 and $4,070 in actuarial present value if the consumer surrenders within the fifth and the tenth years of purchasing insurance. However, these insurers are projected to lose between $32,500 and $34,060 if the consumer does not surrender. The “break-even lapse rate” – that is, the constant rate of lapsation that gives the insurer a zero expected economic profit – equals roughly 12% per year, which, if anything, is actually a bit larger than the actual lapse rates for term policies reported earlier.12 (These results are fairly robust to changes in various assumptions, including interest rates.) Thus, it appears that insurers rely heavily on lapses when making their pricing decisions and are not making economic rents over their entire book of business: rather, policyholders who lapse essentially cross-subsidize policyholders who hold their policies longer, thereby suggesting a competitive market.

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11 For less popular single premium policies, the swing was lower, from -6.5% to +8.7%.
12 As noted in the text, our calculations are based on publicly-available “ultimate” life tables that insurers use for estimating their required reserve requirements for regulatory purposes. However, in practice, insurers base their pricing decisions on more aggressive life tables, using proprietary actuarial models. Moreover, several industry experts told us confidentially that they believe that LIMRA statistics under-report the actual amount of lapsation since insurers that experience large amounts of lapsing choose to not disclose their numbers to the trade organization. Hence, the LIMRA lapse values should be viewed as lower bounds. Additionally, our computations are conservative in that we do not include administrative and marketing costs, which would further increase the break-even lapse rate.
Incidentally, a third source of evidence for lapse-based pricing comes bankruptcy proceedings, which often force a public disclosure of pricing strategies in order to determine the fair distribution of remaining assets between permanent life policyholders with cash values and other claimants. For example, the insurer Conesco relied extensively on lapse-based income for their pricing; they also bet that interest rates earned by their reserves would persist throughout their projected period. Prior to filing for bankruptcy, they attempted to increase premiums – in fact, tripling the amounts on many existing customers – in an attempt to effectively reduce the cash values for their universal life policies. In bankruptcy court, they rationalized their price spikes based on two large blocks of policies that experienced lower-than-expected lapse rates (InvestmentNews 2011). Bankruptcy proceedings have revealed substantial lapse-based pricing in the long-term care insurance market as well (Wall Street Journal 2000); most recently, several large U.S. long-term care insurers dropped their coverage without declaring bankruptcy, citing lower-than-expected lapse rates, which they originally estimated from the life insurance market (InvestmentNews 2012).

![Figure 1: Insurance company’s projected profits when the consumer surrenders after N years (if still alive). Authors’ calculation from 30-year term policies with $750k coverage for a 50-year old 5’10” male non-smoker who weights 165 pounds.](image)

In Canada, life insurance policies are also supported by lapsing. As A. David Pelletier, Executive Vice President of RGA Life Reinsurance Company, argues:

> What companies were doing to get a competitive advantage was taking into account these higher projected future lapses to essentially discount the premiums to arrive at

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13Premiums for universal life type of permanent policies can be adjusted under conditions outlined in the insurance contract, usually pertaining to changes in mortality projections. However, in this case, the bankruptcy court ruled that the Conesco contract did not include provisions for adjusting prices based on lower interest rates or lapse rates. Conesco, therefore, was forced into bankruptcy.

14See, for example, Canadian Institute of Actuaries (2007).
a much more competitive premium initially because of all the profits that would occur later when people lapsed. (Society of Actuaries 1998, p. 12)

But why do policies with cheap premiums and high lapse fees, which cross-subsidize policyholders who do not lapse, give companies such a competitive advantage? In our model, lapse-supported pricing and cross-subsidization from lapsing to non-lapsing policyholders are naturally induced by competition over consumers who frame their risks narrowly. Insurers compete on policy premiums, which are fully internalized by consumers at the time of the purchase. Since lapse income is anticipated by insurers but not by policyholders at the time of purchase (narrow framing), competitive policy premiums fall below the actuarially fair level for those policyholders who hold to maturity.\(^{15}\)

### 2.3 An Inverted Pattern of Policy Loading

Both term and permanent policies are effectively “front loaded” since the level premium is larger than the mortality probability at the time of purchase. This wedge decreases over time as the mortality probability increases with the age of the policyholder. Inflation reinforces the front-loading feature since the premium is typically level in nominal terms. Loadings, therefore, start high and decrease over time (see Figure 2).

![Insurance Loads](image.png)

**Figure 2**: Insurance loads in current dollars under a projected three percent inflation rate (same policies as in Figure 1).

To be sure, policy loads may play an important role, even in a competitive market with “fully rational” consumers. An interesting recent literature – see Hendel and Lizzeri (2003), Daily, Hendel and Lizzeri (2008), and Fang and Kung (2010) – shows how policy loads help enforce continued participation of each member of the insurance pool in the presence of “reclassification risk” where individual policyholders learn more about their mortality likelihood over time. Without a load, policyholders who enjoy a favorable risk reclassification – that is, an increased conditional

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\(^{15}\)In principle, fixed underwriting costs could explain why insurance companies charge front loaded premiums. However, as Hendel and Lizzieri (2003) point out, this hypothesis predicts that front loads would decrease with the face amount of a policy (since the fixed cost as a proportion of the face amount decreases). But, for none of the companies in their sample was the ratio of front load to face amount decreasing. Therefore, fixed underwriting costs cannot account for the front loads in life insurance policies.
life expectancy – will want to drop from the existing risk pool and re-contract with a new pool, thereby undermining much of the benefit from intertemporal risk pooling. Ex-ante identical policyholders, therefore, can increase their welfare by contracting on a dynamic load that punishes those who leave the pool.

If reclassification is the only relevant risk and consumers can borrow, then the load will be constructed to be sufficiently large to prevent any lapsing. With a second “background” risk, such as a liquidity shock, a positive amount of lapsing will occur in equilibrium if rational policyholders now value the ex-ante the option to lapse after a sufficiently negative background shock. For shorthand, we will refer to this setting as the “rational model” since agents are fully informed of (and act upon) both types of shocks.\(^\text{16}\)

A plausibly calibrated rational model, though, faces some challenges in explaining the observed pattern of loadings. Empirically, younger policyholders are mostly subject to non-health related shocks. Appendix D, for example, summarizes a few “snap shots” across different ages of the five-year ahead Markov health transition matrices that are based on the estimates of Robinson (1996).\(^\text{17}\) Younger healthy people are quite likely to remain healthy; health shocks only become material at older ages. Consistently, and more directly related to our context, Fang and Kung (2012) find that younger households tend to lapse for idiosyncratic reasons while health-related shocks play an increasing role at older ages. Figure 3 presents the lapsation rates by age based on data from eleven major life insurance companies in Canada. Young policyholders, who are more likely experience liquidity shocks and less likely to experience health shocks, lapse almost three times more often than older policyholders.

Figure 3: Lapsation rates per policy year by age; 15-year duration policies. Source: Canadian Institute of Actuaries (2007).

The rational model then counterfactually predicts that policies should be back loaded. The reason is that positive loads exist to penalize agents who drop out due to favorable health shocks, thereby ensuring that the pool remains balanced. Charging positive loads for non-health related

\(^\text{16}\)Hendel and Lizzerei (2003) present a model in which there are health shocks only. They show that, in the absence of credit markets, front loads are set according to a trade-off between reclassification risk and consumption smoothing. Daily, Hendel, and Lizzerei (2008) and Fang and Kung (2010) extend this model by incorporating a bequest shock, according to which policyholders lose all their bequest motives.

\(^\text{17}\)See also Jung (2008).
shocks, therefore, is inefficient, as they exacerbate the agent's demand for money and undermine the amount of insurance provision. Since the importance of health-related shocks increases with age, we should expect policy loads to increase (in real value) as people age, contrary to the observed decreasing pattern (Figure 2). Moreover, in the presence of liquidity and health shocks, insurance companies should then lose money on policies that lapse early on. In contrast, the empirical evidence presented earlier demonstrates that insurers made considerable profits on policies that lapse. We prove these results formally in Appendix C, where we extend the rational models of Hendel and Lizzeri (2003), Daily, Hendel, and Lizzeri (2008), and Fang and Kung (2010) by adding an initial period in which consumers are subject to an unobservable liquidity shock. Consumers are then subject to liquidity shocks in the first period, health shocks in the second period, and mortality risk in the third period—a stylized representation of the fact that health shocks are considerably more important later in life.

In contrast, front loaded policies, in which the insurance company profits from consumers who lapse early on, naturally emerge in a model of narrow framing. Because consumers do not take their future liquidity needs into account when buying insurance, competitive policies provide generous coverage to those that do not suffer a liquidity shock at the expense of those that do. Moreover, front loaded premiums increase the policyholder's need for money after a liquidity shock, which allows firms to charge even higher surrender fees. We show that the presence of paternalistic not-for-profit firms does not affect the competitive equilibrium since consumers do not anticipate their future liquidity needs when choosing which policy to buy.

2.4 Opposition to Secondary Markets

Rather than lapsing, a policyholder could instead sell the policy to a third party on the secondary market. This type of contract is called a “life settlement.” In a typical arrangement, the third-party agent pays the policyholder a lump-sum amount immediately and the third party continues to make the premium payments until the policyholder dies. In exchange, the policyholder assigns the final death benefit to the third party. As the National Underwriter Company 2008 writes: “Life settled policies remain in force to maturity causing insurers to live with full term policy economics rather than lapse term economics. This results in an arbitrage in favor of the policyholder when a policy is sold as a life settlement.” (P. 88)

Thus far, the life settlement market is in its infancy. Only $38 billion of life insurance policies were held as settlements as 2008, representing just 0.30% of the $10.2 trillion of individual in-force life insurance in 2008 in the United States, and only 0.34% of total lapses between 1995 through 2008 (Corning Research and Consulting 2009). Nonetheless, the life insurance industry has waged an intense lobbying effort aimed at state legislatures, where life insurance is regulated in the United States, to try to ban life settlement contracts. On February 2, 2010, the American Council of Life Insurance, representing 300 large life insurance companies, released a statement asking policymakers to ban the securitization of life settlement contracts. Life insurance industry organizations have also organized media campaigns warning the public and investors about life settlements. The opposition to life settlements is a bit surprising at first glance. In a standard rational model, the resale option provided by a secondary market should increase the initial de-

\[^{18}\text{In Daily, Hendel, and Lizzeri (2008) and Fang and Kung (2010), individuals live for two periods and are subject to both a health shock and a bequest shock in the first period. In Appendix C, we study the temporal separation of shocks, capturing the idea that non-health shocks are relatively more important earlier in life and health shocks are more important later in life.}\]
mand for life insurance. The market for initial public offerings, for example, would be substantially smaller without the ability to resell securities.

Daily, Lizzeri and Handel (2008) and Fang and Kung (2010) show that secondary markets could undermine dynamic pooling in the presence of reclassification risk in which policyholders receive updates about their mortality risks over time. In their models, consumers are subject to health and bequest shocks. Since policyholders value the option of lapsing for non-reclassification (bequest) purposes, the policy load in equilibrium is smaller than with reclassification risk alone, thereby producing some inefficient reclassification lapsing as well. The secondary market, by offering fairer payouts on lapses, further reduces the competitive size of the load, thereby producing even more inefficient lapsing.19

Figure 4: Lapsation of permanent insurance policies by time held. Source: LIMRA (2011).

However, in their companion empirical paper analyzing the Health and Retirement Survey, Fang and Kung (2012) find that lapsation by younger households appear to be mostly driven by idiosyncratic shocks; health and bequest motives become more important with age. Their evidence seems to be consistent with more anecdotal evidence from other sources. At the aggregate level, income shocks, in particular, appear to be an especially important background shock. Lapse rates spike during times of recessions, high unemployment, and increased poverty. For example, while $600B of coverage was dropped in 1993, almost $1 trillion was dropped in 1994 (a year with record poverty) before returning to around $600B per year through the remainder of the decade. After the 2000 stock market bubble burst, over $1.5 trillion in coverage was forfeited, more than double the previous year; interestingly it appears that lapse rates are now permanently higher after 2000 (ACLI 2011). At the household level, as shown in Figure 4, almost 25% of permanent insurance policyholders stop making premium payments within just the first three years after first purchasing the policies; within 10 years, 40% have lapsed. It is unlikely that concern for a beneficiary, which tends to be more predictable in nature, would change so fast, suggesting a greater weight on liquidity needs. Moreover, as shown in Figure 5, lapses also tend to be prevalent

19In practice, liquidity constraints are enhanced by the structure of level (or one-time) premiums, a simple structure, not conditional on age, that appears to be encouraged by state regulators for estimating required capital reserves. Indeed, level (or single up-front) premiums are the norm.
for smaller policies, which are typically purchased by households with lower income who are more subject to liquidity shocks. In their conceptual paper, Doherty and Singer (2003) suggest that the lobby of life insurance companies against the life settlement market is an attempt to protect their ex-post monopoly power. They do not, however, address why consumers are willing to give insurance companies such an ex-post power in the first place within a highly competitive environment. In our model with narrow framing, consumers are willing to grant ex-post monopoly power to insurance companies because they underestimate the likelihood that they will need to resell their policies in the future. In a competitive equilibrium, lapsing shareholders cross-subsidize policyholders that hold to term. The introduction of a secondary market undermines this cross-subsidy and so insurers lose money on existing contracts written to those households that do not experience background shocks.

Figure 5: Lapsation of permanent insurance policies by size of policy and time held. Source: LIMRA (2011).

3 The Model

We consider a competitive life insurance market in which consumers frame risks narrowly. The model explains each of the stylized facts discussed previously. There are $N \geq 2$ insurance firms indexed by $j = 1, \ldots, N$ and a continuum of households. Each household consists of one head and at least one heir. Because household heads make all decisions while alive, we refer to them as “the consumers.” Consumers are subject to mortality and background risk (“income” or “liquidity” shocks).

3.1 The Timing of the Game

There are three periods: 0, 1, and 2. Period 0 corresponds to the contracting stage. Firms offer insurance policies and consumers decide which one to purchase (if any). Consumption occurs in periods 1 and 2. In period 1, consumers suffer an income loss of $L > 0$ with probability $l \in (0, 1)$. 

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Firms do not observe income losses. In period 2, each consumer dies with probability \( \alpha \in (0, 1) \). Consumers earn income \( I > 0 \) if alive.

To closely examine the role that surrendering plays in providing liquidity, we assume that any other assets that consumers may have are fully illiquid and, therefore, cannot be rebalanced after an income shock. While this is an extreme assumption which greatly simplifies the analysis, our results still go through if part of the investment in assets could be reallocated. All we require is some liquidity motivation for surrendering, consistent with the empirical evidence noted earlier.\(^{20}\)

For notational simplicity, we assume that there is no discounting.

![Figure 6: Timing of the model](image)

An insurance contract is a vector of state-contingent payments

\[
T_j \equiv (t_{1,j}^S, t_{1,j}^{NS}, t_{A,j}^S, t_{D,j}^S, t_{A,j}^{NS}, t_{D,j}^{NS}) \in \mathbb{R}^6,
\]

where \( t_{1,j}^S \) and \( t_{1,j}^{NS} \) are payments in period 1 when the consumer does and does not suffer the income shock, respectively. The terms \( t_{A,j}^S, t_{D,j}^S, t_{A,j}^{NS} \), and \( t_{D,j}^{NS} \) are payments in period 2 when the individual is alive (A) or dead (D) conditional on whether (S) or not (NS) the individual suffered an income shock in period 1. A natural interpretation of these state-contingent payments is as follows. Consumers pay an amount \( t_{1,j}^S \) for insurance in period 0. In period 1, they choose whether or not to surrender the policy. If they do not surrender, the insurance company repays \(-t_{1,j}^{NS}\) if they survive and \(-t_{1,j}^S\) if they die at \( t = 2 \). If they surrender the policy, the insurance company pays a surrender value of \( t_{A,j}^S - t_{D,j}^{NS} \) in period 1. Then, they get paid \(-t_{A,j}^S\) if they survive and \(-t_{D,j}^{NS}\) if they die at \( t = 2 \).

More specifically, the timing of the game is as follows:

**t=0:** Each firm \( j \in \{1, ..., N\} \) offers an insurance contract \( T_j \). Consumers decide which contract to accept (if any). Consumers who are indifferent between more than one contract randomize between them with strictly positive probabilities.

**t=1:** Consumers have initial wealth \( W > 0 \) and lose \( L > 0 \) dollars from a background risk with probability \( l \in (0, 1) \). They choose whether or not to surrender the policy (or, more formally, report a loss to the insurance company). Consumers pay \( t_{1,j}^{NS} \) if they do not surrender and \( t_{1,j}^S \) (possibly negative) if they do.

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\(^{20}\)The assumptions of no consumption in period 0 and the temporal precedence of income uncertainty to mortality uncertainty are for expositional simplicity only. None of our results change if we allow for consumption in period 0 and if we introduce income and mortality shocks in all periods. Although not included in this paper for the sake of space, we have also proven that several of the key results derived herein, including cross-subsidization, persist even in the presence of perfectly liquid outside assets provided that consumers are prudent (a positive third derivative of the utility function). This analysis is available from the authors. In line with our illiquidity assumption, Daily, Lizzeri and Handel (2008) and Fang and Kung (2010) assume that no credit markets exist in order to generate lapsation.
t=2: Consumers die with probability $\alpha \in (0, 1)$. If they are alive, they earn an income $I > 0$; if dead, they make no income. The household of a consumer who purchased insurance from firm $j$ and surrendered at $t = 1$ receives at $t = 2$ the amount $-t_{A,j}^{S}$ if he survives and $-t_{D,j}^{S}$ if he dies. If he did not surrender at $t = 1$, his household instead gets $-t_{A,j}^{S}$ if he survives and $-t_{D,j}^{S}$ if he dies.

Notice that we model surrendering a policy after an income shock as the report of the shock to the company (and the readjustment in coverage that follows). Therefore, we follow the most standard approach in contract theory by assuming two-sided commitment. Nevertheless, our results persist if we assume that only insurers are able to commit. Also, notice that we do not include health shocks in the model. Our intent is to show how narrow framing with income shocks alone can explain the key stylized facts in the life insurance market. We, therefore, do not want to complicate the analysis with additional shocks, which would not overturn our main conclusions.²¹

### 3.2 Consumer Utility

The utility of household consumption when the consumer is alive and dead is represented by the strictly increasing, strictly concave, and twice differentiable functions $u_{A}(c)$ and $u_{D}(c)$, satisfying the following Inada conditions: $\lim_{c \to 0} u'_{A}(c) = +\infty$ and $\lim_{c \to 0} u'_{D}(c) = +\infty$. The utility received in the dead state corresponds the “joy of giving” resources to survivors.

Because other assets are illiquid, there is a one-to-one mapping between state-contingent payments and state-contingent consumption bundles $C_{j} \equiv \left(c_{1,j}^{S}, c_{1,j}^{NS}, c_{A,j}^{S}, c_{A,j}^{NS}, c_{D,j}^{S}, c_{D,j}^{NS}\right)$.²² Thus, there is no loss of generality in assuming that a contract specifies a vector of state-contingent consumption rather than state-contingent payments.

In period 1, consumers decide whether or not to report an income shock. Because companies do not observe income shocks, insurance contracts have to induce consumers to report them truthfully. Those who suffer the shock report it truthfully if the following incentive compatibility constraint is satisfied:

$$u_{A}(c_{1,j}^{S}) + \alpha u_{D}(c_{D,j}^{S}) + (1 - \alpha)u_{A}(c_{A,j}^{S}) \geq u_{A}(c_{1,j}^{NS} - L) + \alpha u_{D}(c_{D,j}^{NS}) + (1 - \alpha)u_{A}(c_{A,j}^{NS}). \quad (1)$$

In words: The expected utility from surrendering must be weakly larger than without surrendering and simply absorbing the loss. Similarly, those who do not suffer the shock do not report one if the following incentive compatibility constraint holds:

$$u_{A}(c_{1,j}^{NS}) + \alpha u_{D}(c_{D,j}^{NS}) + (1 - \alpha)u_{A}(c_{A,j}^{NS}) \geq u_{A}(c_{1,j}^{S} + L) + \alpha u_{D}(c_{D,j}^{S}) + (1 - \alpha)u_{A}(c_{A,j}^{S}). \quad (2)$$

Our key assumption is that consumers do not take background risk into account when purchasing life insurance in period 0. Formally, they attribute zero probability to suffering an income shock at the contracting state. As discussed previously, this assumption is consistent with the idea that individuals bracket risks narrowly and, therefore, do not integrate all risks when making each decision. Since the main purpose of life insurance is to protect against mortality risk, it is natural to assume that consumers who frame risks narrowly focus on mortality risk without

²¹In Appendix C, we include both health and liquidity shocks in the rational model where agents do not narrowly frame their risk decisions.

²²State-contingent consumption is determined by $c_{1,j}^{NS} \equiv W - t_{1,j}^{NS}$, $c_{A,j}^{NS} \equiv I - t_{A,j}^{NS}$, $c_{D,j}^{NS} \equiv -t_{A,j}^{NS}$, $c_{1,j}^{S} \equiv W - L - t_{1,j}^{S}$, $c_{A,j}^{S} \equiv I - t_{A,j}^{S}$, and $c_{D,j}^{S} \equiv -t_{D,j}^{S}$.
merging it with other preexisting (background) risk. As a result, consumers evaluate contracts in period 0 according to the following expected utility function that only includes states in which shocks do not occur:

\[ u_A(c_1^{NS}) + \alpha u_D(c_D^{NS}) + (1 - \alpha) u_A(c_A^{NS}). \]

We will refer to this expression as the consumers’ “perceived expected utility.”

### 3.3 Firms Profits

Each firm’s expected profit from an insurance policy equals the expected net payments from the consumer, which, expressing in terms of consumption, equals the sum of expected reported income minus the sum of expected consumption. Conditional on not surrendering, the sum of expected income equals \( W + (1 - \alpha) I \), whereas expected consumption equals \( c_1^{NS} + \alpha c_D^{NS} + (1 - \alpha) c_A^{NS} \). Similarly, conditional on surrendering, the sum of expected income equals \( W - L + (1 - \alpha) I \) and the sum of expected consumption is \( c_1^S + \alpha c_D^S + (1 - \alpha) c_A^S \).

### 3.4 Equilibrium

We study the subgame perfect Nash equilibria (“SPNE”) of the game. Because consumers do not take the income shock into account when choosing which policy to accept, any offer that is accepted must maximize the firm’s expected profits following an income shock subject to consumers not having an incentive to misreport the shock. Formally, for a fixed profile of consumption in the absence of an income shock \( (c_1^{NS}, c_A^{NS}, c_D^{NS}) \), firms will offer policies that maximize their profits subject to the incentive compatibility constraints (1) and (2). Let \( \Pi \) denote the maximum profit they can obtain conditional on the income shock:

\[ \Pi(c_1^{NS}, c_A^{NS}, c_D^{NS}) \equiv \max_{c_1^{NS}, c_A^{NS}, c_D^{NS}} W - L - c_1^S - \alpha c_D^S - (1 - \alpha) (c_A^S - I) \]

subject to (1) and (2).

Constraint (1) must bind (otherwise, it would be possible to increase profits by reducing \( c_1^S \), \( c_D^S \), or \( c_A^S \)). Therefore, (2) can be rewritten as

\[ u_A(c_1^{NS}) \geq \frac{u_A(c_1^{NS} + L) + u_A(c_1^{NS} - L)}{2}, \]

which is true by the concavity of \( u_A \). Thus, incentive compatibility constraint (2) does not bind.

In period 0 before income shocks are realized, firms are willing to offer an insurance policy as long as they obtain non-negative expected profits. Price competition between firms implies that they will offer policies that maximize the consumers’ perceived expected utility among policies that give zero profits:

\[ \max_{c_1^{NS}, c_A^{NS}, c_D^{NS}} u_A(c_1^{NS}) + \alpha u_D(c_D^{NS}) + (1 - \alpha) u_A(c_A^{NS}) \]

subject to \( \Pi(c_1^{NS}, c_A^{NS}, c_D^{NS}) + (1 - l) [W - c_1^{NS} - \alpha c_D^{NS} - (1 - \alpha) (c_A^{NS} - I)] = 0. \)

Lemma 1 establishes this result formally:

**Lemma 1.** A set of state-dependent consumption \( \{C_j\}_{j=1,...,N} \) and a set of acceptance decisions is an SPNE of the game if and only if:
1. At least two offers are accepted with positive probability,
2. All offers accepted with positive probability solve Program (3),
3. All offers that are not accepted give consumers a perceived utility lower than the solutions of Program (3).

We say that the equilibrium of the game is **essentially unique** if the set of contracts accepted with positive probability is the same in all SPNE (since firms get zero profits in equilibrium, there always exist equilibria in which some firms offer “unreasonable” contracts that are never accepted when there are more than two firms). An SPNE of the game is **symmetric** if all contracts accepted with positive probability are equal: if $C_j$ and $C'_{j'}$ are accepted with positive probability, then $C_j = C'_{j'}$.

The following lemma establishes existence, uniqueness, and symmetry of the SPNE:

**Lemma 2.** There exists an SPNE. Moreover, the SPNE is essentially unique and symmetric.

Because the equilibrium is symmetric, we will omit the index $j$ from contracts that are accepted with positive probability in equilibrium. The following proposition presents the main properties of the equilibrium contracts:

**Proposition 1.** In the essentially unique SPNE, any contract accepted with positive probability has the following properties:

1. $u'_A(c^{S}_1) = u'_D(c^S_D) = u'_A(c^S_D)$,
2. $u'_D(c^{NS}_D) = u'_A(c^{NS}_A) < u'_A(c^{NS}_1)$, and
3. $\pi^S > 0 > \pi^{NS}$.

Condition 1 states that there is full insurance conditional on the income shock occurring. Since insurance companies maximize profits conditional on the income shock subject to leaving the consumers a fixed utility level (incentive compatibility), the solution must be on the Pareto frontier conditional on the shock, thereby equating the marginal utility of consumption in all states after the income shock.

The equality of Condition 2 states that the consumer is fully insured with respect to the mortality risk conditional on the absence of an income shock. Because the equilibrium maximizes the consumer’s utility given zero expected profits to the firm, both consumers and firms are fully aware of the risk of death. Since consumers are risk averse and companies are risk-neutral, firms fully insure the mortality risk. However, the inequality of Condition 2 shows that the insurance policy also induces excessive saving, relative to efficient consumption smoothing that equates the marginal utility of consumption in all periods. Intuitively, shifting consumption away from period 1 increases the harm of the income loss if it were to occur, thereby encouraging consumers to surrender their policies and produce more profits for firms after an income shock. More formally, the excessive savings result follows from the incentive compatibility constraint after an income shock: shifting consumption from period 1 to period 2 reduces the opportunity cost to surrendering. Interestingly, since consumers are fully aware of the no-shock dynamics of the model, this inefficiency is observable to consumers in period 0 when they purchase insurance. Nevertheless, it survives competition because any firm that would attempt to offer a contract that
smooths inter-temporal consumption would be unable to price it competitively since consumers do not believe they will surrender their policies in period 1.

Condition 3 states that firms obtain a strictly positive profit if the consumer surrenders the policy and a strictly negative shock if he does not. Recall, however, that the insurer’s program (3) is solved to produce zero expected profits in equilibrium. Hence, the profits obtained after an income shock are competed away by charging a smaller, cross-subsidized price to policyholders who do not experience an income shock and, therefore, hold their policies to term.

3.5 Efficiency

From a normative perspective, we find it reasonable to use the correct distribution of income shocks when evaluating consumer welfare. Therefore, we say that an allocation is efficient if there is no other allocation that increases the expected utility of consumers (evaluated according to true probability distribution over states of the world) and does not decrease the expected profit of any firm.

Because consumers are risk averse and insurance companies are risk neutral, any efficient allocation should produce constant consumption across all states (full insurance). The equilibrium of the model is inefficient in two ways. First, because the marginal utility of consumption increases after the shock, there is incomplete insurance with respect to the income shock. Of course, this source of inefficiency is standard in models with unobservable income shocks. However, narrow framing further exacerbates the effect of income shocks by transferring consumption from the shock state (where marginal utility is high) to the no-shock state (where marginal utility is low). Second, because consumption is increasing over time when there is no income shock, there is incomplete intertemporal consumption smoothing. This second source of inefficiency is not standard and is produced by consumers narrowly framing their insurance purchase decisions.

3.6 Nonprofit Firms

The model discussed previously assumed that all firms maximize profit. However, several life insurance firms are “mutuals” that, in theory, operate in the best interests of their customers, essentially like nonprofits. The equilibrium of the model is robust to the presence of these sorts of firms as well.

Formally, suppose there are $N \geq 2$ firms, at least one of them being “for profit,” and at least one of them being “paternalistic.” As usual, a for-profit firm maximizes its profits. A paternalistic firm offers contracts that maximize their consumers’ “true expected utility” as long as the firm obtains non-negative profits. Recall that in the (essentially unique) equilibrium of the model without paternalistic firms, contracts that are accepted with a positive probability maximize the consumer’s perceived expected utility subject to the firm getting zero profits. Because accepted contracts are unique, any different contract that breaks even must give the consumer a strictly lower perceived utility and will, therefore, not be accepted. As a result, the model with paternalistic firms has exactly the same essentially unique equilibrium. The presence of a single for-profit firm is enough to ensure that the equilibrium is inefficient. This outcome, therefore, is an example of a market in which competition may magnify biases.\(^{23}\)

\(^{23}\)When there are only for-profit firms, there exist a continuum of equilibria ranked by the Pareto criterion (again, using the true distribution to evaluate the utility of consumers). In the most efficient equilibrium, all accepted contracts maximize the consumers’ “true” utility subject to zero profit. In the least efficient equilibrium,
4 Explaining the Stylized Facts

This section more directly connects our model of Section 3 with the stylized facts noted in Section 2. In some cases, the connection has already been discussed, and so we briefly summarize for the sake of being comprehensive. In other cases, we provide additional derivations to make the points more concrete.

Substantial Lapsing and Lapse-Based Pricing

In the equilibrium of our model, consumers lapse their policies after background shocks. While the existence of lapsation is not inconsistent with a rational model, it is quantitatively hard to reconcile the very large lapsation fees observed in practice and the fact that over three-quarters of policies are not held to term within a model where consumers have rational expectations.

For example, consider a 20-year old male who wants life insurance coverage for the following 20 years. This consumer can either buy one 20-year term policy today or two 10-year policies, one today and one in 10 years. As we argued in Subsection 2.3, individuals in this age group have the highest lapse rates in the population. We collected data on these policies from 10 major life insurance companies and projected the expected payments by setting the probability of lapsation equal to the ones from Figure 3.

Figure 7 compares expected payments from these two possible strategies. Red bars correspond to the total expected premiums in current dollars from buying a 20-year policy, whereas blue bars correspond the expected premium from buying a 10-year term policy and, if there was no previous lapsing until the end of this policy, purchasing another 10-year policy after the first one expires. Purchasing two 10-year term policies costs between 27.8% and 57.7% less than one 20-year policy, while providing the same nominal coverage.

Therefore, consumers with rational expectations about their probability of lapsation would need to be willing to pay significantly more for a 20-year policy relative to 10-year policies. Although health shocks that raise premiums could, in principle, generate an option value from purchasing longer policies, the likelihood that a 20-year old healthy consumer will suffer such a health shock within a decade is too low to explain such a difference (see Tables 1-3). Moreover, because life insurance policies have fixed nominal face values, adding uncertainty about inflation would further reduce the attractiveness of the longer policy (since consumers could, instead, purchase the cheaper 10-year term policy and buy a standard inflation-adjusted bond, thereby hedging against inflation).

Consistent with the empirical evidence reported in Subsection 2.2, although insurance companies do not get extraordinary profits, there is substantial cross subsidization: They make positive profits on consumers who lapse and negative profits on those that do not (Condition 3 of Proposition 1). In contrast, in the competitive equilibrium of a rational model, firms would not choose to magnify the increases in marginal utility after an income shock with additional surrender costs; insurers, therefore, would actually lose money on consumers who lapse (see Appendix C).

\footnote{At least two firms offer the same contracts as in the competitive equilibrium. This is the equilibrium preferred by consumers according to their “perceived utility” (i.e., using the distribution that assigns zero probability to the income shock).}

\footnote{Since our lapsation data only covers 15-year term policies, we conservatively set lapsation rates equal to zero after the 16th year. Any positive lapse rates would make our results even stronger. We assumed a 2.5% inflation rate and a 5% nominal interest rate, although our results are robust to variations in these rates.}
While the existence of lapse-based pricing is consistent with a rational model, it is quantitatively hard to reconcile the very large lapsation fees observed in practice and the fact that over three-quarters of policies are not held to term within a rational expectations model.

**Front Loaded Policies**

Condition 2 of Proposition 1 states that the equilibrium policy shifts consumption into the future. That is, insurance companies offer front-loaded policies: initial premiums are high and later premiums are low. Front loading magnifies the impact of an income shock and induces consumers to surrender their policies, thereby raising the firm’s profits. Of course, these profits are competed away in equilibrium. In contrast, policies should be back loaded in the competitive equilibrium of our rational benchmark (Appendix C).

**Opposition to Secondary Markets**

We now consider the effects of introducing a secondary market for life insurance policies. In this market, individuals may resell their policies to risk-neutral firms, who then become the beneficiaries of such policies.

Suppose $M \geq 2$ firms (indexed by $k = 1, \ldots, M$) enter the secondary market. The game now has the following timing:

$t=0$: Each primary market firm $j \in \{1, \ldots, N\}$ offers an insurance contract $T_j$. Consumers decide which contract to accept (if any). Consumers who are indifferent between more than one contract randomize between them with strictly positive probabilities.

$t=1$: Each consumer loses $L > 0$ dollars with probability $l \in (0,1)$ and chooses whether to report a loss to the insurance company (“surrender”). Each firm in the secondary market

---

25 More specifically, front loading a policy has the cost of providing insufficient consumption smoothing. However, loss from insufficient consumption smoothing is of second-order close to the constant path, whereas the gain from relaxing the incentive-compatibility constraint is of first-order.
$k \in \{1, ..., M\}$ offers a secondary market contract. A secondary market contract is a vector $(r_{1,k}, r_{D,k}, r_{A,k}, r_{NS1,k}, r_{NSD,k}, r_{NSA,k})$ specifying state-contingent net payments from the consumer. Such a policy can be interpreted as the consumer selling the original insurance policy to firm $k$ in the secondary market for a price $t_{s,j} - r_{s,k}$, $s = S, NS$. In exchange, the firm keeps future insurance payments: $t_{A,j} - r_{A,k}$ if the consumer survives and $t_{D,j} - r_{D,k}$ if he dies.

$t=2$: Each consumer dies with probability $\alpha \in (0, 1)$. If alive, he earns income $I > 0$. If dead, he makes no income.

As in Section 3, we can rewrite the contracts offered by firms in both the primary and the secondary markets in units of consumption.

We consider both the short run (transitional) and long run (steady state) impacts of the introduction of the secondary market. We model the transition as a game in which secondary insurer firms unexpectedly enter in period 1, after primary market firms have already sold insurance contracts according to the SPNE of the game from Section 3. We model the steady state as the SPNE of the game in which firms in the primary market know about the existence of a secondary market when offering insurance contracts.

**Short Run**

Consider the continuation game starting at period 1 following the actions taken by firms in the primary market and consumers in the SPNE of the game from Section 3. There are two states of the world in period 1, one in which the consumer suffers an income shock and one in which he does not. We will consider each of these states separately.

First, consider the state in which the consumer does not suffer an income shock. The most attractive contract a consumer can obtain in the secondary market maximizes the consumer’s expected utility subject to the secondary market firm making non-negative profits:

$$\max_{c_1, c_D, c_A} u_A(c_1) + \alpha u_D(c_D) + (1 - \alpha) u_A(c_A)$$

subject to

$$c_1 + \alpha c_D + (1 - \alpha) c_A \leq c_{NS1} + \alpha c_{NSD} + (1 - \alpha) c_{NSA}.$$  

The solution entails full insurance and perfect consumption smoothing: $u'_A(c_1) = u'_A(c_A) = u'_D(c_D)$. Because the original contract had imperfect consumption smoothing, consumers are able to improve upon the original contract by negotiating with firms in the secondary market.

Next, consider the state in which the consumer suffers an income shock. Because firms in the primary market make positive profits if the consumer surrenders (i.e., reports a loss) and negative profits if he does not, it is never optimal for a consumer who will resell a policy in the secondary market to surrender the contract to the primary insurer. Therefore, the best possible secondary market contract solves:

$$\max_{c_1, c_D, c_A} u_A(c_1) + \alpha u_D(c_D) + (1 - \alpha) u_A(c_A)$$

subject to

$$c_1 + \alpha c_D + (1 - \alpha) c_A \leq c_{NS1} - L + \alpha c_{NSD} + (1 - \alpha) c_{NSA},$$

where the zero-profit constraint requires that expected consumption in the new policy cannot exceed the highest expected consumption attainable in the original policy (which is obtained
when the policyholder keeps the original policy and does not report an income loss to the original firm). This program consists of a maximization of a strictly concave function subject to a linear constraint. Therefore, the solution is unique. As in the model without a secondary market, the solution of this program entails full insurance: \( u'_A(c_1) = u'_A(c_A) = u'_D(c_D) \). However, because the primary market firm earns positive profits from the consumer reporting a loss, consumers obtain a strictly higher consumption in all states by renegotiating in the secondary market. As in Section 2, the equilibrium will be such that at least two firms offer the zero-profit full insurance contract and consumers accept it.

Combining these results, we have, therefore, have established the following proposition:

**Proposition 2.** There exists an essentially unique and symmetric SPNE of the short run model. In this SPNE:

1. Consumers always resell their policies in the secondary market.
2. Consumers are fully insured conditional on the shock:
   \[
   u'_A(c_1^{NS}) = u'_A(c_A^{NS}) = u'_D(c_D^{NS}) < u'_A(c_1^S) = u'_A(c_A^S) = u'_D(c_D^S),
   \]
3. Firms in the primary market earn negative expected profits.

Notice that marginal utilities are now constant conditional on the income shock. Consumers, therefore, are fully insured against mortality risk conditional on the realization of the income shock and are strictly better off with the presence of the secondary market.\(^{26}\) The inequality in marginal utilities across different realizations of the income shock reflects the incomplete insurance against income shocks. Primary insurers are worse off with the sudden introduction of the secondary market since original policies cross-subsidize between consumers who report a loss and those who do not. However, no consumer reports a loss in this new equilibrium.

**Long Run**

Next, we consider the SPNE of the full game. Competition in the primary market implies that firms must make zero profits. Moreover, any policy that cross subsidizes between the loss and the no-loss states will be resold in the secondary market leaving the primary market firm with negative profits. As a result, the equilibrium policies must generate zero expected profits in every state in period 1. The only candidate for such an equilibrium has at least one primary market firm offering policies that solves:

\[
\begin{align*}
\mathbf{c}^{NS} & \in \arg \max_{c_1, c_A, c_D} u_A(c_1) + \alpha u_D(c_D) + (1 - \alpha) u_A(c_A) \\
\text{subject to} & \quad c_1 + \alpha c_D + (1 - \alpha) c_A \geq W - \alpha I
\end{align*}
\]

\(^{26}\)This specific welfare conclusion ignores the role of reclassification risk, where agents learn new information over time about their health outlook. As referenced earlier, previous analyses have demonstrated that a secondary market could undermine dynamic risk pooling in the presence of reclassification risk. Our intended main purpose, though, is to simply demonstrate that a secondary market can reverse the impact of narrow framing on consumer welfare.
and
\[ c^S \in \arg \max_{c_1, c_A, c_D} u_A(c_1) + \alpha u_D(c_D) + (1 - \alpha) u_A(c_A) \]
subject to
\[ c_1 + \alpha c_D + (1 - \alpha) c_A \geq W - L - \alpha I, \]
and at least one secondary market firm offering actuarially fair resale policies. As before, these are the only policies accepted with positive probability.

**Proposition 3.** There exists an essentially unique and symmetric SPNE of the long run game. In this SPNE, all contracts accepted with positive probability provide full insurance conditional on the income shock:
\[ u'_A(c^{NS}_1) = u'_A(c^{NS}_A) = u'_D(c^{NS}_D) < u'_A(c^S_1) = u'_A(c^S_A) = u'_D(c^S_D). \]

As in the short run equilibrium, the presence of a secondary insurance market produces full insurance. However, firms now earn zero profits in both markets. Taking into account both short and long runs, it is clear that primary insurers would oppose the rise of secondary markets despite the improvement in efficiency.\(^{27}\)

### 5 Conclusion

This paper shows how narrow framing can explain some of the key stylized facts in the life insurance market. While our analysis focuses on the multi-trillion life insurance market due to its extensive impact on many households, we believe our model is consistent with a more general theory of consumer finance.\(^ {28}\)

In particular, opposition by primary sellers to secondary resellers typically occurs in markets where narrow framing is likely to be prevalent. Besides life insurance, primary sellers of sports and other entertainment tickets have lobbied in the past against secondary markets, arguing that ticket-holders have an exclusive contract with the primary seller and must only resell back to the primary seller (Smetters, 2006). As with life insurance, there exists a background risk (in this case, the future availability to actually attend an event) that might not be fully appreciated at the time of purchase. In contrast, resellers of most financial securities do not face this type of push-back; indeed, primary sellers of financial securities could not reasonably operate without the secondary market. More generally, when the good being purchased is generally intended for direct consumption and resale is only considered after a background shocks, buyers are likely to be most vulnerable to narrow framing. But when the good is more intermediary and must eventually be resold for the purpose of supporting consumption (e.g., financial securities) then narrow framing is less likely.

These results are broadly consistent with the experimental evidence on sportscards trading provided by List (2003, 2004) who demonstrates that deviations from the standard expected

\(^{27}\)Perhaps surprisingly, consumers who frame risks narrowly would not ex-ante favor a regulation that allows insurance to be sold at a secondary market. Therefore, in our model the same behavioral trait that introduces inefficiency in the competitive equilibrium prevents majority voting from implementing an efficiency-enhancing regulation. See, for example, Bisin, Lizzeri, and Yariv (2011) and Warren and Wood (2011) for interesting analyses of political economy based on behavioral economics. They would, of course, favor such a regulation ex-post.

\(^{28}\)See, for example, Bisin, Lizzeri, and Yariv (2011) and Warren and Wood (2011) for interesting analyses of political economy based on behavioral economics.
utility hypothesis was most seen in people who planned to keep the good; in contrast, the rational
(neoclassical) model better described traders who bought goods for the purpose of resale. Our
formalization of narrow framing is also consistent with other behavioral theories such as local
thinking and overconfidence. Future empirical work can attempt to disentangle the exact source
of deviation from expected utility hypothesis.

Appendix A: Current evidence of lapse-based pricing

Data on insurance policy quotes was obtained from the website www.quickquote.com, which
provides term life insurance quotes from ten large U.S. life insurance companies, including Pru-
dential, Legal & General, Metlife, Mutual of Omaha, Lincoln, American General, Protective,
SBLI, Transamerica, and ING. We received quotes for a $750,000 policy (the default) with a
30 year term for a male age 50, non-smoker, weight 165, and a preferred rating class. For the
mortality table, we use the 2008 Valuation Basic Table (VBT) computed by the Society of Actu-
aries that captures the “insured lives mortality” based on the insured population. (The VBT is
commonly used by state regulators for determining reserve requirements. However, in practice,
insurers often use proprietary “best” mortality tables that are more optimistic.) For each policy,
we then calculate the annual lapse rate so that the actuarial (mortality-adjusted) present value of
premiums is equal to the present value of death benefits paid (if any) net of lapsing (i.e., “break
even”). For a nominal discount rate of 7%, we find that a 12.3% annual lapse rate is required
for the median insurer to break even. For a nominal discount rate of 5%, the median break-even
annual lapse rate increases to 13.9%. For a nominal discount rate of 9%, the median break-even
annual lapse rate is equal to 10.7%.

Appendix B: Proofs

Proof of Lemma 1

Necessity:

1. If no offer is accepted, a firm can get positive profits by offering full insurance at a price
slightly above the actuarially fair. If only one offer is accepted, and this offer yields strictly
positive profits, another firm can profit by slightly undercutting the price of this policy. If the
only offer that is accepted in equilibrium yields zero profits, the firm offering it can obtain strictly
greater profits by offering full insurance conditional on the absence of an income shock at a higher
price.

2. Since the consumer is indifferent between any consumption profile conditional on the shock,
firms must choose the profiles that maximize their profits (otherwise, deviating to a profile that
maximizes their profits does not affect the probability that their offer is accepted by raises their
profits).

Firms are willing to provide insurance policies as long as they obtain non-negative profits. If
an offer with strictly positive profits is accepted in equilibrium, another firm can obtain a discrete
gain by slightly undercutting the price of this policy. Moreover, if the policy does not maximize
the consumer’s perceived utility subject to the zero-profits constraint, another firm can offer a
policy that yields a higher perceived utility and extract a positive profit.

3. If a consumer is accepting an offer with a lower perceived utility, either a policy that solves
Program (3) is being rejected (which is not optimal for the consumer) or it is not being offered (which is not optimal for the firms).

To establish sufficiency, note that whenever these conditions are satisfied, any other offer by another firm must either not be accepted or yield negative profits. It remains to be shown that the incentive-compatibility constraint (2) is satisfied in the solution of Program (3). Take a solution to Program (3), and note that allocation \((c_1^S + L, c_D^S, c_A^S)\) gives profit \(\pi^S\), which, as we will show in Proposition 1, is positive. Therefore, the allocation \((c_1^S + L, c_D^S, c_A^S)\) was feasible in Program (3). Since \((c_{1,j}^{NS}, c_{D,j}^{NS}, c_{A,j}^{NS})\) solve this program, it must be the case that
\[
\max_{c_{1,j}^{NS}, c_{D,j}^{NS}, c_{A,j}^{NS}} u_A(c_{1,j}^{NS}) + \alpha u_D(c_{D,j}^{NS}) + (1 - \alpha) u_A(c_{A,j}^{NS}) \geq u_A(c_{1,j}^{S} + L) + \alpha u_D(c_{D,j}^{S}) + (1 - \alpha) u_A(c_{A,j}^{S}).
\]

Hence, (2) is always satisfied by the solution of Program (3).

Before presenting the proof of Lemma 1 and Proposition 1, let us simplify Program (3). It is straightforward to show that the solution of the profit maximization program after the shock features \(c_1^S = c_2^S(A)\). Therefore, the set of contracts accepted in equilibrium are the solutions to the following program:
\[
\max_{c_{1,D}, c_A} u_A(c_1) + \alpha u_D(c_D) + (1 - \alpha) u_A(c_A) \tag{4}
\]
subject to
\[
\Pi(c_1, c_D, c_A) + (1 - l) [W - c_1 - \alpha c_D - (1 - \alpha)(c_A - I)] = 0,
\]
where the function \(\Pi\) is defined as
\[
\Pi(c_1, c_A, c_D) = \max_{x_A, x_D} W - L - (2 - \alpha)x_A - \alpha x_D - (1 - \alpha)I
\]
subject to
\[
(2 - \alpha)u_A(x_A) + \alpha u_D(x_D) \geq u_A(c_1 - L) + \alpha u_D(c_D) + (1 - \alpha) u_A(c_A).
\]

Proofs of Lemma 2 and Proposition 1

Existence of Equilibrium

Let us establish that there exists an SPNE of the game. By Proposition 1, this is equivalent of showing that there exists a solution to Program (4).

First, note that the constraint in Program (5) must be binding. Therefore, it is equivalent to the following program:
\[
\Pi(c_1, c_D, c_A) = \max_{x_A \in [0, -1]} W - L - (2 - \alpha)x_A - \alpha u_D^{-1}\left(V(c_1, c_D, c_D) - (2 - \alpha)u_A(x_A)\right) - (1 - \alpha)I,
\]
where \(V(c_1, c_D, c_D) \equiv u_A(c_1 - L) + \alpha u_D(c_D) + (1 - \alpha) u_A(c_A)\). The derivative with respect to \(x_A\) is \((2 - \alpha)\left[u_D\left(V(c_1, c_D, c_D) - (2 - \alpha)u_A(x_A)\right) - 1\right]\). This converges to \(+\infty\) as \(x_A \to 0\) and to -1 as \(x_A \to u_A^{-1}\left(V(c_1, c_D, c_D)\right)\). Thus, a solution exists and, by the maximum theorem, \(\Pi\) is a continuous function. Also, by the Envelope theorem, \(\Pi\) is a strictly decreasing function.
From the continuity of $\Pi$, it follows that the set of consumption vectors satisfying the constraint of Program (4) is closed. This set is bounded below by $(0, 0, 0)$. Moreover, because
\[
\lim_{c_1 \to \infty} \Pi (c_1, c_D, c_A) = \lim_{c_D \to \infty} \Pi (c_1, c_D, c_A) = \lim_{c_A \to \infty} \Pi (c_1, c_D, c_A) = -\infty,
\]
it follows that the set of consumption vectors satisfying the constraint of Program (4) is also bounded above. Because Program (4) consists of the maximization of a continuous function over a non-empty compact set, a solution exists.

**Characterization of the Equilibrium**

For notational simplicity, let us introduce the function $g$:
\[
g (c_1, c_A, c_D) \equiv l \Pi (c_1, c_D, c_A) + (1 - l) [W - c_1 - \alpha c_D - (1 - \alpha) c_A].
\] (6)

Program (4) amounts to
\[
\max_{c_1, c_D, c_A} u_A(c_1) + \alpha u_D(c_D) + (1 - \alpha) u_A(c_A) \text{ subject to } g (c_1, c_A, c_D) = 0
\]

The first-order conditions are:
\[
u'_A(c_1) - \lambda \frac{\partial g}{\partial c_1} (c_1, c_A, c_D) = 0,
\] (7)
\[\alpha u'_D(c_D) - \lambda \frac{\partial g}{\partial c_D} (c_1, c_A, c_D) = 0, \text{ and}
\] (8)
\[(1 - \alpha) u'_A(c_A) - \lambda \frac{\partial g}{\partial c_A} (c_1, c_A, c_D) = 0.
\] (9)

Thus,
\[
\frac{u'_A(c_1)}{\frac{\partial g}{\partial c_1} (c_1, c_A, c_D)} = \frac{u'_D(c_D)}{\frac{\partial g}{\partial c_D} (c_1, c_A, c_D)} = (1 - \alpha) \frac{u'_A(c_A)}{\frac{\partial g}{\partial c_A} (c_1, c_A, c_D)}.
\] (10)

Program (5) has a unique solution characterized by its first-order conditions and the (binding) constraint. The first-order conditions are
\[
\mu = \frac{1}{u'_A(x^*_A)} = \frac{1}{u'_D(x^*_D)} = \frac{1}{u'_A(x^*_1)} > 0,
\]
where $\mu$ is the Lagrange multiplier associated with Program (5).

Applying the envelope condition to Program (5), we obtain:
\[
\frac{\partial \Pi}{\partial c_1} = -\mu u'_A(c_1 - L) < 0, \quad \frac{\partial \Pi}{\partial c_D} = -\mu \alpha u'_D(c_D) < 0, \quad \frac{\partial \Pi}{\partial c_A} = -\mu (1 - \alpha) u'_A(c_A) < 0.
\]

Using the definition of function $g$ (equation 6), yields
\[
\frac{\partial g}{\partial c_1} = l \frac{\partial \Pi}{\partial c_1} - (1 - l) = -[\mu u'_A(c_1 - L) + 1 - l] < 0,
\]
\[
\frac{\partial g}{\partial c_A} = l \frac{\partial \Pi}{\partial c_A} - (1 - l) (1 - \alpha) = -(1 - \alpha) [\mu u'_A(c_A) l + 1 - l] < 0,
\]
A simple computation shows that
\[
\frac{\partial g}{\partial c_D} = \frac{\partial \Pi}{\partial c_D} - (1 - l) \alpha = -\alpha [\mu u'_D(c_D)l + 1 - l] < 0.
\]

Substituting back in the first-order conditions (10), we obtain
\[
\frac{u'_A(c_1)}{\mu u'_A(c_1 - L) + 1 - l} = \frac{u'_A(c_A)}{\mu u'_A(c_A)l + 1 - l} = \frac{u'_D(c_D)}{\mu u'_D(c_D)l + 1 - l}.
\]

The second equality above states that \(\xi(u'_D(c_D)) = \xi(u'_A(c_A))\), where \(\xi(x) \equiv \frac{x}{\mu u'_A(c_A) + 1 - l}\).

Since \(\xi\) is strictly increasing, it follows that \(u'_D(c_D) = u'_A(c_A)\). Rearranging the first equality above, we obtain
\[
u'_A(c_A) - u'_A(c_1) = \frac{\mu u'_A(c_A)}{1 - l} [u'_A(c_1) - u'_A(c_1 - L)] > 0.
\]

Thus, \(u'_A(c_1) > u'_D(c_D) = u'_A(c_A)\).

It remains to be checked that the second-order conditions are satisfied so that the critical point is indeed the maximum. The Bordered Hessian matrix associated with the Program (4) is
\[
H = \begin{bmatrix}
0 & \frac{\partial g}{\partial c_1} & \frac{\partial g}{\partial c_A} & \frac{\partial g}{\partial c_D} \\
\frac{\partial g}{\partial c_1} & u''_A(c_1) & 0 & 0 \\
\frac{\partial g}{\partial c_A} & 0 & u''_A(c_A) & 0 \\
\frac{\partial g}{\partial c_D} & 0 & 0 & u''_D(c_D)
\end{bmatrix}
\]

We need to calculate the sign of the 2 leading principal minors. That is, we have to check that \(\det(H) < 0\) and \(\det(H_2) > 0\), where
\[
H_2 = \begin{bmatrix}
0 & \frac{\partial g}{\partial c_1} & \frac{\partial g}{\partial c_A} \\
\frac{\partial g}{\partial c_1} & u''_A(c_1) & 0 \\
\frac{\partial g}{\partial c_A} & 0 & u''_A(c_A)
\end{bmatrix}
\]

A simple computation shows that
\[
\det(H) = -\left(\frac{\partial g}{\partial c_1}\right)^2 u''_A(c_A)u''_D(c_D) - \left(\frac{\partial g}{\partial c_A}\right)^2 u''_A(c_1)u''_D(c_D) - \left(\frac{\partial g}{\partial c_D}\right)^2 u''_A(c_1)u''_A(c_A) < 0,
\]
and
\[
\det(H_2) = -\left(\frac{\partial g}{\partial c_1}\right)^2 u''_A(c_A) - \left(\frac{\partial g}{\partial c_A}\right)^2 u''_A(c_1) > 0.
\]

Hence, any critical point is a local maximum.

Since the program consists of an unconstrained maximization and a solution exists, it follows that the unique local maximum is indeed a global maximum. Hence, the solution to Program (3) is unique, which implies that all offers accepted with positive probability in any SPNE are the same in all SPNE (i.e., the equilibrium is essentially unique and symmetric).

In order to verify that \(\pi^S > 0 > \pi^{NS}\), note that offering the same policy before and after the income shock is still feasible for the firms. More specifically, the allocation
\[
c^1_s = c^1_s - L, \ c^2_s(A) = c^2_s(A), \ c^2_s(D) = c^2_s(D)
\]

25
is feasible under the program defining function Π and this allocation gives the same perceived utility for consumers (who only take into account the consumption under no-shock). Since the program defining Π has a unique solution (which is different from offering the policy under no-shock), it must follow that \( \pi^{NS} > \pi^S \). Zero expected profits implies that \( \pi^S > 0 > \pi^{NS} \), which completes the proof.

**Appendix C: Reclassification Risk Insurance**

This section considers a model of reclassification risk based on Hendel and Lizzeri (2003), Daily, Hendel, and Lizzeri (2008), and Fang and Kung (2010). The main distinction between the model considered here and the other ones in the literature is in the timing of shocks. Hendel and Lizzeri (2003) study a model in which consumers are subject to health shocks only. Lack of commitment on the side of the consumer motivates lapsation following positive health shocks. Preventing lapsation is then welfare improving and front-loaded fees (i.e., payments before the realization of the health shock that cannot be recuperated if the consumer drops the policy) are an effective way to do so.

Daily, Hendel, and Lizzeri (2008) and Fang and Kung (2010) introduce bequest shocks in this framework. In their model, there is one period in which both bequest and health shocks may happen. Lapsation is efficient if it is due to a loss of the bequest motive and is inefficient if motivated by a positive health shock. The solution then entails some amount of front loading as a way to discourage lapsation.

As noted before, the composition of shocks changes significantly along the life cycle. Policyholders younger than about 65 rarely surrender due to health shocks whereas health shocks are considerably more important for older policyholders [c.f., Fang and Kung (2012)]. Consistently with this observation, we consider a stylized model in which the period of shocks is broken down in two periods. In the first period, consumers are subject to non-health shocks only. In the second period, they are only subject to health shocks. As a result, optimal contracts are *back loaded*: they do not discourage lapsation in the first period but discourage lapsation in the second period. Because only health-related lapsation is inefficient, lapse fees should be high only in periods in which health shocks are relatively prevalent. Empirically, these periods occur much later in life.

Formally, there are 4 periods: \( t = 0, 1, 2, 3 \). Period 0 is the contracting stage. Consumers are subject to a liquidity shock \( L > 0 \) (with probability \( l > 0 \)) in period 1. They are subject to a health shock in period 2. The health shock is modeled as follows. With probability \( \pi > 0 \), the consumer finds out that he has a high risk of dying (*type H*). With complementary probability, he finds out that he has a low risk of death (*type L*). Then, in period 3, a high-risk consumer dies with probability \( \alpha_H \) and a low-risk consumer dies with probability \( \alpha_L \), where \( 0 < \alpha_L < \alpha_H < 1 \). We model lapsation as motivated by liquidity/income shocks rather than bequest shocks because, as shown by First, Fang and Kung (2012), bequest shocks are responsible for a rather small proportion of lapses, whereas other (i.e. non-health and non-bequest shocks) are responsible for most of it, especially for individuals below a certain age.

The timing of the model is as follows:

- **Period 0:** The consumer makes a take-it-or-leave-it offer of a contract to a non-empty set of firms. A contract is a vector of state-contingent payments to the firm

\[
\left\{ t_{0,s}, t_{1,s}, t_{2,s,h}^{d,s,h} \right\}_{s=NS,NS \ h=NS,NS \ d=DS,DS}
\]


where: \( t_0 \) is paid in period 0 before any information is learned; \( t^s_1 \) is paid conditional on the liquidity shocks in period 1, \( s = S, NS \); \( t^{s,h}_2 \) is paid conditional on the health shock \( h \in \{H, L\} \) in period 2 and liquidity shock \( s \) in period 1; \( t^{d,s,h}_3 \) is paid conditional on being either dead \( d = D \) or alive \( d = A \) in period 3 conditional on previous shocks \( s \) and \( h \).

- Period 1: The consumer observes the realization of the liquidity shock \( s \). He then decides whether to keep the original contract, thereby paying \( t^s_1 \), or obtaining a new contract in a competitive secondary market. The competitive secondary market is again modeled by having the consumer make a take-it-or-leave-it offer a (non-empty) set of firms.

- Period 2: The realization of the health shock is publicly observed. The consumer decides to keep the contract, thereby paying \( t^{s,h}_2 \), or substitute by a new one, obtained again in a competitive environment (in which the consumer makes a take-it-or-leave it offer to firms).

- Period 3: The mortality shock is realized. The consumer receives a payment of \(-t^{d,s,h}_3\).

As before, we assume that consumers and firms discount the future at the same rate and normalize the discount rate to zero. Consumers get utility \( u_A(c) \) of consuming \( c \) units (while alive). Consumers get utility \( u_D(c) \) from bequeathing \( c \) units. The functions \( u_A \) and \( u_D \) satisfy the Inada condition: \( \lim_{c \downarrow 0} u_d(c) = -\infty \), \( d = A, D \).

With no loss of generality, we can focus on period-0 contracts that the consumer never finds it optimal to drop. That is, we may focus on contracts that satisfy “non-reneging constraints.” Of course, this is not to say that the equilibrium contracts will never be dropped in the same way that the revelation principle does not say that in the real world people should be “announcing their types.” To wit, any allocation implemented by a non-reneging contract can also be implemented by a mechanism in which the consumer is given resources equal to the expected amount of future consumption and gets a new contract (from possibly a different firm) in each period. In particular, the model cannot distinguish between lapsing an old contract and substituting it by a new (state-contingent) contract and having an initial contract that is never lapsed and features state-dependent payments that satisfy the non-reneging constraint. However, the model determines payments in each state.

Consistently with actual (whole) life insurance policies, one can interpret the change of terms following a liquidity shock in period 1 as the lapsation of a policy at some pre-determined cash value and the purchase of a new policy, presumably with a smaller coverage. We ask the following question: Is it possible for a firm to profit from lapsation motivated by a liquidity shock? In other words, it is possible for the firm to get higher expected profits conditional on the consumer experiencing a liquidity shock in period 1 than conditional on the consumer not experiencing a liquidity shock? As we have seen in the evidence described in Section 2, firms do profit from such lapses, which are the most common source of lapsation for policyholders below a certain age. However, as we show below, this is incompatible with the reclassification risk model described here.

The intuition for the result is straightforward. The reason why individuals prefer to purchase insurance at 0 rather than 1 is the risk of needing liquidity and therefore facing a lower wealth. If the insurance company were to profit from the consumers who suffer the liquidity shock, it would need to charge a higher premium if the consumer suffers the shock. However, this would exacerbate the liquidity shock. In that case, the consumer would be better off by waiting to buy insurance after the realization of the shock.
As in the text, there is no loss of generality in working with the space of state-contingent consumption rather than transfers. The consumer’s expected utility is

\[
u_A(c_0) + l \left\{ \begin{array}{l}
u_A(c_1^S) + \pi \left[ u_A(c_2^S) + (1 - \alpha_H) u_A(c_3^{S,H,A}) + \alpha_H u_D(c_3^{S,H,D}) \right] \\
+ (1 - \pi) \left[ u_A(c_2^L) + (1 - \alpha_L) u_A(c_3^{S,L,A}) + \alpha_L u_D(c_3^{S,L,D}) \right] \end{array} \right\}
\]

\[
+ (1 - l) \left\{ \begin{array}{l}
u_A(c_1^{NS}) + \pi \left[ u_A(c_2^{NS,H}) + (1 - \alpha_H) u_A(c_3^{NS,H,A}) + \alpha_H u_D(c_3^{NS,H,D}) \right] \\
+ (1 - \pi) \left[ u_A(c_2^{NS,L}) + (1 - \alpha_L) u_A(c_3^{NS,L,A}) + \alpha_L u_D(c_3^{NS,L,D}) \right] \end{array} \right\}.
\]

The equilibrium contract maximizes this expression subject to the following constraints. First, the firm cannot be left with negative profits:

\[
c_0 + l \left\{ \begin{array}{l}c_1^S + \pi \left[ c_2^S + (1 - \alpha_H) c_3^{S,H,A} + \alpha_H c_3^{S,H,D} \right] \\
+ (1 - \pi) \left[ c_2^L + (1 - \alpha_L) c_3^{S,L,A} + \alpha_L c_3^{S,L,D} \right] \end{array} \right\}
\]

\[
+ (1 - l) \left\{ \begin{array}{l}c_1^{NS} + \pi \left[ c_2^{NS,H} + (1 - \alpha_H) c_3^{NS,H,A} + \alpha_H c_3^{NS,H,D} \right] \\
+ (1 - \pi) \left[ c_2^{NS,L} + (1 - \alpha_L) c_3^{NS,L,A} + \alpha_L c_3^{NS,L,D} \right] \end{array} \right\}
\]

\[
\leq W + I [2 - \pi \alpha_H - (1 - \pi) \alpha_L] - l L
\]

Second, allocation has to satisfy the incentive compatibility constraints (which state that the consumer prefers the report of the liquidity shock honestly):

\[
u_A(c_1^S) + \pi \left[ u_A(c_2^S) + (1 - \alpha_H) u_A(c_3^{S,H,A}) + \alpha_H u_D(c_3^{S,H,D}) \right] \\
+ (1 - \pi) \left[ u_A(c_2^L) + (1 - \alpha_L) u_A(c_3^{S,L,A}) + \alpha_L u_D(c_3^{S,L,D}) \right] \geq 0
\]

\[
u_A(c_1^{NS}) + \pi \left[ u_A(c_2^{NS,H}) + (1 - \alpha_H) u_A(c_3^{NS,H,A}) + \alpha_H u_D(c_3^{NS,H,D}) \right] \\
+ (1 - \pi) \left[ u_A(c_2^{NS,L}) + (1 - \alpha_L) u_A(c_3^{NS,L,A}) + \alpha_L u_D(c_3^{NS,L,D}) \right] > 0
\]

and

\[
u_A(c_1^{NS}) + \pi \left[ u_A(c_2^{NS,H}) + (1 - \alpha_H) u_A(c_3^{NS,H,A}) + \alpha_H u_D(c_3^{NS,H,D}) \right] \\
+ (1 - \pi) \left[ u_A(c_2^{NS,L}) + (1 - \alpha_L) u_A(c_3^{NS,L,A}) + \alpha_L u_D(c_3^{NS,L,D}) \right] \geq 0
\]

\[
u_A(c_1^S) + \pi \left[ u_A(c_2^S) + (1 - \alpha_H) u_A(c_3^{S,H,A}) + \alpha_H u_D(c_3^{S,H,D}) \right] \\
+ (1 - \pi) \left[ u_A(c_2^L) + (1 - \alpha_L) u_A(c_3^{S,L,A}) + \alpha_L u_D(c_3^{S,L,D}) \right] \geq 0
\]

The third set of constraints requires contracts to be non-reneging after it has been agreed upon (that is, in periods 1 and 2). The period-2 non-reneging constraints are

\[
u_A(c_2^S) + (1 - \alpha_h) u_A(c_3^{A,NS,h}) + \alpha_h u_D(c_3^{D,s,h}) \geq \max_{\tilde{c}_2} \left\{ \begin{array}{l}u_A(\tilde{c}_2) + (1 - \alpha_h) u_A(\tilde{c}_3) + \alpha_h u_D(\tilde{c}_3) \\
s.t. \tilde{c}_2 + (1 - \alpha_h) \tilde{c}_3 + \alpha_h \tilde{c}_3 = (2 - \alpha_h) L \end{array} \right\}.
\]

(11)
for $h = H, L$ and $s = S, NS$. The period-1 non-reneging constraints are

$$ u_A(c^s_1) + \pi \left[ u_A(c^s_2) + (1 - \alpha_H) u_A(c^{s,H,A}_3) + \alpha_H u_D(c^{s,H,D}_3) \right] $$

$$ + (1 - \pi) \left[ u_A(c^s_2) + (1 - \alpha_L) u_A(c^{s,L,A}_3) + \alpha_L u_D(c^{s,L,D}_3) \right] \geq \max \left\{ c^{s,h}_1 + \pi \left[ c^{s,H}_2 + (1 - \alpha_H) c^{s,H,A}_3 + \alpha_H c^{s,H,D}_3 \right] + (1 - \pi) \left[ c^{s,L}_2 + (1 - \alpha_L) c^{s,L,A}_3 + \alpha_L c^{s,L,D}_3 \right] \leq I \left[ 2 - \pi \alpha_H - (1 - \pi) \alpha_L \right] - \chi_{s=S,L}, \right. $$

subject to

$$ c^{s,h}_1 + (1 - \alpha_h) u_A(c^{A,NS,h}_3) + \alpha_h u_D(c^{D,s,h}_3) \geq \max \left\{ u_A(c^A_2) + (1 - \alpha_h) u_A(c^A_3) + \alpha_h u_D(c^D_3) \right\}, $$

for $s = S, NS$, where $\chi_x$ denotes the indicator function.

We will define a couple of “indirect utility” functions that will be useful in the proof by simplifying the non-reneging constraints. First, for $h = H, L$ we introduce the function $U_h : \mathbb{R}_+ \to \mathbb{R}$ defined as

$$ U_h(W) \equiv \max_{c^A,c^D} \left\{ (2 - \alpha_h) u_A(c^A) + \alpha_h u_D(c^D) \right\}, \text{s.t. } (2 - \alpha_h) c^A + \alpha_h c^D \leq W. $$

It is straightforward to show that $U_h$ is strictly increasing and strictly concave. Next, we introduce the function $U : \mathbb{R}_+ \to \mathbb{R}$ defined as

$$ U(W) \equiv \max_{c,C^L,C^H} \left\{ u_A(c) + \pi U(C^H) + (1 - \pi) U(C^L) \right\}, \text{s.t. } c + \pi C^H + (1 - \pi) C^L \leq W, $$

$$ (2 - \alpha_H) I \leq C^H, \quad (2 - \alpha_L) I \leq C^L. $$

(12)

It is again immediate to see that $U$ is strictly increasing. The following lemma establishes that it is also strictly concave:

**Lemma 3.** $U$ is a strictly concave function.

**Proof.** Let

$$ U_0(W) \equiv \max_{c,C^L,C^H} \left\{ u_A(W - \pi C^H - (1 - \pi) C^L) + \pi U(C^{s,H}) + (1 - \pi) U(C^{s,L}) \right\}, $$

$$ U_1(W) \equiv \max_{c,C^L,C^H} \left\{ u_A(W - \pi C^H - (1 - \pi) C^L) + \pi U(C^{s,H}) + (1 - \pi) U(C^{s,L}) \right\}, $$

and
\[ \mathcal{U}_2(W) \equiv \max_{C^L, C^H} \left\{ \begin{array}{l}
 u_A \left( W - \pi C^H - (1 - \pi) C^L \right) + \pi U(C^{s,H}) + (1 - \pi) U(C^{s,L}) \\
 \text{s.t.} \quad (2 - \alpha_H) I = C^H \\
 \quad (2 - \alpha_L) I = C^L
\end{array} \right\} . \]

Notice that \( \mathcal{U}_0(W) \geq \mathcal{U}_1(W) \geq \mathcal{U}_2(W) \), and \( \mathcal{U}_0, \mathcal{U}_1 \), and \( \mathcal{U}_2 \) are strictly concave. Moreover, it is straightforward to show that there exist \( W_L \) and \( W_H > W_L \) such that:

- \( \mathcal{U}(W) = \mathcal{U}_0(W) \) for \( W \geq W_H \),
- \( \mathcal{U}(W) = \mathcal{U}_1(W) \) for \( W \in [W_L, W_H] \), and
- \( \mathcal{U}(W) = \mathcal{U}_2(W) \) for \( W \leq W_L \).

Moreover, by the envelope theorem, \( \mathcal{U}_0'(W_H) = \mathcal{U}_1'(W_H) \) and \( \mathcal{U}_1'(W_L) = \mathcal{U}_2'(W_L) \). Therefore,

\[ \mathcal{U}'(W) = \begin{cases} 
 \mathcal{U}_0'(W) & \text{for } W \geq W_H \\
 \mathcal{U}_1'(W) & \text{for } W_L < W \leq W_H \\
 \mathcal{U}_2'(W) & \text{for } W < W_L 
\end{cases} . \]

Because \( \mathcal{U}' \) is strictly decreasing in each of these regions and is continuous, it then follows that \( \mathcal{U} \) is strictly concave.

Let \( X^s \) be the sum of the insurance company’s expected expenditure at time \( t=1 \) conditional on \( s \) in the original contract:

\[ X^s \equiv c_1^s + \pi \left[ c_2^s + (1 - \alpha_H) c_3^{s,H,A} + \alpha_H c_3^{s,H,D} \right] + (1 - \pi) \left[ c_2^s + (1 - \alpha_L) c_3^{s,L,A} + \alpha_L c_3^{s,L,D} \right] + (1 - \pi) \chi_{s=s} \equiv X^s . \]

Our main result establishes that in any optimal mechanism the insurance company gets negative profits from consumers who suffer a liquidity shock and positive profits from those who do not suffer a liquidity shock. Expected profits conditional on the liquidity shock \( s = S, NS \) equal

\[ \Pi^s \equiv W + I [2 - \pi \alpha_H - (1 - \pi) \alpha_L] - (c_0 + X^s) . \]

By zero profits, we must have \( \Pi^S + (1-l) \Pi^{NS} = 0 \). We can now prove our main result:

**Proposition 4.** In any equilibrium contract, the insurance company gets negative profits from consumers who suffer a liquidity shock and positive profits from those who do not suffer a liquidity shock:

\[ \Pi^S \leq 0 \leq \Pi^{NS} . \]

**Proof.** Suppose we have an initial contract in which the firm profits from the liquidity shock in period 1 (that is, inequality 13 does not hold). Then, by the definition of \( \Pi^s \), we must have that the total expenditure conditional on \( s = NS \) exceeds the one conditional on \( s = S \): \( X^{NS} > X^S \).

Consider the alternative contract that allocates the same consumption at \( t = 0 \) as the original one but implements the best possible renegotiated contract at \( t = 1 \) conditional on the liquidity shock. More precisely, consumption in subsequent periods is defined by the solution to

\[ \max_{(c_1^s, c_2^s, c_3^{s,H,A}, c_3^{s,H,D}, c_3^{s,L,A}, c_3^{s,L,D})} \left\{ \begin{array}{l}
 u_A \left( c_1^s + \pi \left[ u_A \left( c_2^{s,H} + (1 - \alpha_H) u_A \left( c_3^{s,H,A} + \alpha_H u_D \left( c_3^{s,H,D} \right) \right) \right] \\
 \quad + (1 - \pi) \left[ u_A \left( c_2^{s,L} + (1 - \alpha_L) u_A \left( c_3^{s,L,A} + \alpha_L u_D \left( c_3^{s,L,D} \right) \right) \right] 
\end{array} \right\} . \]

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subject to
\[
\begin{align*}
{c^s_1} + \pi \left[ c^s_2 + (1 - \alpha_H) c^s_2 + \alpha_H c^s_2 \right] + (1 - \pi) \left[ c^s_0 + (1 - \alpha_L) c^s_0 + \alpha_L c^s_0 \right] & \leq \chi_{s=S} L,
\end{align*}
\]
\[
\begin{align*}
u_A(\tilde{c}_2^s) + (1 - \alpha_h) u_A(\tilde{c}_3^s) + \alpha_h u_D(\tilde{c}_3^s) & \geq \max_{\{c\}} \begin{cases} u_A(\tilde{c}_2^s) + (1 - \alpha_h) u_A(\tilde{c}_3^s) + \alpha_h u_D(\tilde{c}_3^s) \text{ s.t. } \tilde{c}_2(1 - \alpha_h) \tilde{c}_3 + \alpha_h \tilde{c}_3 = (2 - \alpha_h) I, \quad h = L, H. \end{cases}
\end{align*}
\]

By construction, this new contract satisfies the non-reneging and incentive compatibility constraints. We claim that the solution entails full insurance conditional on the shock: \( u'_A(c^s_h) = u'_A(c^s_h) = u'_D(c^s_h) \) for all \( s, h \) (starting from any point in which this is not satisfied, we can always increase the objective function while still satisfying both the zero-profit condition and the non-reneging constraints by moving towards full insurance). Let \( C^{s,h} \equiv c^s_h + (1 - \alpha_h) c^s_h + \alpha_h c^s_h \) denote the total expected consumption at periods 2 and 3. Then, \( c^s_h \) and \( c^{d,s,h} \) maximize expected utility in period 2 conditional on the shocks \( s, h \) given the total expected resources:

\[
\begin{align*}
u_A(\tilde{c}_2^s) + (1 - \alpha_h) u_A(\tilde{c}_3^s) + \alpha_h u_D(\tilde{c}_3^s) & = \max_{c,A,D} \begin{cases} u(c) + (1 - \alpha_h) u_A(c^A) + \alpha_h u_D(c^D) \text{ s.t. } c + (1 - \alpha_h) c^A + \alpha_h c^D \leq C^{s,h} \end{cases} \\
& = \max_{c,A,D} \begin{cases} (2 - \alpha_h) u_A(c^A) + \alpha_h u_D(c^D) \text{ s.t. } (2 - \alpha_h) c^A + \alpha_h c^D \leq C^{s,h} \end{cases} = U_h(C^{s,h}).
\end{align*}
\]

The non-reneging constraints (15) can be written as

\[
U_h(C^{s,h}) \geq U_h((2 - \alpha_h) I), \quad h = L, H.
\]

Using the fact that \( U_h \) is strictly increasing, they can be further simplified to

\[
(2 - \alpha_h) c^s_h + \alpha_h c^{d,s,h} \geq (2 - \alpha_h) I, \quad h = L, H.
\]

With these simplifications, we can rewrite Program (14) as

\[
\max_{c^s_1, C^{s,H}, C^{s,L}} u_A(c^s_1) + \pi U(C^{s,H}) + (1 - \pi) U(C^{s,L})
\]

subject to

\[
\begin{align*}
{c^s_1} + \pi C^{s,H} + (1 - \pi) C^{s,L} & \leq \chi_{s=S} L, \\
(2 - \alpha_H) I & \leq C^{s,H}, \\
(2 - \alpha_L) I & \leq C^{s,L}.
\end{align*}
\]

By equation (12), this expression corresponds to \( U \left( I \left[ 2 - \pi \alpha_H - (1 - \pi) \alpha_L \right] - \chi_{s=S} L \right) \).

The consumer’s expected utility from this new contract (at time 0) equals

\[
u(c) + lU(I \left[ 2 - \pi \alpha_H - (1 - \pi) \alpha_L \right] - L) + (1 - l)U(I \left[ 2 - \pi \alpha_H - (1 - \pi) \alpha_L \right]).
\]

The utility that the consumer attains with the original contract is bounded above by the contract that provides full insurance conditional on the amount of resources that the firm gets
at each state in period 1: $X^S$ and $X^{NS}$ (note that this is an upper bound since we do not check for incentive-compatibility or non-reneging constraints). That is, the utility under the original contract is bounded above by

$$u(c_0) + lU(X^S - L) + (1 - l)U(X^{NS}).$$

(17)

By zero profits, the expected expenditure in the original and the new contracts are the same. Moreover, because $X^S < I[2 - \pi \alpha_H - (1 - \pi) \alpha_L]$, it follows that the lottery $\{X^S - L, l; X^{NS}, 1 - l\}$ is a mean-preserving spread of the lottery

$$\{I[2 - \pi \alpha_H - (1 - \pi) \alpha_L] - L, l; I[2 - \pi \alpha_H - (1 - \pi) \alpha_L], 1 - l\}.$$

Thus, strict concavity of $U$ yields:

$$lU(X^S - L) + (1 - l)U(X^{NS}) <$$

$$lU(I[2 - \pi \alpha_H - (1 - \pi) \alpha_L] - L) + (1 - l)U(I[2 - \pi \alpha_H - (1 - \pi) \alpha_L]).$$

Adding $u(c_0)$ to both sides and comparing with expressions (16) and (17), it follows that the consumer’s expected utility under the new contract exceed his expected utility under the original contract, thereby contradicting the optimality of the original contract.

Therefore, in any equilibrium, firms cannot profit from consumers who suffer a liquidity shock and cannot lose money from those that do not.

### Appendix D

Tables 1-3 show “snap shots” across different ages of five-year ahead Markov health transition matrices based on hazard rates provided by Robinson (1996). State 1 represents the healthiest state while State 8 represents the worst (death). As the matrices show, younger individuals are unlikely to suffer negative health shocks and the ones who do experience such shocks typically recover within the next 5 years (with the obvious exception of death, which is). Older individuals are more likely to suffer negative heath shocks, and those shocks are substantially more persistent.

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Table 1: Probability of five-year ahead changes in health states at age 25.
Markov Transition Matrix (50 year old Male; 5 years)

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Table 2: Probability of five-year ahead changes in health states at age 50.

Markov Transition Matrix (75 year old Male; 5 years)

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Table 3: Probability of five-year ahead changes in health states at age 75.

References


