Transferring the Sticky:
Individual Annuity Demand and Demographic Risk

Roman N. Schulze • Thomas Post • Helmut Gründl

Preliminary version. Please do not cite!
(July 2006)

Abstract    Demographic risk—the risk that mortality laws change in a nondeterministic way (stochastic mortality)—and its implications for various corporate risk management decisions has been the subject of recent scientific discussion. For example, Lin and Cox (2005), Dowd, Blake, and Cairns (2006) study mortality-based securities as possible risk management tools. Gründl, Post, and Schulze (2006) analyze natural hedging opportunities between term life insurance and annuity contracts. In our contribution, we show that demographic risk is also a key determinant in individual risk management decisions. We demonstrate this for the case of retirement risk management decisions, particularly focusing on how individual annuity demand is influenced by demographic risk. Demographic risk turns out to be a sticky risk, i.e., a risk that is only hardly transferable. We show that, whether the existence of demographic risk leads to an increase in individuals’ annuity demand or not, is determined by objective factors (such as the exposure of the government pension and/or the insurance industry with respect to demographic shocks) and not by subjective factors (such as individuals’ risk aversion). Subjective factors only determine the intensity of the annuity demand reaction with respect to demographic risk. We find that the consideration of demographic risk may both alleviate, but also intensify the annuity puzzle.

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Initiated by Yaari’s seminal work the study of optimal individual annuitization decisions gained increasing interest in the economics literature. Yaari (1965) showed that, in a perfect market setting, expected utility maximizers with no utility of bequest would manage the uncertainty regarding their lifetime by annuitizing their entire wealth. Under more general assumptions on utility functions this result was recently confirmed by Davidoff, Brown, and Diamond (2005). Empirical studies found, however, that only a small portion of private wealth is used to purchase annuities. Several explanations were brought forward to explain this “annuity puzzle”: due to adverse selection and transaction costs annuities are unfairly priced (Friedman and Warshawsky, 1988; Mitchell et al., 1999), constant annuity payouts in combination with borrowing and short-sale constraints may induce a suboptimal consumption profile (Brown, 2001), the existence of bequest motives (Yaari, 1965), the crowding-out effect of government pensions (Mitchell et al., 1999; Brown and Poterba, 2000) and intra-family risk sharing (Kotlikoff and Spivak, 1981; Post, Gründl, and Schmeiser, 2006).

This paper analyzes the impact of stochastic mortality, also called demographic risk, on individual annuity demand. Demographic risk refers to the fact that mortality laws and life tables describing lifetime uncertainty may change in a nondeterministic way (see Olivieri, 2001; Cairns, Blake, and Dowd, 2006). Such unexpected changes in mortality may result from, e.g., medical innovations (leading to lower mortality rates, see, e.g., Hayflick, 2000; Held, 2002; Olshansky, Hayflick and Carnes, 2002) or eating habits (in combination with an increase in the prevalence of obesity leading to higher mortality rates, see Swiss Re, 2004). The importance of demographic risk for risk management decisions of insurance companies is highlighted by recent publications (see, e.g., Lin and Cox, 2005; Cowley, and Cummins, 2005; Dowd, Blake, and Cairns, 2006; Gründl, Post, and Schulze, 2006). We show that demographic risk is also a key determinant in individual risk management, especially for annuitization, decisions.

Our investigation utilizes an intertemporal expected utility framework with borrowing and short-selling constraints and uninsurable government pension income risk. The risk-averse individual has to optimize decisions regarding consumption, saving, and how to allocate savings between a risky asset, a risk
free asset, and annuities. In accordance with Olivieri (2001) and Gründl, Post, and Schulze (2006) demographic risk is modeled as a mean-preserving shock on future survival probabilities.

We show that demographic risk does not influence optimal decisions if it is stochastically independent of other sources of income risk of the individual. We make clear that this is a direct consequence of the preference assumptions under which optimal decisions are derived. This result is in accordance with Rothschild and Stiglitz (1976) in the context of informational asymmetry, and also with Franke, Schlesinger, and Stapleton (2005, 2005a) in the context of—amongst other—exchange rates and portfolio returns.

However, for most individuals demographic risk will not be independent from other sources of income risk. For example, Gründl, Post, and Schulze (2006) (henceforth abbreviated GPS (2006)) made clear that in the solvency situation of an annuity-providing life insurer is strongly influenced by demographic risk, too. Depending on the structure of the life insurer’s contract portfolio and asset allocation, mortality shocks can lead to an at least partial insurer default. This links the distribution of annuity payment to demographic risk. Furthermore, government pension payments—especially in case of pay-as-you-go systems—may depend on demographic developments. Also, the performance of the individual’s investment into the risky asset might depend on the development of future mortality (“asset meltdown”). Finally, via the insurer’s investment into the risky asset and its link to its solvency situation the performance of the risky asset may also indirectly influence distribution of annuity payments.

After integrating these dependencies in our framework, the individual’s optimal decisions become strongly dependent on demographic risk. We show that, whether the existence of demographic risk leads to an increase in individuals’ annuity demand or not, is determined by objective factors (such as the exposure of the government pension and/or the insurance industry with respect to demographic shocks) and not by subjective factors (such as individuals’ risk aversion). Subjective factors only determine the intensity of the annuity demand reaction with respect to demographic risk. We find that
the consideration of demographic risk may both alleviate, but also intensify the annuity puzzle.

This paper is organized as follows. In section 1, we first formalize our model. As the model will be analytically tractable only under very restrictive assumptions, we afterwards will calibrate our model parameters in order to further analyze the implications of our model via numerical calculations. Our results are presented in section 2. Section 3 summarizes and derives policy implications.

1. The Model

1.1 Formalization

We consider a two period framework. Let $w_1$ denote the individual’s known initial wealth in period 1, and $c_i \geq 0, i = 1, 2$, the individual’s nominal consumption in period $i$. Denoting by $u$ the individual’s one–period utility function with respect a real monetary argument (period consumption), we assume that from the perspective of period 1, the individual evaluates total two period utility additively separably by

$$u(c_1) + \beta \cdot I_{Ind} \cdot u(c_2 \cdot \delta),$$

(1)

where $I_{Ind}$ is a Bernoulli distributed random indicator variable with success probability $p$ taking value 0, if the individual dies before period 2 (implying that there are no bequest motives), and taking the value 1 if the individual survives, so that $p$ is the individual’s survival probability. We assume that $p$ is representative for the whole population and subject to demographic risk. Similarly as done by Olivieri (2001) and GPS (2006) we model demographic risk as a symmetrically distributed additive deviation $\Delta p$ from individuals’ mean survival probability $p_0$. Thus, the survival probability $p$ is random and given by $p = p_0 + \Delta p$. By $\beta \in (0, 1)$ we denote the individual’s subjective

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1 We will summarize model parameters and their calibrations in Table 3 at the end of this section.
discount factor (expressing time preference), and $\delta > 0$ is the one–period deflator.

Let $s_R$ and $s_f$ be the amounts of money the individual invests into some risky and some risk free asset in period 1 providing him with a risky return of $s_R \cdot R$ and a risk free return of $s_f \cdot r_f$ in period 2, respectively. We assume both $s_R$ and $s_f$ to be nonnegative, implying that we allow the individual neither to short-sell the risky asset, nor to go into debt. We further assume that the individual can decide to put some money $\pi \geq 0$ into a life annuity, providing him with an annuity payment in period 2 given he survives until period 2, i.e., if $I_{ind}$ realizes as 1. Total savings, i.e., $s_R + s_f + \pi$, will henceforth be denoted as $S$.

Let us assume that the annuity payment in period 2 is subject to default risk of the insurance company having sold the contract, and that in the case of default the individual only receives a fraction $\psi \in [0, 1]$ of every € of the agreed upon survival benefit (depending, of course, on the premium $\pi$). We can understand $\psi$ as the fraction of the claims against the insurer that can be recovered by some guarantee fund. Let us denote by $d(r, \varphi) \in [0, 1]$ the insurer’s probability of ruin given realizations $r$ and $\varphi$ of $R$ and $\Delta p$. With respect to the influences of the actual outcomes of $R$ and $\Delta p$, we demand that the first derivative of $d$ with respect to its first argument is negative, while the sign of the derivative of $d$ with respect to its second argument is positive. The first of these assumptions implies that the insurer’s probability of ruin is the higher the poorer is the performance of the risky asset which appears to be plausible if we assume that the risky asset is part of the insurer’s asset portfolio, too. The second assumption implies that the insurer has rather underwritten annuity than term-life business. In case the derivative of $d$ with respect to its second argument is equal to zero, the insurer appears to be immune in his solvency situation with respect to demographic shocks.

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2 As discussed by GPS (2006), the actual dependency of the insurer’s ruin probability on the demographic shock depends on the insurer’s underwriting policy. For example, if the insurer holds a liability portfolio which mainly consists of annuity contracts the relationship between $d$ and $\Delta p$ should be positive, while for an insurer that mostly sold term life insurance contracts natural hedging effects lead to a rather negative relationship. Furthermore, an insurer may hold mortality derivatives (see, e.g., Cowley and Cummins, 2005). Then it depends on degree of coverage and the basis risk of these instruments in how far the insurer is vulnerable to demographic risk.
Note that, while $d$ is an indicator for the insurer’s idiosyncratic risk of payment default, $\psi$ may be interpreted as an indication of the unprotected exposure of the whole insurance industry against systematic risks such as asset return risk and, especially, demographic risk. In case, $\psi$ is very low, this means that the exposure to the systematic risks is very high (which could be the case, if the guarantee fund is funded too low as to provide sufficient cover against systematic risks, which hit many life insurers at the same time). On the contrary, a high $\psi$ may mean that there is some (unlimited) government guarantee to support the insurance industry in case of a systematic default.

Let us further denote by $A \geq 0$ the actuarially fair survival payment of the annuity. Accounting for the insurer’s possibility of ruin and assuming, symmetric mortality and, as in Doherty and Schlesinger (1990), ruin probability beliefs between the individual and the insurer,\(^3\) for a given ruin probability $d$ and a given survival probability $p$ the actuarially fair survival payment $A$ is determined via the pricing formula

$$\pi = r_f^{-1} \cdot p \cdot [(1 - d) \cdot A + d \cdot \psi \cdot A].$$

(2)

Thus the annuity price is the higher the lower the insurer’s ruin probability and the higher the default pay-off fraction is, i.e., the annuity price increases as product quality increases (Doherty and Schlesinger, 1990). Rearranging and now accounting for the stochastic character of the arguments of $d$ and $p$ itself, we achieve for the actuarially fair survival payment

$$A(\pi) = \frac{r_f \cdot \pi}{\int \int (p_0 + \varphi)\left[1 - (1 - \psi) \cdot d(r, \varphi)\right] \cdot dF(r, \varphi)}$$

(3)

where $F$ shall denote the joint distribution function of $R$ and $\Delta p$, and the expected value operator is with respect to $F$. The fair survival payment $A$ thus depends—beyond the individual’s decision on $\pi$—also on the default pay-off fraction $\psi$, and on the joint distribution of the risky asset return $R$ and the survival probability shock $\Delta p$. To implement premium loadings, let us further

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\(^3\) For an analysis of asymmetric information on the probability of ruin see Cummins and Mahul (2003).
introduce the parameter $\lambda \in [0, 1]$. Instead of the fair amounts $A(\pi)$ and $\psi \cdot A(\pi)$ that are paid out depending on the insurer’s solvency situation, only $\lambda \cdot A(\pi)$ and $\lambda \cdot \psi \cdot A(\pi)$ are paid out. A certain value for $\lambda$ thus stands for a multiplicative premium loading of $100 / \lambda \%$.

If now $I_{Ins}$ is a Bernoulli distributed indicator variable with success probability $1 - d(r, \phi)$ for given realizations $r$ and $\phi$ of $R$ and $\Delta p$ (i.e., which is 0 if the insurer defaults and is 1 if it does not) we can write the payment, let us call it $L(\pi)$, the individual actually receives given he survives until period 2 as

$$L(\pi) = \lambda \cdot A(\pi) [I_{Ins} + (1 - I_{Ins}) \cdot \psi]$$

(4)

which is conditionally (on the realization of $d$) binarily distributed and purely random as soon as $\psi > 0$ and $d(r, \phi) > 0$. Unconditionally, also the success probability of $I_{Ins}$ is random which transfers to $L$, too, so that $L(\pi)$ in (4) actually should be written as $L(\pi | R = r, \Delta p = \phi)$.

Let further, for a given realization $\phi$ of $\Delta p$, $Y(\phi)$ denote the individual’s income from the pay-as-you-go government pension system receivable in period 2, given the individual will be still alive then. Since pay-as-you-go pension systems strongly depend on the development of demographic variables (see, e.g., Aaron, 1966; Kotlikoff, 1979), we assume that $Y'(\phi) \leq 0$, which means that an increase (decrease) in the general (and individual) survival probability will lead to a drop (rise) or constancy in the pension payment.

The individual’s (from the perspective of period 1) random consumption in period 2 is thus given by

$$c_2 = s_R \cdot R + s_f \cdot r_f + L(\pi) + Y,$$

(5)

where $s_R$, $s_f$, and $\pi$, together with first period consumption $c_1$ are linked by the budget constraint $w_1 = c_1 + s_R + s_f + \pi$, and each of these four variables is subject to its nonnegativity constraint. The individual’s evaluation of total utility is thus given by
\[ u(c_1) + \beta \cdot I_{Ind} \cdot u((s_R \cdot R + s_f \cdot r_f + L(\pi) + Y) \cdot \delta) \]  \hspace{1cm} (6)

which, of course, is also random. To determine optimal decisions on consumption, investment, and annuity purchase, we assume that the individual maximizes the expected value of total utility, which is given as

\[
\begin{align*}
    &u(c_1) + \beta \cdot \int \int (p_0 + \varphi) \cdot \\
    &\cdot u(\delta \cdot [s_R \cdot r + s_f \cdot r_f + L(\pi | R = r, \Delta p = \varphi) + Y(\varphi)]) dF(r, \varphi) \\
\end{align*}
\]  \hspace{1cm} (7)

subject to \( w_1 = c_1 + s_R + s_f + \pi \) and \( c_1, s_R, s_f, \pi \geq 0 \).

Due to the complexity of the interrelations of influencing factors in the model and also the no-short-selling and borrowing constraints, most of the necessary calculations are not tractable analytically. We therefore determine optimal solutions numerically, what makes it necessary to calibrate our model first.

1.2 Calibration

In this section, we define the base values assigned to our model parameters. Section 2.3 contains a thorough analysis of what consequences alternative calibrations of these parameters have.

To express the risk averse individual’s intrain temporal risk preferences, we use utility function with constant relative risk aversion (CRRA), given by parameter \( \gamma > 0 \), thus for \( c > 0 \) we have \( u(c) = c^{(1-\gamma)/(1-\gamma)} \). The parameter of constant relative risk aversion \( \gamma \) is set to 3, the intertemporal subjective discount factor \( \beta \) is set to 0.95 (see, e.g., Laibson, Repetto, and Tobacman 1998). For the individual’s initial wealth, we set \( w_1 = 100 \).

To model demographic risk, we use three-point distributions as done in GPS 2006. For the state space of \( \Delta p \) we set \((-0.01, 0, +0.01)\), which means that the individual’s survival probability—starting from \( p_0 \)—will either increase by 1
percentage point, will stay constant, or will drop by 1 percentage point.\textsuperscript{4} Onto the set of measurable subsets of this state space, we set five alternative probability measures, which we choose as done in GPS, 2006. Table 1 contains the probability distributions used.

### Table 1 Probability distributions of $\Delta p$

<table>
<thead>
<tr>
<th>probability measure</th>
<th>probability that $\Delta p$ is</th>
<th>probability that $\Delta p$ is</th>
<th>probability that $\Delta p$ is</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>– 0.01 (decrease in survival probability)</td>
<td>0 (no demographic shock)</td>
<td>+ 0.01 (increase in survival probability)</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.025</td>
<td>0.950</td>
<td>0.025</td>
</tr>
<tr>
<td>3</td>
<td>0.050</td>
<td>0.900</td>
<td>0.050</td>
</tr>
<tr>
<td>4</td>
<td>0.100</td>
<td>0.800</td>
<td>0.100</td>
</tr>
<tr>
<td>5</td>
<td>0.200</td>
<td>0.600</td>
<td>0.200</td>
</tr>
</tbody>
</table>

In all distributions, main probability mass—depending on the probability model, between 60\% and 100\%—is assigned to the state of nature when $\Delta p$ realizes as 0, i.e., if $E(p) = p_0$ turns out to be the individual’s actual survival probability. The remaining mass then is distributed equally to the “deviating” states of nature, when the individual’s survival probability realizes as $p_0 – 0.01$ or $p_0 + 0.01$, respectively. For the mean survival probability we set $p_0 = 0.85$.

We vary the default pay-off fraction $\psi$ between 0 and 1. The loading factor $\lambda$ is set to 0.9.

The one-period return of the risk free asset is set to $r_f = 1.05$ (see GPS 2006). The one-period deflator $\delta$ is set to 0.9744, which is the inverse of the mean inflation rate given by the German consumer price index (CPI) between 1950 and 2003 which is about 1.0263.\textsuperscript{5}

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\textsuperscript{4} Due to lack of empirical data on this field, this number is—at least to some extent—arbitrary. We tested the sensitivity of our results with respect to this parameter. It turned out that the tendencies of the results stayed the same.

\textsuperscript{5} GPS (2006) used this time span to estimate the risk free and the risky return.
For the return $R$ of the risky asset we initially (sections 2.1 and 2.2) assume that its conditional distribution given any realization of the demographic shock is normal. We set mean and standard deviation for every conditional risky return distribution to 1.1567 and 0.3192, respectively, implying stochastical independence between $R$ and $\Delta p$ (see GPS, 2006). Introducing the asset meltdown hypothesis into our model (section 2.3), we will then drop this independence assumption. Instead, we assume the distribution of the risky return to be negatively correlated with the demographic shock.\textsuperscript{6} We model this, assuming that, again, conditional distributions of $R$ given a realization of $\Delta p$ are normal. Keeping conditional standard deviations constant at the level used before, i.e., at 0.3192, we now, however, model conditional expected values to depend on the actual realization of $\Delta p$. We do this by assuming that the conditional expectation of $R$ decreases by 2 percentage points to 1.1367 if $\Delta p$ realizes as $+0.01$ (i.e., improvement of the survival probability leads rather to a drop in asset returns) and increases to 1.1767, if $\Delta p$ realizes as $-0.01$. Table 2 summarizes the possible probability distributions of $R$.

Table 2 Probability distributions of risky return $R$

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>independence of $R$ and $\Delta p$</th>
<th>negative dependence of $R$ and $\Delta p$ (asset meltdown)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.01$</td>
<td>N(1.1567, 0.3192)</td>
<td>N(1.1767, 0.3192)</td>
</tr>
<tr>
<td>$0$</td>
<td>N(1.1567, 0.3192)</td>
<td>N(1.1567, 0.3192)</td>
</tr>
<tr>
<td>$+0.01$</td>
<td>N(1.1567, 0.3192)</td>
<td>N(1.1367, 0.3192)</td>
</tr>
</tbody>
</table>

For the payment from the government pension system in period 2, we assume the functional relationship

$$Y(\varphi) = w_1 \cdot \mathcal{Y} \cdot (1 + \alpha_Y \cdot \varphi)$$  \hspace{1cm} (8)

\textsuperscript{6} This rationale for this assumption comes from the idea, that an increase in the survival probability will lead to a slower than expected decumulation of assets (less pressure on asset prices due to selling activities) held by retirees and vice versa. That demographic variables may have an effect on asset prices is not generally accepted in the literature. For further discussion see, e.g., Abel (2001, 2003); Poterba (2001); Brooks (2002).
to be true, where the parameter $\alpha_Y \geq 0$ determines the degree of reaction of the government pension payment on a given realization $\varphi$ of the demographic shock $\Delta p$. The parameter $\vartheta \geq 0$ is some pension factor determining the fraction of initial wealth the individual expects to receive as government pension in period 2.\(^7\) A positive demographic shock on the individual’s survival probability will thus lead to a proportional drop in the individual’s pension payment in period 2 (starting with $E[Y(\Delta p)] = w_1 \cdot \vartheta$), while a negative demographic shock will lead to a proportional pension increase. We set $\vartheta$ to 0.15. The severity of the change in the pension payment for a given demographic shock $\varphi$ is determined by $\alpha_Y$, which—depending on, if $Y$ reacts on the demographic shock or not—we set $\alpha_Y$ to 1 and 0, respectively.

To model the insurer’s ruin probability $d(r, \varphi)$ for every pair of realizations $(r, \varphi)$ of $(R, \Delta p)$, we choose the log-linear functional form

$$d(r, \varphi) = \exp(\alpha_{\Delta p} \cdot \varphi + \alpha_R \cdot r + \text{const}),$$  \hspace{1cm} (9)$$

where $\alpha_{\Delta p}$ and $\alpha_R$ determine the degree of reaction of the insurer’s ruin probability on the demographic scenario and the performance of the risky asset. From the partial derivative conditions above, we demand $\alpha_R$ and $\alpha_{\Delta p}$ to be nonpositive and nonnegative, respectively. In cases, where there is no dependency of the insurer’s ruin probability on demographic risk or the performance of the risky asset, the respective reaction parameters are set to 0. Besides the natural requirement $0 \leq d(r, \varphi) \leq 1$, for every pair of realizations $(r, \varphi)$ of $(R, \Delta p)$, we additionally impose the restrictions that, first, if the realization of $R$ drops below the 0.1%-percentile of the risky asset return distribution and at the same time $\Delta p$ realizes as 0 (no survival probability shift), the insurer’s ruin probability is equal to 3%; and that, second, every ruin probability induced by a realization of $R$ increases (decreases) tenfold, if $\Delta p$ realized as $+0.01 (-0.01)$, i.e., if there is an increase (decrease) in the survival probability. In the—with respect to the insurer’s solvency situation—worst case of an asset return below about 0.126 (i.e., a loss of 87.4%) and an increase in the survival probability, the insurer will fail to meet the

\(^7\) By doing this, the CRRA feature of the one-period utility function makes our results scaleable in the period 1 wealth to pension factor ratio.
individual’s annuity payment claims with a probability of 30%. In cases, when $R$ realizes at the other end of its support, and demographic risk realizes as a reduction in survival probability, the insurer’s ruin probability is approximately 0.\(^8\) For the unconditional (expected) ruin probability $E[d(R,\Delta p)]$ (i.e., the a priori ruin probability of the insurer from the perspective of period 1) we assume that it agrees with some regulator fixed maximum ruin probability which, as done in GPS, 2006, we set to 0.01, i.e. 1% (this is what we need the fitting variable $\text{const}$ in (9) for).

The following Figure 1 summarizes the endogenous dependencies in the model:

![Diagram of model dependencies](image)

**Fig. 1** Summary of model dependencies

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\(^8\) Of course, this calibration is to some extent arbitrary. We checked the consequences of changes in these two assumptions (3% and tenfold increase) and found robustness of the tendencies of our results.
The variables and parameters used in our model and their calibration are summarized in Table 3.

Table 3 Summary of variables and parameters, and their calibration

<table>
<thead>
<tr>
<th>symbol</th>
<th>variable/parameter</th>
<th>value/specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>actuarially fair survival payment</td>
<td>see formula (3)</td>
</tr>
<tr>
<td>$\alpha_Y$</td>
<td>reaction parameter of government pension w.r.t. demographic shock</td>
<td>${0, 1}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>subjective discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$c_i$</td>
<td>consumption in period $i$</td>
<td>decision variable</td>
</tr>
<tr>
<td>$d$</td>
<td>insurer’s ruin probability</td>
<td>$d(R, \Delta p)$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>one-period deflator</td>
<td>1/1.0263</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>parameter of relative risk aversion</td>
<td>3</td>
</tr>
<tr>
<td>$L$</td>
<td>actual annuity payment</td>
<td>see formula (4)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>annuity loading factor</td>
<td>0.9</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>demographic shock</td>
<td>see Table 1</td>
</tr>
<tr>
<td>$p$</td>
<td>survival probability</td>
<td>$p_0 + \Delta p$</td>
</tr>
<tr>
<td>$p_0$</td>
<td>mean survival probability</td>
<td>0.85</td>
</tr>
<tr>
<td>$\pi$</td>
<td>annuity premium</td>
<td>decision variable</td>
</tr>
<tr>
<td>$\psi$</td>
<td>default pay-off fraction</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>$R$</td>
<td>risky return</td>
<td>see Table 2</td>
</tr>
<tr>
<td>$r_f$</td>
<td>risk free return</td>
<td>1.05</td>
</tr>
<tr>
<td>$S$</td>
<td>total savings</td>
<td>$s_R + s_f + \pi$</td>
</tr>
<tr>
<td>$s_f$</td>
<td>risk free investment</td>
<td>decision variable</td>
</tr>
<tr>
<td>$s_R$</td>
<td>risky investment</td>
<td>decision variable</td>
</tr>
<tr>
<td>$\varrho$</td>
<td>government pension factor</td>
<td>0.15</td>
</tr>
<tr>
<td>$u$</td>
<td>one-period utility function</td>
<td>CRRA</td>
</tr>
<tr>
<td>$w_1$</td>
<td>initial wealth</td>
<td>100</td>
</tr>
<tr>
<td>$Y$</td>
<td>government pension payment</td>
<td>see formula (8)</td>
</tr>
</tbody>
</table>

1.3 Solving Technique

We solve the optimization problem (7) using the MATHEMATICA® 5.1 implemented nonlinear optimizer NMaximize. To keep the optimization
problem tractable, we discretized the risky return density function using Gaussian quadrature method as done by, e.g., Cocco, Gomes, and Maenhout (2005).

2. Results

As introduction and model section made clear, our model incorporates complex dependencies between demographic risk, government pension payments, the insurer’s ruin probability, annuity payouts and the performance of the risky asset. To gain insight into how demographic risk per se is perceived by an expected utility maximizer, in section 2.1 we first analyze a situation where neither government pension payments, nor the risky return and the insurer’s ruin probability depend on demographic risk. After this, in section 2.2.1 we investigate the situation including all dependencies illustrated by Figure 1, but neglecting for the moment for the possibility of a dependency between demographic risk and the performance of the risky asset (asset meltdown). Since this kind of dependency is discussed controversially in the scientific literature (see, e.g., Abel, 2001, 2003; Poterba, 2001; Brooks, 2002), we will analyze that case separately in section 2.2.2. In section 2.3, the parameters used in our calibration are varied to generalize our results. In section 2.4, we then derive policy implications for potential insurance buyers as well for the insurance industry.

2.1 Demographic risk as an independent multiplicative background risk

From (7) it becomes apparent that, if

(i) pension system payments do not react on demographic developments (or the considered individual just does not participate in it),

(ii) annuity providers—even if with default risk—appear to be immunized against demographic shocks with respect to their solvency situation, and
(iii) demographic shocks and the risky asset return are stochastically independent from each other,

a mean preserving demographic risk does not influence individuals’ decisions at all. In this case, the survival probability in (7) can be simply drawn out and evaluated separately via the expected value operator. In case—and that is what we assumed—that \( E(p) = p_0 \), (7) has the same form and will lead to the same optimal consumption, saving and asset allocation decisions as it would, if demographic risk did not exist at all. The individual thus behaves risk neutrally towards this intertemporal risk. In fact, for the individual the risk to have used the wrong (survival) discount factor when determining optimal decisions (and thus to have consumed, e.g., too much if his survival probability turns out to go up) does not matter at all. To put it in other words: the pure existence of demographic risk does not produce any wish for risk protection, and thus does not influence the individual’s annuity purchase decision.

Even if the shock was not mean preserving but only symmetric around some mean deviation (or trend), it rather had the effect of a change in the discounting of tomorrow’s consumption than of an (additive) background risk: while the presence of a background risk generally reduces risky investment (Kimball, 1993) and increases insurance demand (Eeckhoudt, and Kimball, 1992), the introduction of demographic risk in the manner considered so far does not influence investment and insurance purchasing decisions. This result, on the first glance, seems to be surprising, is, however, a standard implication of the individual’s assumed preference structure (see Rothschild and Stiglitz, 1976; Franke, Schlesinger, and Stapleton, 2005, 2005a).9

The individual’s risk neutral perception of demographic risk changes, as soon as at least one of the three conditions (i) to (iii) above does not hold. In what follows, we will study the impacts if, first, assumptions (i) and (ii) and afterwards all three assumptions are dropped.

9 Note that for these results, it is not necessary that the insurer’s ruin probability is also stochastically independent from the performance of the risky asset return \( R \).
2.2 A first analysis of the model with dependencies on the basis of numerical examples

2.2.1 The insurer’s ruin probability depends on demographic risk and the risky return; government pension payments depend on demographic risk

In the situation considered in this section, we now assume that the insurer’s ruin probability depends on demographic risk and the risky return. Furthermore, government pension payments depend on demographic risk. Here, demographic risk becomes a sticky risk. In states of nature where the survival probability increases due to a demographic shock, the government pension system performs poor and the ruin probability of the insurer increases, i.e., also the annuity has a higher probability to perform poorly. To complete the distress exactly those cases are weighted more heavily within the expected utility evaluation (see formula (7)). On the one hand, this results from the fact that for those cases the survival probability, i.e., the weight in the expected utility evaluation, is high. On the other hand, due to the concavity of $u$ the worsening in the solvency situation of the insurance company when survival probability increases is perceived more badly than a reduction in the insurer’s ruin probability is appreciated.

For high values of the default pay-off fraction $\psi$ (and low values of $\alpha_1$) these effects are somehow alleviated, since the annuity payout (and government pension payout) reacts less on demographic risk. Thus both parameters indicate the stickiness of demographic risk.

The annuity demand reaction function w.r.t. the strength of demographic risk is shown in figure 2 for a default pay-off fraction of $\psi = 0.6$. This reaction function is henceforth abbreviated by $ADRF$. 
Annuity demand $\pi$ and annuity demand in proportion to total savings $\pi / S$ dependent on $\text{Pr}[\Delta p = 0]$, i.e., the probability that the demographic shock is 0 ($ADRF$); default pay-off fraction $\psi = 0.6$.

In this situation, it can be seen that insurance demand increases in absolute terms and also in proportion to total savings with the strength of demographic risk, i.e. with decreasing probability that the demographic shock is 0.\(^{10}\) This increase in insurance demand as demographic risk increases is the reaction one would intuitively expect. The actual causal mechanisms leading to such positively sloped $ADRF$, however, are rather complex. As demonstrated in section 2.1, there is no incentive for the individual to protect herself against demographic risk per se, for example, by buying annuities. Furthermore, due to the additional dependency between the insurer’s ruin probability, the annuity default and demographic risk, the annuity can only hardly be used to hedge against this risk. Things have to be different. As government pension

\(^{10}\) In our results, total savings react only very slightly on demographic risk. The reaction is in absolute terms and relative terms almost equal. For the rest of our contribution we only show relative values, i.e., portfolio shares.
payments negatively depend on the realization of the demographic shock, demographic risk increases the background risk stemming from the government pension system and therefore increases the individual’s wish to create a safer asset allocation (cp. Kimball, 1993; Elmendorf and Kimball, 2000).\footnote{Total savings decrease slightly with increasing demographic risk. This is the normal (prudent) reaction of a CRRA investor to an increase in background risk, see Deaton (1991); Elmendorf and Kimball (2000).} For this, the individual shifts savings out of the risky asset, as illustrated in Figure 3. To answer the question into which other assets savings should be shifted, the individual compares the specific payment characteristics of the risk free asset and the annuity. In case, the annuity is risk free itself (and not loaded too heavily), it will clearly dominate the risk free asset due to the inherent mortality premium on top of the risk free return (see, e.g., Brown, 2001). If the annuity becomes risky, it now depends on the comparison between the situation-dependent certainty equivalent of the annuity with the risk free investment. If $\psi$ is high, this comparison ends in favor of the annuity leading to a zero demand for the risk free investment.\footnote{Note that—although risky and dependent on demographic risk—government pension income is partly risk free, and thus serves to some extent as a substitute for risk free investment (see Cocco, Gomes, and Maenhout, 2005). Without government pension income, i.e., $\vartheta = 0$, the individual indeed invests a part of his savings risk free.}
Fig. 3 Risky investment in proportion to total savings $s_R / S$ dependent on $\Pr[\Delta p = 0]$, i.e., the probability that the demographic shock is 0; default pay-off fraction $\psi = 0.6$

If the default pay-off fraction $\psi$ of the annuity shrinks, the risk free investment becomes comparatively more attractive, and finally, it will be used for saving. This is the case in Figure 4, where $\psi$ is 0; i.e., in case of ruin of the insurer, the individual receives no payment from the annuity at all.\textsuperscript{13}

\textsuperscript{13} This extreme case is what Wakker, Thaler and Tversky (1997) called probabilistic insurance.
**Fig. 4** Annuity, risky, and risk free investment in proportion to total savings $S$ dependent on $\text{Pr}[\Delta p = 0]$, i.e., the probability that the demographic shock is 0; default pay-off fraction $\psi = 0$

**ADRF** now decreases with increasing demographic risk.\(^{14}\) Moving from $\text{Pr}[\Delta p = 0] = 1$ to $\text{Pr}[\Delta p = 0] = 0.95$, i.e., introducing demographic risk, the emerging (government pension) background risk induces the individual to reduce annuity demand and to shift savings into the risky asset, which now, for $\psi = 0$, appears to be the less risky investment opportunity (for $\psi = 0.6$ this shift did not take place). The risk free investment, however, for this demographic probability model is still not attractive enough (or in other words: the background risk is not severe enough to induce risk free savings). In fact, the individual would even go into debt, if the borrowing constraint did not keep her from doing so. For lower values of $\text{Pr}[\Delta p = 0]$, higher demographic risk and the thereby induced background risk now leads to the situation that any portfolio of annuity and the risky asset alone is too risky for

\[^{14}\] This reaction of the individual to demographic risk can be observed for $\psi < 0.2$. Inside the interval $0.2 \leq \psi \leq 0.3$ the annuity demand curve reverses its slope, by changing its shape from a decreasing function to a parabola, and from there ($0.3 < \psi$) to an increasing function.
the individual and she begins to shift savings from annuities and the risky investment into the risk free investment. Due to the high systematicness of demographic risk, now $ADRF$ decreases. Note, that this decrease is lower in the region where the borrowing constraint is nonbinding. This can be explained as follows: the slope of the $ADRF$ region where the borrowing constraint is nonbinding can be seen as the individual’s “normal” reaction on demographic risk. If there was no borrowing constraint, for $\Pr[\Delta p = 0] = 1$ the individual would go into dept. However, as this is not possible, the individual seeks another way to get some risk into her portfolio: she is getting more tolerant to demographic risk. This leads to a less strong reaction to demographic risk in the constraint regions.

From a comparison of annuity demand shown in Figures 2 and 4 it can also be seen, that a reduction in the default pay-off fraction $\psi$ does not only reverse the portfolio decisions for different strengths of demographic risk. Also, given a certain value of $\Pr[\Delta p = 0]$, a decrease in the default pay-off fraction $\psi$ leads to a reduction in annuity demand, which is the normal reaction: a reduction of $\psi$ does neither change the expected value of the return from the annuity nor its actuarial fairness. However, it makes the return more volatile, increases the interval between possible payments, and increases the annuity’s exposure to demographic risk (hereby also increasing the positive dependence on the government pension payment; see formula (4)). Due to risk aversion and the prudence feature of $u$ this leads to a lower annuity demand (cp. Kimball, 1993; Elmendorf and Kimball, 2000).

### 2.2.2 The insurer’s ruin probability depends on demographic risk and the risky return; government pension payments depend on demographic risk; the risky return depends on demographic risk (asset meltdown)

In this section the asset meltdown hypothesis is introduced into the model. Until now, only the pay-off from the annuity was linked—via the insurer’s ruin probability—on both systematic risks: demographic risk and the performance of the risky asset. So far these risks were assumed to be stochastically independent from each other. The asset meltdown hypothesis now links demographic risk and the return of the risky asset directly, in a way
that a combination of both risks becomes more undesirable to the individual. Furthermore, if the individual decides to hold the risky asset, also government pension payments become riskier. This is because the dependency between demographic risk and the return of the risky asset implicitly links also government pension payments to the performance of the risky asset (see Figure 1). Finally, an asset meltdown increases the stickiness of demographic risk, since the states of nature that are weighed more in the expected utility evaluation (increases in the survival probability) additionally come along with rather poor returns from the risky asset.

Consequently, the tendencies shown in Figures 2 and 4 in section 2.2.1 become stronger pronounced if the asset meltdown hypothesis is introduced. Figure 2a shows $ADRF$ for the case of default pay-off fraction $\psi = 0.6$.

**Fig. 2a** Annuity demand proportion to total savings $S$ dependent on $Pr[\Delta p = 0]$, i.e., the probability that the demographic shock is 0 ($ADRF$); default pay-off fraction $\psi = 0.6$

The additional dependence of both, the risky asset and the government pension payments on the performance of the risky asset return, leads—in the case $\psi$ is
rather high—to a higher demand for the rather safe risky asset, the annuity (cp. 
Viceira, 2001; Cocco, Gomes, and Maenhout, 2005). This is because the just 
introduced dependence in the first instance disturbs the payoff characteristics 
of the risky asset. The annuity also before was linked to both demographic and 
risky return risk, so that the new dependence affects the (for $\psi$ high in any 
case rather safe) annuity only slightly.

If the annuity becomes more and more risky as $\psi$ decreases, $ADRF$ goes down 
(shown in Figure 4a) and, as before (see Figure 4), the individual shifts some 
of her wealth into the risk free asset (not shown here).

**Fig. 4a** Annuity demand proportion to total savings $S$ dependent on $Pr[\Delta \rho = 0]$, i.e., the probability that the demographic shock is 0 ($ADRF$); default pay-
off fraction $\psi = 0$

Both figures also contain the independence case discussed in section 2.1, 
where annuity demand did not react on demographic risk at all. The fact that—
compared to this case—annuity demand increases (decreases) for higher 
(lower) values of the default pay-off fraction $\psi$ underlines the arguments made 
above. For higher values of $\psi$ (Figure 2a) the individual reacts to the increase
in risk stemming from the introduction of dependencies by shifting savings from the risky asset to the comparatively less riskier annuity. For lower values of \( \psi \), again, the reaction reverses (see Figure 4a).

### 2.3 A deeper analysis of impact of model parameters on ADRF

The numerical examples of section 2.2 revealed that there are several influences that have to be considered regarding the individual’s annuity demand reaction w.r.t. demographic risk (ADRF). In particular, we observed that ADRF may have a negative or positive slope, i.e., in some situations the individual reacts on demographic risk by decreasing, in other cases by increasing the portfolio share of annuities. The sign of the slope of ADRF seems to be controlled by the default fraction pay-off parameter \( \psi \). In what follows we will enter a deeper analysis of the mechanisms of our model.

When we vary the parameters \( \gamma, \beta, \lambda, p_0, \alpha_Y \), the strength of the demographic shock, the strength of the mean return shift in case of the asset meltdown, and \( \theta \) we observe fundamental differences in the kind of the influences of the respective parameters: some parameters (\( \gamma \) and \( \beta \)) control only the intensity of the individuals reaction to demographic risk, i.e., the magnitude of the slope of ADRF, whereas other parameters (\( \psi, \lambda, p_0, \alpha_Y \), the strength of the demographic shock, the strength of the mean return shift in case of the asset meltdown, and \( \theta \)) control also the direction of reaction of the individual, i.e., the sign of the slope of ADRF. Moreover, emphasizing our results of section 2.2.1, the question if the borrowing constraint binds or not also turned out to be a factor that influences the magnitude of the slope (but not the sign) of ADRF. In the following we will discuss these influences in detail. We start in section 2.3.1 with parameters that do not influence the sign of the slope of ADRF, next, in section 2.3.2 we examine the other parameters, that are even able to change the sign of the slope of ADRF. Section 2.3.3 draws general conclusions from the preceding sections.
2.3.1 Parameters influencing only the magnitude of the slope of $ADRF$

An increase in the relative risk aversion parameter $\gamma$ generally leads to safer asset allocations (see Gollier, 2001, chapter 4). From the discussion of Figures 2 and 4 in section 2.2.1, for the way the individual chooses to achieve this safeness, it is of great importance, first, whether the annuity is rather safe ($\psi$ high) or not ($\psi$ low) and, if $\psi$ is low, second, whether the borrowing constraint is binding or not. If $\psi$ is rather high ($ADRF$ increasing) and, thus, the annuity appears to be more attractive for the individual than the risk free asset, an increased wish for a safe asset allocation will lead to an even stronger shift out of the risky asset into the (safer) annuity than for a lower $\gamma$. The increasing $ADRF$ becomes steeper. In case, $\psi$ is rather low ($ADRF$ is decreasing) and the borrowing constraint is binding, our discussion of Figure 4 showed that individual shows some risk tolerance towards demographic risk. As risk aversion goes up, however the degree of this additional risk tolerance, is much more sensitive to an increase in demographic risk as when $\gamma$ is low. Thus, the individual will react more strongly on increasing demographic risk: The falling $ADRF$ becomes steeper. Only, if the borrowing constraint is nonbinding and the individual can react on increasing demographic freely, we find a decrease in reaction intensity: for a higher $\gamma$, the individual invests a higher amount into the risk free asset and chooses a lower exposure to demographic risk (via purchasing annuities) than if $\gamma$ is low. Due to her lower exposure, the individual’s adjustments in the annuity portfolio share will be less pronounced than for lower $\gamma$. Thus $ADRF$ becomes flatter.

The influence of the subjective discount factor $\beta$ on the optimal decisions depends on whether the individual receives government pension income or not. Generally, an increase in the subjective discount factor $\beta$ leads to higher total savings since the individual puts a larger weight on future consumption. In case the government pension income is 0 ($\vartheta = 0$) the individual’s asset allocation is independent of $\beta$ for CRRA utility (see, e.g., Gollier, 2001, chapter 19). In case the individual receives pension income ($\vartheta > 0$), the pension income position works to some extent as a substitute for risk free savings (see, e.g., Cocco, Gomes, and Maenhout, 2005). Higher total savings due to an increase in $\beta$ naturally lead to an increase in the relation of total savings to pension income. This relative reduction of a risk free position
induces the individual to choose a safer asset allocation in his savings, or in other words: the individual behaves as if she was more risk averse. Thus the effects of an increase in $\beta$ onto the slope of the annuity demand curves are the same as in case of an increase in $\gamma$. Table 4 summarizes the results of this subsection.\textsuperscript{15}

Table 4 Influence of variation of model input parameters $ADRF$

<table>
<thead>
<tr>
<th>influence of increase in</th>
<th>magnitude of absolute slope of $ADRF$ in case the borrowing constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>relative risk aversion $\gamma$</td>
<td>$\downarrow$ $\uparrow$</td>
</tr>
<tr>
<td>subjective discount factor $\beta$ (for $\vartheta &gt; 0$)</td>
<td>$\downarrow$ $\uparrow$</td>
</tr>
</tbody>
</table>

2.3.2 Parameters influencing the sign of the slope of $ADRF$

Whereas $\gamma$ and $\beta$ control the behavior in a given risk situation, the parameters $\psi$, $\lambda$, $p_0$, $\alpha_Y$, the strength of the demographic shock, the strength of the mean return shift in case of the asset meltdown, and $\vartheta$ (which we will analyze here) control the situation itself, thus leading to changes in the way the individual reacts on demographic risk, i.e., the sign of the slope of $ADRF$. At this, it turns out that only for variations in $\vartheta$, the strength of the mean return shift in case of the asset meltdown, and $\psi$ it is of importance, if the borrowing constraint is binding or not.

A decrease in the loading factor $\lambda$ leads to a proportional drop in annuity payments for a given annuity premium which makes the annuity—compared to alternative investment opportunities—less attractive in all states of nature. Annuity demand shrinks (see, e.g., Mitchell, 1999; Brown and Poterba, 2000).

\textsuperscript{15} Note that, as made clear from the discussion in section 2.1, that if the demographic shock—instead of being mean-preserving—is only symmetric around some mean shift in survival probability, this can be decomposed into a deterministic increase in $\beta$ and a mean preserving shock. An additional assumption of a trend for changes in survival probability would thus have the same effect as an increase in the subjective discount factor $\beta$.
An increase in the volatility of annuity payouts with increasing demographic risk is penalized now more heavily. If the slope of $ADRF$ already had a negative sign (compare Figure 2), a decrease in $\lambda$ makes $ADRF$ even steeper. The slope of a positive $ADRF$ (compare Figure 4) becomes flatter, the individual shifts savings with increasing demographic risk more slowly to the now less attractive annuity. If the annuity actuarially becomes too unfair, the slope of $ADRF$ even reverses and becomes negative. Thus a decrease in $\lambda$ turns $ADRF$ clockwise.

Regarding a variation of the mean survival probability $p_0$ we have to consider two effects: on the one hand, from formula (7) it follows that an increase in $p_0$ is equivalent to an increase in the mean “overall” discount rate $\beta \cdot p_0$ which leads to the same tendencies as in the case of an increase in $\beta$. On the other hand, from formula (3) follows that an increase in $p_0$ leads to a drop in the mortality credit of the annuity. Thus the expected return from the annuity shrinks similar to the case of a decrease in $\lambda$. Again, in all states of nature the annuity becomes less attractive compared to alternative investment opportunities, and annuity demand decreases (see, e.g., Mitchell et al., 1999). In our calibration the second effect dominates the first, and thus an increase of $p_0$ leads to the same (however more moderate) tendencies as a decrease in $\lambda$, $ADRF$ turns clockwise.

An increase in the reaction parameter of government pensions w.r.t. demographic shocks, $\alpha_y$, makes government pension payments more volatile, i.e., background risk and the stickiness of demographic risk increase. Since annuity payouts are positively correlated with government pension payments, the annuity becomes less attractive relative to other (uncorrelated) assets, similar as in the case of a decrease in $\lambda$. Consequently, the tendency of change in the slope of annuity demand curves is the same as in the case of a decrease of $\lambda$. $ADRF$ turns clockwise.

Same as for $\alpha_y$, also an increase in the strength of the demographic shock (in the base case it was set to $\pm 0.01$) leads to an increase in background risk. It also makes the stickiness of demographic risk more pronounced, since bad states of nature are weighted even more heavily in the expected utility evaluation. Thus an increase in the strength of the demographic shock leads to
the same tendencies as an increase in $\alpha_Y$, just more pronounced: $ADRF$ turns clockwise.

Increases in the strength of the mean return shift in case of the asset meltdown link the risky asset to demographic risk and make it even more risky. Here, the effect on the $ADRF$ depends on whether the borrowing constraint binds or not. If the borrowing constraint binds the only way for the individual to react on stronger demographic risk is to reduce the risky position in favor of the annuity, $ADRF$ turns counterclockwise (compare Figure 2a). If the borrowing constraint does not bind, the individual increases her risk free position in disfavor of the annuity the more, the stronger demographic risk and mean return shift in case of the asset meltdown are, $ADRF$ turns clockwise (compare Figure 4a).

Section 2.2 already indicated, that the default pay-off fraction $\psi$ is important for the $ADRF$. Next, we examine this parameter in detail. We observed, that this parameter influences $ADRF$ dependent on if the borrowing constraint binds or not. For low $\psi$ we saw in section 2.2, that the risk free asset is an attractive investment, and thus the borrowing constraint does not bind, the annuity is a rather risky asset. $ADRF$ was shown to have a negative slope. Increasing $\psi$ here leads to a steeper $ADRF$, it turns clockwise. Although the risk return trade-off of the annuity improves, he is less willing to hold this asset with increasing demographic risk. The reason for this is that we are in the case where $\psi$ is low, i.e, demographic risk—when it occurs—hits the individual more strongly. When increasing a $\psi$ on a low level, the risk exposure of the annuity stays still high. Nevertheless, especially for $Pr[\Delta p = 0] = 0$, the individual increases the position of the annuity with increasing $\psi$ due to the better risk return trade-off. But, since the individual is now more invested into the annuity in the first place he reacts more sensitively to demographic risk. If demographic risk is introduced, $Pr[\Delta p = 0] > 0$, the higher but nevertheless still low $\psi$ is not able to compensate the individuals fear of demographic risk. This changes as soon $\psi$ takes values high enough to dominate the risk free asset, i.e., to make the borrowing restriction bind. With increasing $\psi$ $ADRF$ turns counterclockwise, and finally the sign of its slope even turns positive (see Figure 2).
For variations in the government pension income factor $\vartheta$, the influence on $ADRF$ is less clear-cut. Figure 5 contains the individual’s annuity demand reaction on the strength of demographic risk for given values of $\vartheta$. It plots the difference of the annuity share in the individual’s savings between the cases where the probability of a zero demographic shock $\Pr[\Delta p = 0]$ is 0.6 or 1 against $\vartheta$.\(^{16}\)

![Figure 5: Demographic risk induced changes in the annuity demand proportion of total savings $S$ depending on the government pension income factor $\vartheta$](image)

Here, the value of the default-payoff fraction parameter $\psi$ is of great importance. Looking at a government pension income factor $\vartheta = 0.15$ and the curve where $\psi = 0.6$ we see that the sign of the slope of $ADRF$ is positive (the curve in Figure 5 takes positive values), which refers to the case shown in Figure 2. When moving to the right, i.e., increasing government pension income, the sign of slope of $ADRF$ reverses at about $\vartheta = 0.43$ (the curve for $\psi = 0.6$ in Figure 5 takes negative values). That means, for higher $\vartheta$ (opposite to

\(^{16}\) Thus we look on the left and right boundary of the abscissa of Figures 2 and 4. Other pairs of $\Pr[\Delta p = 0]$ delivered the same tendencies.
Figure 2), the higher the demographic risk is the less the individual invests into the annuity. For $\vartheta > 0.71$ the curve is equal to 0. Here, the individual in both probability models does not buy any annuities (therefore, also the difference in the portfolio shares is 0). The large pension income crowds out private annuity demand (see, e.g., Mitchell et al., 1999; Brown and Poterba, 2000). The reason for the over-all negative slope of the curve $\psi = 0.6$ in Figure 5 is the increasing exposure of the individual to demographic risk as $\vartheta$ increases. The positive dependence of the annuity on pension income makes annuities less attractive, the higher the individual’s government pension income is. At $\vartheta = 0.43$ this effect becomes so strong, that the total risk situation—which so far was perceived by the individual so safe that she reacted on demographic risk by an increase in annuity demand—now turns into a rather risky situation where demographic risk reduces annuity demand. $ADRF$ turns clockwise.

For low values of the default pay-off fraction $\psi$, e.g., for $\psi = 0$, in Figure 5, the sign of the slope of $ADRF$ stays negative but the impact of an increase in government pension income is more complex. To understand the effects, in Figure 6, we additionally plotted the risk free investment of the individual.
For very low values of $\vartheta$ (between 0 and about 0.15), increasing demographic risk leads to a harsh reduction in the annuity portfolio share which even gets more pronounced when $\vartheta$ increases. The reason is the same as above: the annuity payment for $\psi = 0$ reacts very strongly on demographic risk and thus is strongly positively correlated with government pension income (this explains the highly negative value of the curve for very low values of $\vartheta$). As $\vartheta$ increases (in-between the interval [0, 0.15]), the individual, on the one hand, reacts by reducing annuity demand more strongly with increasing demographic risk (which is due to the described higher exposure to demographic risk). This is the same effect we observed for $\gamma$ and $\beta$. On the other hand, the individual reduces the risk free investment (due to the higher absolute risk free income part in the government pension income payment). At one point, however, due to the borrowing constraint, a further reduction of the risk free investment is not possible any more (see $\vartheta$ around 0.15 to 0.20 in Figures 5 and 6). That means, although—due to the higher risk free part of the pension income—the individual becomes less risk averse, she cannot react by simply reducing her risk free investment. Instead, she wants to increase the

**Fig. 6** Risk free investment in proportion to total savings $S$ dependent on the government pension factor $\vartheta$; default pay-off fraction $\psi = 0$
riskiness of the remaining portfolio. In the course of this, she becomes less sensitive to the positive correlation between annuity and pension income, leading to a less strong decrease in her demand curve with respect to demographic risk (the curve in Figure 5 becomes less negative). This is what we already observed for decreasing $\gamma$ and $\beta$. This effect, however, then again is caught up by the described effect of the increasing exposure to demographic risk as $\theta$ more and more increases. This again leads to a reversion of the curve in Figure 5, meaning that the individual again becomes more sensitive to the increasing positive correlation between annuity and pension income as demographic risk rises. For $\theta > 0.705$, also here the individual reduces annuity demand to 0 for both demographic models.

The influence of variations of model input parameters on $ADRF$ discussed in this section is summarized in Table 4.

**Table 4** Influence of variation of model input parameters on $ADRF$

<table>
<thead>
<tr>
<th>influence of increase in</th>
<th>turning direction of $ADRF$ in case the borrowing constraint</th>
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<tbody>
<tr>
<td></td>
<td>does not bind</td>
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<tr>
<td>loading factor $\lambda$</td>
<td>$\bigcirc$</td>
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<tr>
<td>mean survival probability $p_0$</td>
<td>$\bigcirc$</td>
</tr>
<tr>
<td>reaction of government pensions w.r.t. demographic shocks $\alpha_Y$</td>
<td>$\bigcirc$</td>
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<tr>
<td>strength of demographic shock</td>
<td>$\bigcirc$</td>
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<tr>
<td>strength of mean return shift in case of the asset meltdown</td>
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</tr>
<tr>
<td>default pay-off fraction $\psi$</td>
<td>$\bigcirc$</td>
</tr>
<tr>
<td>government pension income factor $\theta$ for given a low $\psi$</td>
<td>indeterminate</td>
</tr>
<tr>
<td>government pension income factor $\theta$ for given a high $\psi$</td>
<td>$\bigcirc$</td>
</tr>
</tbody>
</table>
2.3.3 Summary of section results

As made clear in the previous sections, model parameters mainly can be classified into two groups. On the one hand, there are those parameters influencing the individuals risk perception ($\gamma$ and $\beta$). On the other hand, there are those parameters that influence the risk situation itself ($\psi$, $\lambda$, $p_0$, $\alpha_Y$, the strength of the demographic shock, the strength of the mean return shift in case of the asset meltdown, and $\vartheta$). While the parameters from the first group, which are all subjective parameters, only determine the intensity of the individuals reaction on demographic risk, the parameters from the second group, which are all objective parameters, also influence the direction into which the individual’s annuity demand curve reacts on demographic risk. In the numerical examples presented in section 2.2, we demonstrated this by varying $\psi$ in a way that, in one case ($\psi = 0.6$), the annuity was perceived rather safe, while in the other case ($\psi = 0$), the annuity was received rather risky due to its exposure to the systematic risks. As we have seen, the sign of the slopes of the individual’s respective ADRF changed. From the analyses in section 2.3.2, we now know that this result also would have appeared, if we had varied one (or more) of the other objective parameters of the second group (i.e., parameters influencing the risk situation) such that the annuity exposure to systematic risks in general, and demographic risk in particular is increased. Any constellation of objective parameters of the second group thus set up a risk situation determining the sign of the slope of the individual’s ADRF. The intensity of reaction within this risk setting is then determined by the individual’s subjective risk perception parameters. That means that even if $\psi$ is rather high, a high value of the government pension reaction parameter $\alpha_Y$ can lead to a negatively sloped ADRF. This turn in the ADRF then cannot be reversed anymore, no matter how low are the individual’s risk aversion or weight of tomorrow’s consumption utility, i.e., how far ever $\gamma$ or $\beta$ are decreased within their domains.

2.4. Policy implications

What are the implications that can be derived from our results? The first and most important one is that demographic risk per se does not increase the
demand for annuities. It rather depends on the riskiness of the whole situation in which the potential insurance buyers decide on how much of their wealth they are willing to annuitize. Parameters influencing the risk situation can be

- such that are individual specific (e.g., the extent of claims on government pension payments, or the expected survival probability),
- such that are valid for all potential insurance buyers (e.g., the extent to which systematic insurance defaults can be absorbed, or in how far asset returns and/or the stability of the government pension system is exposed to demographic shocks), and also
- such that are controllable by the insurer (e.g., exposure to systematic risks or choice of premium loading).

All objective parameters together determine, if a potential insurance buyer will react with an increased or an decreased demand for annuities if new information on the characteristics of demographic risk become available, e.g., by the publication of a new study in this field. At this, it is less the trends that are expected for future developments in mortality, but rather the precision of the survival probability forecasts (with respect to both the extent and the probability of possible deviations) that deliver information of interest.

For most people (future) government pension income will be very high when compared to present wealth. For those people that already have a high exposure to demographic risk, annuities, e.g., become more attractive with increasing demographic risk, if the guarantee fund is working that well, that it takes away—at best—any of the insurance industry’s exposure to demographic risk (in case, also an asset meltdown is considered as possible, the guarantee fund additionally should also reduce systematic exposure of annuity payments to asset return risks), or the insurer itself provides for a very robust solvency situation. For other potential insurance buyers that have a rather low or no government pension income at all, also at lower degrees of safeness in the whole risk situation, higher uncertainty about future mortality can even lead to an increase in annuity demand. In so far, our results may both alleviate, but also intensify the annuity puzzle. In any case, potential insurance buyers should be aware to check their exposure to demographic risk when it
comes to an annuity purchasing decision. At this, the direction of their demand reaction does not depend on their subjective risk perception characteristics.

If additionally considering the costs that come along with the respective measures, our model can help the insurance industry finding answers for example to the following questions:

- In how far can strategic decisions with respect to premium policy, or to take measures that aim at an decrease in the idiosyncratic (via asset liability management) or the systematic risk exposure (via financing guarantee funds) serve to stimulate annuity demand in dependence of the severity of demographic risk?
- In how far will an increased research activity with respect to the actual changes in mortality rates lead to an increase or a decrease in annuity demand?
- What will be the consequences, if the asset meltdown hypothesis turns out to be true?

3. Summary

In our paper, we studied the impact of the uncertainty of future mortality developments (demographic risk) on individuals’ annuity demand. Within an expected utility maximization framework, we showed that demographic risk does not influence annuity purchasing decisions if it is stochastically independent from all other sources of income risk which, however, for most potential insurance buyers is not true. We showed that, whether the existence of demographic risk leads to an increase in individuals’ annuity demand or not, is determined by objective factors (such as the exposure of the government pension system and/or the insurance industry with respect to demographic shocks) and not by subjective factors (such as individuals’ risk aversion). Subjective factors only determine the intensity of the annuity demand reaction with respect to demographic risk. Our results show that the consideration of demographic risk may both alleviate, but also intensify the annuity puzzle.
References


