Brokers and the Insurance of Non-Verifiable Losses

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Abstract

How do insurance markets spread the risk when events are unknown or even unknowable? We argue that the insurance market is organized to write incomplete contracts such that these risks can be spread, even though complete contracts cannot be written. Both policyholders and insurers hostage their reputations when they engage in trade. The force of these reputation investments leads the parties to negotiate for a settlement when an event that is not covered by the insurance contract occurs. By choice of brokers, the parties can leverage the reputation stakes and thus influence the payoffs for un-contracted events. Thus, we see the role of ex post negotiation as helping to complete markets where otherwise insurance would not have been available. This view contrasts with other recent analyses in which ex post negotiation is seen as a degradation of the insurance market.

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1 Introduction

At the heart of our paper are non-verifiable or non-contractible events. By these we mean events that are incapable of inclusion in the policy because they cannot be anticipated; or events which are simply too complex to include in the contract, or events or circumstances which could be included but, due to time inconsistency or the prospect of new information, the parties believe it preferable to bargain after the fact.\(^1\) Non-verifiable losses can also refer to the size of the loss. An insurance policy might be specific about how a claim is to be settled (damage to a home or its contents might be limited to the repair cost or the cost of replacement with something of similar condition), but the insurer and insured might wish to leave open the possibility that settlement can be more generous as idiosyncratic circumstances dictate, or less generous if there is suspicion of claim fraud (which is difficult to prove).

Consider unanticipated losses. The parties consider the possibility that some unanticipated losses might occur. These unanticipated losses are not insurable in a formal contract because they cannot be specified and, even if they could be specified, they might be unsuited to insurance perhaps because they would incite severe ex post moral hazard, or because they are undiversifiable. However, there is another class of unanticipated losses for which, had they been anticipated, the parties would concur that they are insurable. Let us simply describe these as ex post insurable.\(^2\) If the parameters by which they could be deemed ex post insurable could be pre-specified, then there is nothing to prevent the parties from conditioning the insurance on these parameters. For example, suppose that the only thing that separated an ex post insurable from an ex post uninsurable event, was diversification. An ex post insurable event might be one that hit only one policyholder but an event impacting many policyholders would be ex post uninsurable. If things were this simple, a contract could be written conditioning coverage on the insurer’s surplus (rather like a mutual contract). But the circumstances that determine whether an event is ex post insurable may not be that easy to pre-specify or that easy to anticipate. For example, consider toxic mold, which burst onto the insurance scene as an unanticipated loss recently. Not only can it be an undiversifiable loss but, going forward, its coverage carries significant moral hazard. The fear is that insurance may be seen as a substitute for proper

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1Some policies specify the perils and losses that are covered. If a loss occurs that is not specified, then it is not covered. Other policies work in the opposite direction, they cover everything that is not included. The latter does provide a structure for including the unanticipated, but does so at a cost – it is open ended and becomes very difficult to price. Moreover, having such open policies complicates the insurer’s financial and risk management.

2Berliner (1982) defines insurable losses as satisfying the following criteria. Losses are anticipated, measurable and, after the fact observable. There must be low correlation and little moral hazard or adverse selection. Losses that are not anticipated but satisfy the other conditions are considered ex post insurable.
repair and maintenance of property. It may not be practical to write into contracts enforceable exclusions based, not only on the peril which is unanticipated, but on the moral hazard it might engender. Table 1 illustrates the criteria often listed in insurance textbooks for insurability (the loss is anticipated, measurable, etc.) and suggests how such factors might determine whether an unanticipated loss is ex post insurable. Thus we suggest that a surprise event may be insurable going forward if it is measurable (ex post), there is low correlation, low moral hazard, and information is symmetrically distributed between the parties.

<table>
<thead>
<tr>
<th>Classification of Losses</th>
<th>Ex ante insurable</th>
<th>Ex post insurable</th>
<th>Ex post uninsurable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anticipated</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Measurable</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Observable</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Low correlation</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Low moral hazard</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Symmetric information</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 1

Consider the following examples of incomplete contracts. The first involves reinsurance. Traditionally, the reinsurance market has been considered to be one based on relationships. Contracts were not very detailed and relationships between the insurer and reinsurer tended to persist over many years. In this relationship, the parties link their fortunes. If an unusual or uncovered loss arises, the reinsurer will often pay without raising a fuss. But there is a corresponding obligation of “payback”. The payback feature makes the contract something like a debt contract. Jean-Baptiste and Santomero (2000) argue that long term implicit contracts are efficient because they allow evolving information to be included in pricing. But reinsurance contracts may be incomplete in other ways. Contracts rarely specify the underwriting and claims settlement practices to be adopted by the primary insurer. The reinsurer will usually monitor the relationship but permit the primary insurer considerable flexibility in exploiting its core skills. Finally, reinsurance contracts often are not as specific in defining coverage as are the direct policies from which they are derived. This allows some ex post flexibility for the reinsurer to respond to losses that may not be covered. For example, despite the war exclusion on many reinsurance policies, reinsurers uniformly
responded to the 9/11 losses of their primary insurance clients without seeking to evoke this exclusion.

The feature of the reinsurance market that is of particular interest here is that it is largely a brokered market. If a reinsurer behaves badly to the primary insurer, the broker will know of it. Thus, the consequence for the reinsurer might be not only a loss of that contract, but a diversion of other business from the broker to other reinsurers. This leveraging of reputation enhances the “hold-up” power of the primary insurer.3

The second example of contract incompleteness lies in the procedures used to inaugurate coverage in the brokered reinsurance and high-end commercial insurance markets. Coverage is often syndicated across a number of carriers. To set this in place, the broker approaches a number of insurers asking for a commitment to provide coverage for a portion of the risk. To make such a commitment, the underwriter signs a binder (or slip at Lloyds) indicating how much coverage is offered. The interesting thing about this binder is the almost total lack of specificity about the coverage.4 Sometimes, the binder will refer to a particular policy form indicating the general form of coverage. Other times the binder will simply commit the underwriter to the same type of coverage offered by another insurer. But sometimes, the coverage is still to be agreed. This practice permits coverage to be arranged, or renewed without waiting for the contract to be issued. Thus, coverage need not be delayed pending the agreement of all contract details. The potential downside is that the lack of specificity offers leeway for dispute.

As a third illustration of incomplete insurance, some insurers such as Chubb Insurance company have made and protected a reputation for going the “extra mile” to ensure its personal (and other) policyholders are happy with their claims settlements. The strategy is to resolve ambiguity over amount or coverage more in the policyholder’s favor. Thus, there is a willingness to go beyond the narrow limits of the contract to ensure that the policyholder is adequately compensated. This flexibility introduces a degree of contract incompleteness, which is resolved after the fact in the claim negotiation. Chubb’s distribution system is a set of independent agents and brokers. These intermediaries “own” the renewal rights and can advise clients to move business if they become unhappy with Chubb’s claims performance. Thus, Chubb entrusts its

3 While we are stressing the relationships in the reinsurance markets, there are signs of an apparent shift to a more commoditized approach, which is highlighted by, though not confined to, insurance securitization. In contrast to relationship-based transactions, insurance securitizations are embodied in precise contractual form leaving little room for discretionary performance and ex post bargaining.

4 In a dispute between the leaseholders and insurers of the World Trade Center, the central issue is whether there was one “occurrence” or two. This is of importance because policies had not been issued at the time of the loss and coverage offered under binders was specific enough to give policy limits per occurrence but not specific enough to avoid dispute as to the meaning of “occurrence”. Had a policy been issued, then there would have probably been a precise definition of the term.
reputation to these agents and brokers in order to pre-commit to a “policyholder friendly” claim settlement strategy.

In each of these illustrations, the relationship is brokered. This is important in shaping the discretion the parties can exercise. For example, suppose Chubb fails to make a suitably generous offer to settle a claim, despite the fact that it promotes itself in this basis and charges higher premiums to cover such settlements. The broker can now exercise discretion in its response. If it suspects that the claim is fraudulent or inflated, it might condone Chubb’s lowball offer or refusal to pay. On the other hand, if the broker believes that Chubb is failing to meet the reasonable expectations of a deserving policyholder, it might withdraw business from Chubb. Thus, the reputation costs to Chubb from its unsatisfactory claim offer could be very high.

There is a corresponding mechanism to discipline the policyholder. If the policyholder plays fast and loose on its claims, the broker may withdraw its services – indeed the broker’s reputation with insurers is one of its most valuable assets and brokers may have a financial incentive to drop troublesome policyholders.

Alternatively, suppose insurance was offered on a binder but this minimal document was not specific in the terms of coverage. A loss occurs before the policy issued and the insurer baulks on payment. In choosing how to react to this breakdown, the broker must decide whether the event was one, which in principle is insurable and which a reasonable person would be expected to be covered. If the answer is affirmative, the broker might extract a large reputation toll from the insurer. But if the policyholder “is trying it on” – to hold up the insurer to pay for a loss that is not an insurable loss (e.g. poor business performance) – the broker will not impose a reputation cost on the insurer for refusing to pay.

By acting as a guardian of reputations, the broker can help secure a market for non-contractible losses. The formal and informal incentive structure for the broker reinforces this value-creating role. There is some ambiguity as to whether an insurance broker is the agent of the policyholder or of the insurer. The policyholder chooses whether to seek the services of a broker who will advise on insurance, on the choice of insurers, and will place contracts with specific insurers. The policyholder also is concerned with the treatment of its claims and usually expects the broker to become involved. The broker’s involvement can range from a monitoring of the settlement to exerting pressure on the insurer to resolve contractual ambiguity. Thus, the ability of the broker to create a market for non-verifiable losses is likely to enhance the demand for its services. The issue of legal agency is clouded by the payment for the service and by the incentives this creates. The normal default structure is for the broker to receive a commission from the insurer based
on the premium. However, many policyholders, particular large commercial clients, negotiate a fee with the broker (for which the proportional commission is an offset) related to the perceived value added. Such an arrangement provides a mechanism for the broker to be directly rewarded for creating insurance coverage for non-verifiable losses and for disciplining insurers that behave poorly towards their policyholders.

But brokers also act on behalf of insurers. Insurers often have contingent fees for brokers under which an additional compensation is paid to the broker based on the revenues and/or profitability of the book of business the broker holds with the insurer. Insurers typically compete amongst each other in the design of these profit and revenue sharing schemes and there is evidence that brokers do indeed respond to these incentives in their placement and cancellation decisions, (see Wilder, 2002).

In our formal model below, brokers respond to these bilateral incentives in the following way. Brokers offer an implicit deal to policyholders that they will use their leverage over insurers when unusual claims arise. Insurers for their part understand that such leverage will be used and that this adds value for the policyholder. Thus, insurers price for this effect. When a verifiable loss occurs it is paid according to the contract. When a non-verifiable loss arises, the broker determines whether the loss is ex post insurable or not. If the broker determines the loss to be ex post insurable, the insurer and policyholder bargain for a settlement. If the insurer fails to make a reasonable offer, the broker inflicts reputation penalty on the insurer. With such a breakdown in bargaining, the broker no longer feels that its clients are getting good service from this insurer and it diverts business to rivals or extracts concessions from the misbehaving insurer in the form of lower future prices on the business it places. If the broker determines the loss to be ex post uninsurable, no claim is made. While we do not go this far in our model, the broker could inflict a reputation penalty on the policyholder for trying to push an ex post uninsurable loss onto the insurer. For example, if the broker feels that the loss is due to fraud, or is unreasonably inflated (i.e., uninsurable), the broker could inflict a reputation cost on the policyholder. The broker now is less willing to represent such policyholders because they may hurt its contingent fees from the insurer or compromise its reputation with the insurer and therefore its ability to place higher quality policyholders. This would be an extension of our model and we consider formally only the reputational stake of the insurer.

The role of the broker in our model in some ways resembles the role of the courts in Anderlini et al (2003a, b). Both courts and brokers must exercise judgment in deciding whether an unanticipated loss should be covered under the contract and in both cases they are guided by the efficiency gains this “precedent” implies
for future contracts. In Anderlini et al, the court’s desire for efficiency gain is implied by its adoption of a social welfare function. In our model, brokers seek future efficiency gain because they can capture rents directly from value added. However, there are differences. In our case, there is a modeled efficiency gain in the current period. The parties can choose to contract with one or another of a number of competing brokers. In making this choice, the degree of hold up, and therefore the terms of ex post bargaining, can be bounded by the parties. The second difference lies in the nature and expertise of the institutions. The nature of the ex post judgment (whether a revealed loss is ex post insurable) is technically quite complex. Courts, by their nature, have no core skills to address such problems and rely on the expertise imported by the parties in an adversary process. Brokers do have these core skills and derive rents directly from the creation of value. But perhaps the biggest difference between our model and theirs lies in its purpose. Whereas they address unanticipated events that frustrate the purpose or outcome of a contract (i.e., they are an unintended nuisance), we address a situation where the parties know unanticipated events can happen and we are trying to expand the domain of the contracting relationship to encompass such events (i.e., to provide insurance). This is why, for us, it is important to address efficiency gains in the current period.

2 “Insuring” Non-Verifiable Losses without Brokers

In this section, we consider a simple world in which an individual is exposed to a loss, $L$, with probability $p$. The loss may be verifiable (and therefore insurable under a conventional policy) with a probability $q$ or non-verifiable (and therefore not insurable by a conventional contract) with a probability $1 - q$. The non-verifiable loss is observed by the parties, but cannot be contracted upon. For example, this may be a loss that could not have been anticipated at the time the contract was written but, once it has occurred, it is clearly observed by all parties.

Now consider the following infinite period problem. The policyholder buys a contract from a competitive insurer to cover a proportion $\alpha$ of the verifiable loss (probability $pq$). However, the premium exceeds the actuarial value of the verifiable loss, $P > pq\alpha L$. Moreover, assume there is an expectation that the policyholder will renew this coverage indefinitely. On such a contract, and with these renewal expectations, the insurer would make rent having a present value of $(P - pq\alpha L)/r$ where $r$ is the discount rate.

The expected rent only will be realized if the policyholder actually renews the contract. This hoped for
rent provides some “hold up” power to the policyholder. Suppose a non-verifiable loss occurs. The size of the loss is $L$, but the policyholder believes (for reasons we will examine presently) it would be appropriate for the insurer to make a non-contractual payment of $b$ to the policyholder to help defray its loss. Moreover, the policyholder makes a credible threat to withdraw its business, if the insurer fails to make a payment of $b$. If the threat is indeed credible, then the insurer will pay as long as the payment is less than the jeopardized rent; i.e., $b \leq (P - \alpha pq L)/r$.

Consider now a steady state in which, in any future year, policyholders might have an expectation that a payment of $b$ will be made against the unverifiable loss. In this case, future rents are reduced by the expectations of future “type-$b$” payment. Thus, assuming $b$ to be constant over time, the insurer will pay in the face of a credible threat if

$$b \leq \frac{P - pq \alpha L - p (1 - q) b}{r}.$$

If the policyholder has all the bargaining power, then the premium $P$ which includes future “type-$b$” payment is given by

$$P = pq \alpha L + p (1 - q) b + rb.$$

The premium is the sum of the actuarially fair value, $pq \alpha L + p (1 - q) b$, and a loading, $rb$. This loading represents the future rents that provides the insurer with the incentive to pay the transfer $b$ in case of a unverifiable loss.

Continuing to assume that the policyholder’s threat to cancel is credible, we can ask the following question. Would the policyholder and insurer wish to engage in this partially incomplete contract in which there is conventional insurance coverage together with the ability of the policyholder to hold-up the insurer to pay non verifiable, and therefore not contracted losses? If so, then such an arrangement provides explicit insurance for contractible losses and implicit insurance for the non-verifiable (non-contractible) losses.

We show in Appendix A, that it is never optimal to provide full coverage for either the verifiable or unverifiable loss, i.e. $b^* < L$ and $\alpha^* < 1$ for all $r$. Furthermore, as long as the discount rate is not too high, there will be a demand for partially incomplete contracts which carry partial implicit coverage of the unverifiable loss, i.e. there exists a critical discount rate $\bar{r} (\alpha^*) > 0$ such that $b^* > 0$ if $r < \bar{r} (\alpha^*)$ and $b^* = 0$ if $r \geq \bar{r} (\alpha^*)$. Last, it is optimal to get insurance for at least one type of loss, i.e. either $\alpha^* > 0$ and/or $b^* > 0$. 

8
The intuition behind these results are as follows. In order to generate a “hold-up” and thereby payments for unverifiable losses the policyholder has to pay a loading $rb$. The premium is therefore unfair and full “insurance” of the unverifiable loss is not optimal.\(^5\) Purchasing full coverage of the verifiable loss would then lead to a higher marginal utility in the state of the unverifiable event. In this case, the policyholder finds it optimal to transfer wealth into that state by not buying full coverage of the verifiable loss and thereby reducing the premium.

This result is powerful, for it reveals a structure for transfer of unverifiable losses to an insurer. However, this analysis has several weaknesses.

- The first, and most important, weakness is the assumption that the policyholder can impose a credible threat to withdraw its insurance if the insurer fails to make the anticipated payment, $b^*$.

- A second weakness is that we have provided little structure to the ex post bargain. We simply gave all the bargaining power to the policyholder who then extracts a transfer $b$ bounded by the future rents. More realistically, we might expect that the parties would bargain to divide the “gains from trade” that arise from continuing their relationship.

- The third weakness is that, in case of multiple possible loss sizes, if the future rents brought by the policyholder are small, then this mechanism only provides for small transfer. Thus the policyholder suffering a $1$ million non-verifiable loss, but generating rents with a present value of $1,000$, will be able to recover at most $1,000$.

We will now provide more market structure to address these weaknesses. We will allow the parties to bargain after the loss to determine the transfer, $b$. We will introduce brokers who are able, and motivated, to inflict reputational losses on parties that fail to participate in the bargain. Moreover, because the reputational costs that brokers can impose on insurers are determining by the value of the book of business the broker holds, rather than the rents from an individual policy, such bargaining can provide much transfers for non verifiable losses.

\(^5\) The result still holds if there is no verifiable loss and is therefore robust with respect to different correlation structures between the verifiable and unverifiable loss.
3 "Insuring" Non-Verifiable Losses with Brokers and Ex-Post Bargaining

Policyholders are exposed to the same losses as in the previous section and, being risk averse, have the same desire to transfer their verifiable and non-verifiable losses to an insurer. Now, however, they use brokers to place their business with a carrier. For their part, brokers seek to facilitate their clients’ wishes to create a market for non-verifiable losses, in addition to their usual function of finding coverage for the verifiable losses. For their part policyholders choose brokers on the basis of their reputations for assembling partially incomplete contract and for assisting the parties to engage in ex post bargaining for non-verifiable losses. If a broker fails to encourage bargained transfer for non-verifiable losses, it will lose clients. Presently, we will address the issue of whether the compensation for brokers provides appropriate motivation. Each broker deals with several insurers and has a book of business with each insurer.

The brokers are said to “own” the book of business which means that, while the clients will finally choose which insurer writes the business, the broker is free to make recommendations to its clients. The very fact that clients use brokers, implies that they deliver some value and we will assume that the brokers recommendation to its clients are influential. Thus, if a broker is unhappy with the performance of the insurer, it can inflict a costly penalty on that insurer by recommending to its clients to place the business elsewhere. The particular behavior the broker seeks to penalize is the failure for the insurer to make a payment on the non verifiable loss. Under the threat of such a penalty, the insurer and policyholder engage in Nash bargaining. If the bargaining breaks down, the broker executes the penalty.

3.1 Ex-Post Bargaining

Under Nash bargaining the parties maximize the product of the “gains from trade”. The gain for the insurer from bargaining is \( R - b \) where \( R \) is the reputational penalty extracted by the broker for failure and \( b \) is the payment to the policyholder. The “gain” for the policyholder is measured as the increase in utility from the settlement, i.e. \( u(w_0 - P - L + b) - u(w_0 - P - L) \). The Nash bargaining solution, NBS, shares the joint “gains from trade”. Thus, as a benchmark, the resulting payment for the case a risk neutral policyholder would by \( b^* = \frac{1}{2} R \). However, with risk aversion, the policyholder has more to lose and this reduces his/her bargaining power. Thus, the NBS \( b^* \) is decreasing in the degree of risk aversion of the policyholder and

10
$b^* < \frac{1}{2} R$.

Ultimately, the policyholder and insurer are interested in the amount of transfer $b$ provided for the non-verifiable loss. To illustrate this suppose the policyholder is risk neutral which implies $b = \frac{1}{2} R$. Suppose that the broker wishes ex post to induce the parties to bargain to attain a transfer $b$. The broker then imposes the penalty $R = 2b$.

### 3.2 The Ex-Ante Partially Incomplete Contract

We can now think of the ex ante problem. The insurer and the policyholder are brought together through the medium of the broker to arrange explicit coverage for the verifiable loss and an expectation of implicit coverage for the non-verifiable loss. Clearly the level of implicit coverage, $\alpha$, is a decision variable to be set. The client can also choose to go through a large or small broker. A large broker (more specifically a broker with a large book of business with an insurer) has the power to extract a large reputational penalty from that insurer. In this way, the choice of broker provides a bound on the penalty that the broker can extract for failure to pay non-verifiable losses. However, since the insurer will charge for staking its reputation in this way, and therefore the policyholder will prepay for “coverage” of non-verifiable losses, the parties are unlikely to wish to enter an arrangement in which $b$ is maximized. Rather, we can imagine that there is some optimal payment $b^*$ which is related to the degree of risk aversion of the policyholder and the size of the loss $L$. The task of the broker ex ante is to intermediate in the assembly of a partially incomplete contract in which $\alpha$ is set explicitly at $\alpha^*$ and to signal that it will impose an ex post reputational penalty, $R^*$, to induce insurer/policyholder bargaining resulting in a settlement of $b^*$. In this sense, the broker needs to develop a reputation for ex post intercession when non-verifiable losses arise.

In Appendix B we show the solution to the ex ante problem. We set this up as a program in which the parameters $R$ and $\alpha$ are chosen to maximize policyholder’s expected utility subject to the insurer making non negative long-term rents and to ex post Nash bargaining. This structure can be thought of as an explicit choice of coverage $\alpha$ by the policyholder and the choice of a signal $R$ by the broker that it intends to enforce should a non-verifiable loss arise. We will consider incentive compatibility for the broker presently.

The Appendix shows that there is an interior optimal transfer on the non-verifiable loss, i.e. $0 < b^* < L$, and full insurance on the verifiable loss is not optimal, i.e. $\alpha^* < 1$. The desire for a positive ex post transfer on the non verifiable loss is understandable because the policyholder is risk averse. As both parties split the
gains from trade there is an implicit “loading” in the coverage for the unverifiable loss. It is thus not optimal for the policyholder to commit to impose a high reputational loss which then generates full coverage. We show in the Appendix that full coverage is optimal if the policyholder holds all the bargaining power. The intuition behind the optimality of partial insurance of the verifiable loss is the following. A high premium implied by full insurance would lead to a weaker bargaining position for the policyholder and to a higher marginal utility in the state where the unverifiable event occurs.

It is interesting to examine whether the broker has an incentive to participate ex ante in such an incomplete contract and then to enforce the penalty $R^*$ after the loss. The Pareto structure to the optimization program maximizes the welfare of one party (the policyholder) subject to a reservation level of profit for the other (the insurer). This suggests that any compensation structure for the broker that is aligned with this program might engage the broker in participating in the optimal program. Since we have specified a competitive solution in which zero profits are made by the insurer, the optimal broker compensation would be a linear increasing function of the policyholder’s expected utility. We will comment on whether such compensation structures are encountered in practice. In the meantime, we note that such structures are incentive compatible. If the broker compensation is paid it will have an incentive to induce bargaining to by imposition of a penalty of $R^*$ and its failure to do so will involve loss of reputation and loss of business. In the present case, it might appear that the broker who gets paid in proportion to the policyholder’s expected utility, might impose $R > R^*$ to provide a short term gain in utility and commission. However, such “over-insurance” will be picked up by insurers who will raise premiums. This is sub-optimal for policyholders who will, therefore, switch brokers.

3.3 Broker Compensation and Incentives in Practice

Brokers are compensated in several ways. The most common form is a commission which is expressed as a percentage of the premium. These commissions are paid by insurers and their levels vary between insurers and lines of business. Brokers will often negotiate fees with their clients in lieu of these commissions. The commission will then be offset against the fee. These negotiated fees are usually found for large corporate clients. In addition brokers sometimes receive contingent commissions from insurers. These are based on some metrics of the book of business the broker has with the insurer. Most common are profit based

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contingent commissions and contingent commissions based on premium volume which have recently attracted attention though these are mainly used by the mega brokers. Additional triggers, such as growth of business and retention rates are sometimes also used for contingent commissions. The final major elements of broker revenues are fees for specific ancillary services such as captive management, claim management, actuarial services and loss modeling.

In terms of our model with competitive partially incomplete contracts, fees in lieu of commissions seem to fit the incentive bill. There is no standard for calculating such fees; the parties are free to negotiate. However, negotiations often are based on the value added to the client. Insofar as the broker is able to assemble the partially incomplete contract, including the expectation of ex post bargaining for non-verifiable losses, the broker will be rewarded. Failure for the broker to stimulate ex post bargaining will, at best, lead to a downward negotiation of the fee and, at worst, a loss of the business.

While we have modeled a competitive market, the basic insights should carry over to the non-competitive case. The creation of a market for non-verifiable losses, adds value to the risk-averse policyholder who will be willing to pay loaded premiums for such coverage. The additional rents received by insurers from incomplete contracts (compared with simple contracts covering only verifiable losses), provide a lower bound on the value created for their clients. For this setting, compensation that allows the broker to share in the insurer’s rents provides an incentive for the broker to participate in partially incomplete contracts. Contingent commissions based on insurer’s profit have this structure. However there is a time inconsistency. After a non-verifiable loss, the insurer’s profit will be reduced for that year which will in turn reduce the broker’s contingent commissions. However, a satisfactory bargained settlement will preserve the demand for future partially incomplete contracts and the future rents which will generate future contingent commissions.

The argument that contingent commissions facilitate the assembly of partially incomplete contracts challenges the recent attacks on these compensation structures. The New York attorney general, Elliot Spitzer, has criticized contingent commissions. These commissions are characterized as “kickbacks” from the insurer that compromise the broker’s obligations to its clients (the policyholders). While we agree that profit based contingent commissions do provide the myopic broker to discourage claims, this is countered by a longer term reward for the broker who is able to innovate with new products that add value. Insofar as part of this

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7 The investigations of the New York Attorney General, Elliot Spitzer into Marsh McLennan Inc. focused largely on volume based contingent commissions. Since that investigation, the leading brokers Marsh, Aon, and Willis have abandoned these contingent commissions. Almost all other smaller brokers continue to receive contingent commissions which are based mostly on profit.
added value is captured by insurers in the form of rents, the broker will be rewarded by higher commissions. Contrary, if the value added accrued purely as a surplus to the consumer, then brokers would share by virtue of negotiated fees.

Figure 1 illustrates a contingent commission structure. The accumulating rents are shown. However, brokers can reduce these rents by imposing a reputational penalty and the parties bargain for $b^*(R)$ in the shadow of this reputational penalty. This bargain reduces the present value of the rents as shown. Ex ante, a partially incomplete contract with an expected bargain of $b^*(R)$ maximizes the present value of rents net of the transfer $b^*$ (shown as $PV - b^*$). If many such relationships are established with policyholders and these are intermediated with a broker who receives say a proportional contingent commission, then the portfolio of partially incomplete contracts that maximizes the present value of the net rents also will maximize the present value of the contingent commissions.

![Figure 1: Sharing rents with contingent commissions.](image)

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8 Cummins and Doherty 2005 also argue that profit based contingent commissions help reduce the costs of asymmetric information which provides additional benefits to policyholders.

9 While a proportional contingent commission aligns the interests of the monopoly insurer and broker in maximizing net rents, many contingent commission structures are not proportional. A common form is progressive with the marginal commission increasing.
4 Conclusion

Several observers have noticed a recent, and supposedly, disturbing trend in insurance markets. Apparently, insurers are now more likely to dispute large claims, to offer less than 100 cents on the dollar, or to try to get away without paying. Richard and Barbara Stewart (2001) have labeled this the “loss of certainty effect” and Kenneth Abraham (2001) has talked of the “de facto big claims exclusion”. One reason for such disputes is that large claims threaten insurer solvency and such offers may be seen to resemble workouts in which distressed non-insurance firms negotiate with creditors. But the issue here is with the willingness, not the ability, to pay. These writers see the “big claims exclusion” as degradation of the insurance market because risk-averse consumers will place a lower value on such uncertain insurance. Indeed, they see a potential downward spiral of the insurance market if this practice continues.

The loss of certainty may be characterized as ex-post bargaining over a settlement rather than a straightforward appeal to the policy conditions. Yet such bargaining should not be a surprise when claims are unusual and it is unclear whether they are really covered. For example, it is a matter of real dispute whether many environmental losses (e.g. for cleanup of Superfund sites) are really covered and, if so, how the many policies in force over the long gestation period of such losses should contribute. Indeed, losses of this nature and duration were probably not anticipated when the policies were written and therefore the policy wording is simply unclear.

In this paper we provide a very different view of these trends in the context of incomplete contract theory. In a world with rapidly evolving technology and shifting sociopolitical institutions, we might expect to be exposed to new types of losses. As with more traditional losses, there may be a comparative advantage in the transfer of such risk from individuals and firms to insurers and reinsurers whose capital and portfolio structure enables them to absorb such unknown losses at lower cost. But, the novelty of these losses, presents a problem. If the nature of losses cannot be anticipated with any precision (or if the variety of such potential losses is wide) then it may simply be infeasible to write enforceable contracts to share risk. Can we then find a way of arranging the affairs of individuals and potential insurers such that there is sharing of risk, despite the absence of enforceable insurance?

We define a particular role for brokers in potentially completing insurance markets with non-contractible risks. Brokers are the repositories of the reputation of insurers and policyholders. If non-verifiable losses occur the parties can bargain over a settlement. By its subsequent behavior, the broker can influence
the outcome of this bargaining. For example, if an insurer fails to reach a satisfactory bargain with its policyholder, the broker might be less inclined to place future business with that insurer. Thus, the policyholder can hold-up the insurer against this reputation cost. Ex-ante, policyholders have some degree of choice over whether they do business in the brokerage market and in their choice of broker. This, in turn, permits them some degree of control over their prospective bargaining position with their insurer and thus some control over the transfer of non-verifiable risk.

The extent to which ex-post Nash bargaining permits effective hedging in the case of multiple possible non-verifiable loss sizes rests on the information available, the utility function of the policyholder and on the structure of the reputation cost function. In principle, there exists a reputation function that would induce a full transfer of non-verifiable risk though Nash bargaining. But this function is complex and requires the broker to have sufficient market clout, and full knowledge of realized losses and of the policyholder’s risk preferences. Of course, by making the assumptions too strong, we can always argue that the losses were contractible. With weaker assumptions, there can still be risk transfer. However, this requires that the reputation function be positively related to the size of the non-verifiable loss.

We are also able to determine the limits on such risk sharing of non-verifiable losses. If the broker is unable to condition the reputation of the insurer on the occurrence or size of the non-verifiable loss, then Nash bargaining will increase the policyholder’s risk. However, it would seem an unlikely set of circumstances. The stylized model with increasing reputation costs does seem to correspond with the functioning of the insurance market place. Brokers usually have some access to loss estimates, they do indeed shop around risks and no doubt policyholders do take refuge behind the bargaining clout of their brokers when it comes to negotiating unusual claims. And brokers do place business, not only according to price, policy conditions and solvency, but also factor in the claim settlement records of insurers (see Harrington and Niehaus, 2004, p. 504).
A Appendix

The policyholder and insurer write an insurance contract under which the policyholder pays a premium \( P \) to the insurer with the understanding that a fraction \( \alpha \) of the loss is covered by the insurer. Suppose that a loss of severity \( L \) occurs with probability \( p \) and that the event and severity is observable. Given a loss, the loss is either verifiable with probability \( q \) or not verifiable with probability \( 1 - q \). If the loss is verifiable the insurer pays coverage \( \alpha L \) to the policyholder according to the terms of the contract. If the loss is not verifiable, the contract is not enforceable in front of a court and the insurer will only pay a transfer \( b \) to the policyholder if it has an incentive to do so. The policyholder can provide these incentives by paying a premium above the actuarially fair rate and thereby creating rents for the insurer. The distribution of final level of wealth of the policyholder is given by

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<td>( p(1 - q) )</td>
<td>( w_0 - P - L + b )</td>
</tr>
</tbody>
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A.1 Ex-Post Transfer for Non-verifiable Loss

The rents for the insurance company from paying the transfer \( b \) to the policyholder and thereby continuing the relationship are

\[-b + \frac{P - E[I]}{r}\]

where \( r \) is the risk-free rate and \( I \) is the indemnity, i.e. \( \alpha L \) in case of a verifiable event and \( b \) in case of a non-verifiable event. Its expected value is thus

\[E[I] = pq\alpha L + p(1 - q)b.\]

The incentive constraint for the insurer to pay the transfer \( b \) is thus

\[-b + \frac{P - E[I]}{r} \geq 0.\]

In this version of the model, we assume that the market observes whether the insurance company shirks such that the threat is credible. In a perfectly competitive insurance market where the policyholder has all the bargaining power we thus have

\[-b + \frac{P - E[I]}{r} = 0\]

i.e.

\[P = pq\alpha L + p(1 - q)b + rb.\]

The premium is thus the sum of the actuarially fair rate, \( pq\alpha L + p(1 - q)b \), and a loading, \( rb \).

A.2 Ex-Ante Decisions

Ex-ante the policyholder chooses the level of insurance coverage, \( \alpha L \), of the verifiable loss and the transfer, \( b \), of the non-verifiable loss to maximize expected utility. We first examine these two choice variables separately as a function of the other to then make inference about the optimal joint solution.

Proposition 1 Suppose the transfer \( b \) related to the non-verifiable event is fixed. If the transfer provides less than full insurance then it is optimal to also partially insure the verifiable loss, i.e. \( \alpha^*(b) < 1 \) for all
0 ≤ b < L, and without a transfer it is optimal to buy some coverage of the verifiable loss, i.e. α∗(0) > 0. If the transfer provides full or more than full insurance then it is optimal to fully insure the verifiable loss, i.e. α∗(b) = 1 for all b ≥ L. If the policyholder exhibits preferences consistent with constant absolute risk aversion, then the optimal insurance coverage is increasing in the transfer, i.e. \( \frac{dα∗(b)}{db} > 0 \).

**Proof.** The policyholder solves the following program

\[
\max_α E[u(w)]
\]

where

\[
E[u(w)] = (1 - p) u(w_0 - P) + pq(u_0 - P - (1 - α) L) + p(1 - q) u(w_0 - P - L + b),
\]

\( P = pqαL + p(1 - q)b + rb, \)

and b is fixed. The first derivative of expected utility is

\[
\frac{∂E[u(w)]}{∂α}|_{α=1} = pqLp(1 - q) (u'(w_0 - P) - u'(w_0 - P - L + b)).
\]

The second derivative is negative and expected utility therefore globally concave in α. The FOC thus determines the unique global maximum α∗ = α∗(b). Evaluating the first derivative at α = 1 yields

\[
\frac{∂E[u(w)]}{∂α}|_{α=1} = pqLp(1 - q) (u'(w_0 - P) - u'(w_0 - P - L + b)).
\]

If 0 ≤ b < L then \( \frac{∂E[u(w)]}{∂α}|_{α=1} < 0 \) and thus α∗(b) < 1. If b ≥ L then α∗(b) = 1. Evaluating the first derivative at α = 0 and b = 0 yields

\[
\frac{∂E[u(w)]}{∂α}|_{α=0, b=0} = (1 - p) pqL (u'(w_0 - P - L) - u'(w_0 - P)) > 0,
\]

i.e. α∗(0) > 0. Taking the total differential of the FOC with respect to α and b yields

\[
\frac{∂^2 E[u(w)]}{∂α^2}|_{α=α∗(b)} dα + \frac{∂^2 E[u(w)]}{∂α∂b}|_{α=α∗(b)} db = 0
\]

which implies

\[
\frac{dα∗(b)}{db} = - \frac{∂^2 E[u(w)]}{∂α∂b}|_{α=α∗(b)},
\]

and therefore

\[
\text{sign} \left( \frac{dα∗(b)}{db} \right) = \text{sign} \left( \frac{∂^2 E[u(w)]}{∂α∂b}|_{α=α∗(b)} \right).
\]
The cross derivative is given by

\[
\frac{\partial^2 E[u(w)]}{\partial \alpha \partial b} = pqL \left[ (1 - p) (p (1 - q) + r) u''(w_0 - P) - (1 - pq) (p (1 - q) + r) u''(w_0 - P - (1 - \alpha) L) + p (1 - q) (p (1 - q) + r) u''(w_0 - P - L + b) \right]
\]

where \( R_a \) is the coefficient of absolute risk aversion which is constant under CARA. The FOC for \( \alpha^* (b) \) then implies

\[
\frac{\partial^2 E[u(w)]}{\partial \alpha \partial b} |_{\alpha=\alpha^*(b)} = pqLR_a (1 - q) u'(w_0 - P - L + b) > 0
\]

and therefore \( \frac{d\alpha^*(b)}{db} > 0 \). □

**Proposition 2** Suppose insurance coverage \( \alpha \) is fixed. It is then optimal to not generate full coverage of the non-verifiable loss, i.e. \( b^*(\alpha) < L \) for all \( \alpha \in [0,1] \). For each \( a \) there exists a discount factor \( \bar{r}(\alpha) > 0 \) such that it is optimal to generate a transfer if the discount rate is below \( \bar{r}(\alpha) \) and not to generate a transfer otherwise, i.e. \( b^*(\alpha) > 0 \) if \( r < \bar{r}(\alpha) \) and \( b^*(\alpha) = 0 \) if \( r \geq \bar{r}(\alpha) \). If the policyholder exhibits preferences consistent with constant absolute risk aversion, then the optimal level of transfer is decreasing in the discount rate \( r \).

**Proof.** The policyholder solves the following program

\[
\max_b E[u(w)]
\]

where \( \alpha \) is fixed. The first derivative of expected utility is

\[
\frac{\partial E[u(w)]}{\partial b} = \left[ (1 - p) (p (1 - q) + r) u'(w_0 - P) - (1 - pq) (p (1 - q) + r) u'(w_0 - P - (1 - \alpha) L) + p (1 - q) (p (1 - q) + r) u'(w_0 - P - L + b) \right].
\]

The second derivative is negative and expected utility is thus globally concave. The FOC thus determines the unique global maximum \( b^* = b^*(\alpha) \). Evaluating the first derivative at \( b = L \) yields

\[
\left. \frac{\partial E[u(w)]}{\partial b} \right|_{b=L} = -ru'(w_0 - P) + pq (p (1 - q) + r) (u'(w_0 - P) - u'(w_0 - P - (1 - \alpha) L)) < 0
\]

for all \( \alpha \in [0,1] \), i.e. \( b^*(\alpha) < L \). Evaluating the first derivative at \( b = 0 \) yields

\[
\left. \frac{\partial E[u(w)]}{\partial b} \right|_{b=0} = \left[ (1 - p) (p (1 - q) + r) u'(w_0 - P) - (1 - pq) (p (1 - q) + r) u'(w_0 - P - (1 - \alpha) L) + p (1 - q) (p (1 - q) + r) u'(w_0 - P - L) \right].
\]

For any \( r \geq 1 - p (1 - q) \) we have

\[
\left. \frac{\partial E[u(w)]}{\partial b} \right|_{b=0} < 0
\]

for all \( \alpha \in [0,1] \), i.e. \( b^*(\alpha) = 0 \) for all \( r \geq 1 - p (1 - q) \). For \( r = 0 \) we get

\[
\left. \frac{\partial E[u(w)]}{\partial b} \right|_{b=0,r=0} = \left[ (1 - p) (p (1 - q) (u'(w_0 - P - L) - u'(w_0 - P)) + pq (p (1 - q) (u'(w_0 - P - L) - u'(w_0 - P - (1 - \alpha) L))) \right] > 0
\]
for all $\alpha \in [0,1]$, i.e. $b^*(a) > 0$ for $r = 0$. Differentiating $\frac{\partial E[u(w)]}{\partial b}|_{b=0}$ with respect to $r$ yields
\[
\left. \frac{\partial}{\partial r} \frac{\partial E[u(w)]}{\partial b} \right|_{b=0} = (- (1 - p) u'(w_0 - P) - pqu'(w_0 - P - (1 - \alpha) L) - p(1 - q) u'(w_0 - P - L)) < 0.
\]

This implies that for each $\alpha$ there exists a unique $\bar{r}(\alpha) > 0$ such that $b^*(\alpha) > 0$ for all $r < \bar{r}(\alpha)$ and $b^*(\alpha) = 0$ for all $r \geq \bar{r}(\alpha)$. Taking the total differential of the FOC with respect to $r$ and $b$ yields
\[
\left. \frac{\partial^2 E[u(w)]}{\partial b^2} \right|_{b=b^*(r)} db + \left. \frac{\partial^2 E[u(w)]}{\partial b \partial r} \right|_{b=b^*(r)} dr = 0
\]

which implies
\[
\left. \frac{db^*(r)}{dr} = - \frac{\left. \frac{\partial^2 E[u(w)]}{\partial b \partial r} \right|_{b=b^*(r)}}{\left. \frac{\partial^2 E[u(w)]}{\partial b^2} \right|_{b=b^*(r)}} \right.
\]

and therefore
\[
\text{sign} \left( \frac{db^*(r)}{dr} \right) = \text{sign} \left( \left. \frac{\partial^2 E[u(w)]}{\partial b \partial r} \right|_{b=b^*(r)} \right).
\]

The cross derivative is given by
\[
\left. \frac{\partial^2 E[u(w)]}{\partial b \partial r} \right|_{b=b^*(r)} = \begin{vmatrix}
(1 - p) (p(1 - q) + r) bu''(w_0 - P) + pq (p(1 - q) + r) bu''(w_0 - P - (1 - \alpha) L) \\
\quad -p(1 - q) (1 - p(1 - q) - r) bu''(w_0 - P - L + b) \\
\quad (1 - p) u'(w_0 - P) - pqu'(w_0 - P - (1 - \alpha) L) - p(1 - q) u'(w_0 - P - L + b)
\end{vmatrix}
\]

where $R_a$ is the coefficient of absolute risk aversion which is constant under CARA. The FOC then implies
\[
\left. \frac{\partial^2 E[u(w)]}{\partial b \partial r} \right|_{b=b^*(r)} = - (1 - p) u'(w_0 - P) - pqu'(w_0 - P - (1 - \alpha) L) - p(1 - q) u'(w_0 - P - L + b)
\]
\[
< 0
\]

and therefore $\frac{db^*(r)}{dr} < 0$. ■

We now combine our results of the previous two propositions to examine the optimal ex-ante decision on both, the insurance coverage $\alpha$ and the transfer $b$.

**Proposition 3** In the model outlined above, it is optimal for the policyholder to partially insure both losses the verifiable and the non-verifiable, i.e. $b^* < L$ and $\alpha^* < 1$ for all $r$. Furthermore, there exists a critical discount rate $\bar{r}(\alpha^*) > 0$ such that it is optimal to generate a transfer if the discount rate is below $\bar{r}(\alpha^*)$ and not to generate a transfer otherwise, i.e. $b^* > 0$ if $r < \bar{r}(\alpha^*)$ and $b^* = 0$ if $r \geq \bar{r}(\alpha^*)$. Last, it is optimal to get insurance for at least one type of loss, i.e. either $\alpha^* > 0$ and/or $b^* > 0$.

**Proof.** In Proposition 2 we have shown that $b^*(\alpha) < L$ for all $\alpha \in [0,1]$ which implies $b^* < L$. Proposition 1 then implies $\alpha^* < 1$. Proposition 1 also shows that there exists $\bar{r}(\alpha^*)$ such $b^* > 0$ if $r < \bar{r}(\alpha^*)$ and $b^* = 0$ if $r \geq \bar{r}(\alpha^*)$. Proposition 2 proves $\alpha^* > 0$ for all $r \geq \bar{r}(\alpha^*)$, i.e. either $\alpha^* > 0$ and/or $b^* > 0$. ■
**B Appendix**

In this model, we consider a different mechanism that generates a transfer \( b \) from the insurer to the policyholder in case of a non-verifiable loss. Ex-ante, the policyholder chooses a broker who threatens the insurer with a reputational loss \( R \) if the insurer fails to pay a transfer \( b \). The policyholder and insurer then bargain over the transfer under the shadow of the reputational loss. We assume that the premium \( P \) is actuarially fair including the transfer \( b \) in case of a non-verifiable loss. The distribution of final level of wealth of the policyholder is the same, i.e.

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<td>( w_0 - P - L + b )</td>
</tr>
</tbody>
</table>

where \( P = pq\alpha L + p(1-q)b \). Ex-ante the policyholder chooses the level of insurance coverage \( \alpha \) and the reputational loss \( R \) to the insurer which results in a transfer \( b \).

**B.1 Ex-Post Bargaining Game**

Ex-post the policyholder and insurer engage in a Nash-bargaining game that results in a transfer \( b \) from the insurer to the policyholder. The Nash-bargaining solution is given by the maximization problem

\[
\max_b Z(b) = (R - b)(u(w_0 - P - L + b) - u(w_0 - P - L)).
\]

In the following proposition we derive some properties of the Nash-bargaining solution.

**Proposition 4** For each \( R \), there exists a unique Nash-bargaining solution \( b^*(R) \) with \( b^*(0) = 0 \) and \( 0 < b^*(R) < \frac{1}{2}R \) for all \( R > 0 \). Furthermore, \( b^*(R) \) is decreasing in the policyholder’s degree of risk aversion.

**Proof.** The first and second derivative of \( Z(b) \) are given by

\[
Z'(b) = (R - b)u'(w_0 - P - L + b) - (u(w_0 - P - L + b) - u(w_0 - P - L))
\]

\[
Z''(b) = (R - b)u''(w_0 - P - L + b) - 2u'(w_0 - P - L + b).
\]

For all \( b \geq R \) we have \( Z'(b) < 0 \) which implies \( b^*(R) \leq R \). For all \( b \leq R \), \( Z''(b) < 0 \) and any solution to the FOC \( Z'(b) = 0 \) is the unique global maximum. Furthermore

\[
Z'\left(\frac{1}{2}R\right) = \frac{1}{2}Ru'(w_0 - P - L + b) - \left(u\left(w_0 - P - L + \frac{1}{2}R\right) - u(w_0 - P - L)\right) < 0
\]

by concavity of \( u \). Concavity of \( Z(b) \) for all \( b \leq R \) implies \( b^*(R) < \frac{1}{2}R \) for all \( R > 0 \). If \( R = 0 \), then \( b = 0 \) satisfies the FOC and therefore \( b^*(0) = 0 \). Suppose a policyholder with utility function \( v \) is more-risk-averse than a policyholder with utility function \( u \). Then there exists an increasing, concave transformation \( g \) such that \( v(w) = g(u(w)) \) for all \( w \). Let \( b^*_v(R) \) and \( b^*_u(R) \) be the Nash-bargaining solution for those
policyholders. The first derivative of $Z_v (b)$ of the more risk-averse policyholder evaluated at $b_u^* (R)$ yields

$$Z'_v(b_u^*(R)) = (R - b_u^*(R)) v' (w_0 - P - L + b_u^*(R)) - v (w_0 - P - L + b_u^*(R)) - v (w_0 - P - L)$$

$$= \left[ (R - b_u^*(R)) g' (u (w_0 - P - L + b_u^*(R))) u' (w_0 - P - L + b_u^*(R)) \right]$$

$$= \left[ \begin{array}{c}
\frac{(R - b_u^*(R)) g' (u (w_0 - P - L + b_u^*(R))) u' (w_0 - P - L + b_u^*(R))}{(R - b_u^*(R)) g' (u (w_0 - P - L + b_u^*(R))) u' (w_0 - P - L + b_u^*(R))}
\end{array} \right]$$

$$= 0.$$ The inequality follows from $g$ being increasing and concave and the last equality follows from the FOC for $b_u^*(R)$. As $Z_v (b)$ is concave for all $b \leq R$ we deduce $b_u^*(R) < b_v^*(R) < \frac{1}{R}$. ■

### B.2 Ex-Ante Decisions

The following proposition derives the policyholder’s optimal choice with respect to insurance coverage $\alpha$ and the reputational loss $R$.

**Proposition 5** In the model outlined above, it is optimal for the policyholder to put the insurer’s reputation at stake and not fully insure the verifiable loss, i.e. $b^* > 0$ and $\alpha^* < 1$. Furthermore, it is not optimal to generate full insurance of the non-verifiable loss, i.e. $b^* < L$.

**Proof.** Suppose first that $\alpha \in [0,1]$ is fixed. Then the policyholder solves

$$\max_R E [u (w)]$$

where

$$E [u (w)] = (1 - p) u (w_0 - P) + p qu (w_0 - P - (1 - \alpha) L) + p (1 - q) u (w_0 - P - L + b^*)$$

subject to

$$(R - b^*) u' (w_0 - P - L + b^*) - (u (w_0 - P - L + b^*) - u (w_0 - P - L)) = 0 \quad (1)$$

and

$$P = p\alpha L + p (1 - q) b^*.$$ We have

$$\frac{\partial b^*}{\partial R} = \frac{u' (w_0 - P - L + b^*)}{(1 - p (1 - q)) Z'' (b^*) - p (1 - q) u' (w_0 - P - L + b^*) + u' (w_0 - P - L)} > 0.$$ by implicitly differentiating the FOC (1) and

$$\frac{\partial P}{\partial R} = p (1 - q) \frac{\partial b^*}{\partial R} > 0.$$
The first derivative of expected utility is

\[
\frac{\partial E[u(w)]}{\partial R} = \left[ - (1 - p) \frac{\partial P}{\partial R} u'(w_0 - P) - pq \frac{\partial P}{\partial R} u'(w_0 - P - (1 - \alpha) L) \right] + p(1 - q) \left( \frac{\partial b^*}{\partial R} - \frac{\partial P}{\partial R} \right) u'(w_0 - P - L + b^*) \\
= - \frac{\partial b^*}{\partial R} p (1 - q) \left[ (1 - p) u'(w_0 - P) + pq u'(w_0 - P - (1 - \alpha) L) \right] - (1 - p (1 - q)) u'(w_0 - P - L + b^*) \\
= - \frac{\partial b^*}{\partial R} p (1 - q) \left[ (1 - p) (u'(w_0 - P) - u'(w_0 - P - L + b^*)) + pq (u'(w_0 - P - (1 - \alpha) L) - u'(w_0 - P - L)) \right]
\]

Evaluating the first derivative at \( R = 0 \) yields

\[
\frac{\partial E[u(w)]}{\partial R} \bigg|_{R=0} = - \frac{\partial b^*}{\partial R} \bigg|_{R=0} p (1 - q) \left[ \frac{(1 - p) (u'(w_0 - P) - u'(w_0 - P - L))}{+pq (u'(w_0 - P - (1 - \alpha) L) - u'(w_0 - P - L))} \right] > 0.
\]

It is therefore ex-ante optimal to commit to bargain ex-post, i.e. \( R^* > 0 \) for all \( \alpha \in [0,1] \), or equivalently \( b^* > 0 \).

Now suppose \( R \geq 0 \) is fixed. Then the policyholder solves

\[
\max_\alpha E[u(w)]
\]

subject to

\[
(R - b^*) u'(w_0 - P - L + b^*) - (u(w_0 - P - L + b^*) - u(w_0 - P - L)) = 0.
\]

and

\[
P = pq \alpha L + p(1 - q) b^*.
\]

We have

\[
\frac{\partial P}{\partial \alpha} = pq L + p(1 - q) \frac{\partial b^*}{\partial \alpha}
\]

and

\[
\frac{\partial b^*}{\partial \alpha} = \frac{pq L (Z''(b^*) + u'(w_0 - P - L + b^*) + u'(w_0 - P - L))}{(1 - p (1 - q)) Z''(b^*) - p (1 - q) (u'(w_0 - P - L + b^*) + u'(w_0 - P - L))}
\]

by implicitly differentiating the FOC (1). The first derivative of expected utility is

\[
\frac{\partial E[u(w)]}{\partial \alpha} = \left[ - (1 - p) \frac{\partial P}{\partial \alpha} u'(w_0 - P) - pq \frac{\partial P}{\partial \alpha} u'(w_0 - P - (1 - \alpha) L) \right] + p(1 - q) \left( - \frac{\partial P}{\partial \alpha} + \frac{\partial b^*}{\partial \alpha} \right) u'(w_0 - P - L + b^*)
\]

Evaluating the first derivative at \( \alpha = 1 \) yields

\[
\frac{\partial E[u(w)]}{\partial \alpha} \bigg|_{\alpha=1} = \left[ \frac{(1 - p) \frac{\partial P}{\partial \alpha} u'(w_0 - P) - pq \frac{\partial P}{\partial \alpha} u'(w_0 - P)}{+p(1 - q) \left( - \frac{\partial P}{\partial \alpha} + \frac{\partial b^*}{\partial \alpha} \right) u'(w_0 - P - L + b^*)} \right]
\]

\[
= p(1 - q) \left( \frac{\partial b^*}{\partial \alpha} - \frac{\partial P}{\partial \alpha} \right) (u'(w_0 - P - L + b^*) - u'(w_0 - P))
\]

23
Furthermore
\[
\frac{\partial b^*}{\partial \alpha} = \frac{pqL (Z'' (b^*) + u' (w_0 - P - L + b^*) + u' (w_0 - P - L))}{(1 - p (1 - q)) Z'' (b^*) - p (1 - q) (u' (w_0 - P - L + b^*) + u' (w_0 - P - L))}
\]
\[
\frac{\partial P}{\partial \alpha} = \frac{pqLZ'' (b^*)}{(1 - p (1 - q)) Z'' (b^*) - p (1 - q) (u' (w_0 - P - L + b^*) + u' (w_0 - P - L))}
\]
and thus
\[
\frac{\partial b^*}{\partial \alpha} - \frac{\partial P}{\partial \alpha} = \frac{pqL (u' (w_0 - P - L + b^*) + u' (w_0 - P - L))}{(1 - p (1 - q)) Z'' (b^*) - p (1 - q) (u' (w_0 - P - L + b^*) + u' (w_0 - P - L))} < 0.
\]
This implies
\[
\frac{\partial E \left[ u (w) \right]}{\partial \alpha} \bigg|_{\alpha = 1} < 0
\]
and therefore \( \alpha^* < 1 \) for all \( R \geq 0 \).

Full insurance for the non-verifiable loss is generated by a reputational loss
\[
\bar{R} = \frac{u' (w_0 - P) L + (u (w_0 - P) - u (w_0 - P - L))}{u' (w_0 - P)}.
\]
Evaluating the first derivative at \( R = \bar{R} \) yields
\[
\frac{\partial E \left[ u (w) \right]}{\partial R} \bigg|_{R = \bar{R}} = -\frac{\partial b^*}{\partial R} \bigg|_{R = \bar{R}} p (1 - q) pq \left( u' (w_0 - P - (1 - \alpha^*) L) - u' (w_0 - P) \right) < 0
\]
as \( \alpha^* < 1 \). As \( b^* \) is decreasing in \( R \) we conclude \( b^* < L \) for all level of \( \alpha \in [0, 1] \).

**B.3 All Bargaining Power with Policyholder**

If all the bargaining power is with the policyholder, then the insurer pays the transfer \( b \) if and only if
\[
b \leq R.
\]
This results in an ex-post transfer \( b = R \). Ex-ante, the policyholder solves
\[
\max_{\alpha, R} E \left[ u (w) \right]
\]
where
\[
E \left[ u (w) \right] = (1 - p) u (w_0 - P) + pq u (w_0 - P - (1 - \alpha) L) + p (1 - q) u (w_0 - P - L + R)
\]
and
\[
P = pq\alpha L + p (1 - q) R.
\]
As both insurance and the transfer are priced at an actuarially fair rate, the optimal choice is \( \alpha^* = 1 \) and \( R^* = L \).
References


