Can Social Welfare Explain the
Non-Participation Puzzle in Asset Markets?

Fangyi Jin\textsuperscript{1}
Department of Economics, Room F330
University of Konstanz
78457 Konstanz, Germany
Tel: +49(0) 7531 88 4326 Fax:+49(0) 7531 883120
Email: fangyi.jin@uni-konstanz.de
www.wiwi.uni-konstanz.de/jackwerth/FangyiJin/fangyi_jin.htm

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Abstract

This paper quantifies the effects of the means-tested social welfare (insurance) program on individuals’ savings behavior and portfolio choices by solving numerically a life-cycle model in the presence of social welfare and zero income risk. The paper shows that less wealthy households tend to either choose to quit the asset markets completely or increase the portfolio risk. Social welfare depresses savings through two channels: the jump effect induces poor households to increase current consumption dramatically; the gradual effect imposes a declining consumption-wealth ratio in wealth. Both effects can drive the non-participation in asset markets. Serving like a put option, social welfare gives less wealthy households the incentive to increase the portfolio risk conditional on participation. Last but not least, the paper calibrates the non-participation rate by using the PSID data and shows that social welfare plays a crucial role in explaining the non-participation puzzle in asset markets.
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1. Introduction

This paper introduces zero income risk and a means-tested social welfare (insurance) program into the standard life-cycle model and solves numerically for the optimal portfolio and consumption decisions. Based on this model, the paper calibrates the non-participation rate (the ratio of non-participants in asset markets over total households) of the 1984 wave of the PSID data and compares it with the true rate. With reasonable parameters and restriction of minimum investment, the non-participation rate is predicted to be 25% for high school graduates. The accuracy of the prediction amounts to 77% and is statistically significant. The result shows that social welfare is a main factor in explaining the non-participation puzzle in asset markets.

The non-participation puzzle is one of the puzzling facts in the empirical studies of household portfolio choices. Many households, particularly younger and poorer ones, appear to hold no equities at all. Bertaut and Haliassos (1997) report that in the 1983 Survey of Consumer Finances (SCF), only 20% of households held stocks or mutual funds directly. About another 15% are likely to have held stocks in defined contribution (DC) pension plans. Mankiw and Zeldes (1991) report a similar rate of stockownership in the 1984 Panel Study of Income Dynamics (PSID). Therefore non-participation in the stock market is a robust empirical finding. This fact is inconsistent with simple frictionless models of optimal portfolio choice which stem from the Merton’s (1969) continuous-time theory of optimal lifetime consumption and portfolio choice. These models all suggest households to hold positive amount of stocks, or in a more general sense risky assets, regardless of age and wealth.

The most popular explanation in the present literature to this puzzle is attributed to the fixed participation cost. Campbell, Cocco, Gomes, and Maenhout (1999), Gomes and Michaelides (2002), and Vissing-Jorgensen (2002) demonstrate that the low demand for saving can lead to a decision of non-participation in asset markets if the
investors must bear some fixed cost which makes risky assets unattractive. This explanation implies the importance of saving motive. Although saving motive has been extensively studied in the consumption literature, it is unfortunately not yet fully investigated in this literature. Therefore, this paper is motivated to explain the non-participation puzzle by focusing on investors’ saving motive.

Social welfare provides the minimum living expense and is means-tested. It is a social insurance conditional on not only a certain event such as unemployment but also wealth. To my best knowledge, social welfare has not been modeled in the household portfolio optimization problems. Yet the government spending on means-tested programs amounts for 1 to 1.3 percent of GDP in US (Feldstein 2005). Empirical data show that about 5% households receive zero income other than social welfare, transfer from relatives, and other transfers. Interestingly, about 0.5% households receive zero income including all possible income sources. If this is not a data error, then it has to be the case in which households are unemployed but not poor enough to be qualified to social welfare. This shows that personal income is not strictly positive. The above facts motivate this paper to model zero income risk and social welfare at the same time.

This paper is related to Carroll (1992, 1997) and Michaelides (2003) from the consumption literature and Cocco, Gomes, and Maenhout (2005) from the household portfolio literature, but extends their studies in several important dimensions. First, social welfare is modeled as the insurance conditional both on unemployment and wealth. Carroll (1992, 1997) introduces the zero income risk and suggests that the typical poor households have a precautionary saving motive when they face some disastrous income risk. Michaelides (2003) varies the level of available insurance during unemployment and shows that households’ saving motive is much lower with insurance than without. However, insurance like social welfare is also strictly conditional on wealth. I compare my results with the ones in Michaelides (2003) and find that social welfare has a special jump effect which induces investors to jump to consume all when they are poor enough. On the other hand, the gradual effect which makes the saving motive exhibit an increasing pattern in wealth can be found in both
cases. I further show that employed investors are dominated by the *gradual* effect while unemployed investors are impacted by both effects.

Regarding the portfolio choice, wealth-dependent social welfare results in the gambling behavior for relatively poor and/or unemployed investors. The risk taking is non-linearly and non-monotonically decreasing in wealth. In Cocco, Gomes, and Maenhout (2005), social insurance is assumed to be independent of wealth and thus the risk taking is linearly and monotonically decreasing in wealth. Such non-linear pattern of the conditional portfolio choices in wealth is backed up by the PSID 1984 wave. The average share of risky assets conditional on participation is found higher for the first quartile than second quartile in wealth.

Second, I assume the conditional probability of unemployment to be different for the employed and the unemployed.

Third, I relate the non-participation to the social welfare and test the power of prediction by comparing the calibrated results with the observed behavior for each household. Other papers usually simulate households’ behaviors based on policy functions, and compare the simulated distribution with the sample. This paper applies the policy functions directly on each observation in the data and a piecewise match is statistically tested.

Gomes and Michaelides (2003) include internal habit formation preferences in a life-cycle model and find that habit formation increases saving motive because the intertemporal consumption smoothing motive is stronger. Polkovnichenko’s model (2006) predicts more conservative portfolios for some low to moderately wealthy households with additive and internal habit formation preferences. However, in most cases households still invest in risky assets. The most recent work on rank-dependent preferences explored by Polkovnichenko (2005) suggests that the loss aversion can lead to the non-participation in the equity market. The introduction of labor income is well understood analytically from the work by Merton (1971), Bodie, Merton and Samuelson (1991), Heaton and Lucas (1997), and Jagannathan and Kocharlakota (1996). The riskless labor income serves just like the minimum consumption in the framework of HARA utility, which leads to decreasing relative risk aversion and an
increasing saving motive in wealth. Therefore, riskless labor income also yields the result that poor households consume most of their wealth and may not have sufficient wealth to afford the participation cost of risky assets. However, labor income is typically risky as studied by Koo (1995), Heaton and Lucas (1997), Viceira (2001), and Cocco, Gomes, and Maenhout (2005). Real estate risk is considered in Cocco (2005). Any additional risky assets will squeeze out the demand for stocks.

The paper is organized as follows. In section 2, I present the set-up of the model. In section 3, the parameterization and the estimation of the labor income process are presented. Section 4 presents the policy functions of consumption and portfolio. Section 5 reports the calibrated non-participation rates. Comparative static analysis is documented in Section 6. Section 7 concludes.

2. The Preference Model

2.1 The Standard Life-cycle Model

The benchmark discrete-time model follows Merton (1969) in a multi-period optimization set up. The investor has a time-separable power utility:

\[ U_i = E_i \sum_{t=1}^{T} \delta^{t-1} \left( \prod_{j=0}^{t-2} p_j \right) \left[ p_{t-1} \frac{C_t^{1-\gamma}}{1-\gamma} + b(1-p_{t-1}) \frac{D_t^{1-\gamma}}{1-\gamma} \right] \]  

(1)

The investor lives for T periods. \( C_t > 0 \) is the level of date t consumption. \( \gamma > 0 \) is the coefficient of relative risk aversion. \( D_t \) is the amount of wealth the investor bequeaths to his descendants at death. \( b \) controls the intensity of the bequest motive. For the benchmark model, the bequest utility is assumed to be zero, so \( b = 0 \). \( \delta < 1 \) is the time preference parameter. \( p_t \) denotes the probability that the investor is alive at \( t+1 \), conditional on being alive at date \( t \).

2.2 Investments

The investor can invest in a riskfree asset and a risky asset. The riskfree asset has a constant gross real return of \( R_f \) and the corresponding log return is \( r \). The risky
asset has a gross real return $R_t$. It follows a geometric Brownian motion and the log return has constant mean $\mu$ and volatility $\sigma$. For a given proportion allocated to risky assets $\alpha_t$, the log returns of the portfolio are normally distributed over each discrete time step of length $\Delta t$ (the time length between $t$ and $t+1$) with mean

$$
\mu_{a,\Delta t} = \left[ \alpha \mu + (1-\alpha) r - \frac{1}{2} \alpha^2 \sigma^2 \right] \Delta t
$$

and volatility $\sigma_{a,\Delta t} = \alpha \sigma \sqrt{\Delta t}$.

2.3 The Labor Income Process

Each period, the investor is facing the zero income risk, e.g., being long-term unemployed. Another risk of the labor income comes from the permanent or temporary shock on the expected labor income. I refrain to model these risks because as shown by Cocco, Gomes, and Maenhout (2005), the labor income subject to permanent or temporary shocks acts more like the riskless labor income as long as the labor income is defined to include any short-term benefits from compensation or social insurance. Zero income risk can only be insured conditional on wealth. For an employed investor, with probability $1 - \lambda_e$, she will receive a deterministic labor income $\bar{Y}$. With probability $\lambda_e$, she will be long-term unemployed and receive zero income other than means-tested social welfare. For a long-term unemployed investor, she will be re-employed with probability $1 - \lambda_u$ and stay unemployed with probability $\lambda_u$. Unlike the usual assumption of the equal probabilities of two states (e.g., Carroll 1997), here $\lambda_e$ is different from $\lambda_u$. This assumption of the different conditional unemployment probability is in line with the fact that those who have been unemployed for a long time will be much more difficult to get placement in the labor market than those who are being employed. Formally, the labor income process is given by:
\[
Y_i = \begin{cases} 
Y_i & \text{with prob. } 1 - \lambda_e \text{ if } Y_{t-1} > 0 \\
1 - \lambda_u & \text{if } Y_{t-1} = 0 \\
0 & \text{with prob. } \lambda_e \text{ if } Y_{t-1} > 0 \\
\lambda_u & \text{if } Y_{t-1} = 0 
\end{cases} 
\quad (2)
\]

### 2.4 Social Welfare

The social welfare insures an investor to have a minimum wealth \( W_{sw} \). Investors who are eligible for receiving the social welfare have to be long-term unemployed, which means \( Y_i \) is 0, and have wealth \( W_i \) less than \( W_{sw} \). Formally, the wealth held in period \( t \) after being adjusted by the social welfare is:

\[
W_i' = W_i + \max(0, W_{sw} - W_i) I(Y_i = 0) 
\quad (3)
\]

where \( I(\cdot) \) is the indicator function.

### 2.5 The Investor’s Optimization Problem

In each period \( t \), the investor starts the period with wealth \( W_i \). If the investor is employed, then labor income \( Y_i \) is realized. If the investor is unemployed, then the wealth is adjusted by the social welfare depending on \( W_i \) and \( Y_i \) is correspondingly zero. Following Cocco, Gomes, and Maenhout (2005), I denote cash-on-hand in period \( t \) by

\[
X_i = W_i' + Y_i , \quad (4)
\]

conditional on the employment status. This is understood as the wealth that includes labor income and possible social welfare earned in period \( t \). In the numerical solutions, \( \log(X_i) \) coincides with the value of the grid. Then the investor makes the decision on consumption and the allocation to the risky asset in a portfolio composed of a risky asset and a riskless asset. The intertemporal budget constraint is then given by:

\[
W_{t+1} = R_{t+1} (W_i' + Y_i - C_i) , 
\quad (5)
\]
where \( R^p_{t+1} \) is the portfolio return held from period \( t \) to period \( t+1 \):

\[
R^p_{t+1} = \alpha_t R_{t+1} + (1 - \alpha_t)\overline{R}.
\] (6)

The problem the investor faces is to maximize (1) subject to constraints (2) through (6): the labor income process, the investments process, and the social welfare. I do not impose the borrowing constraint in the benchmark model because according to the 1998 SCF (Kennickell, Starr-McCluer, and Surette 2000), low-income households do have non-collateralized debt. As long as the borrowing is not too extreme, the borrowing constraint is not necessary. In the calibration for employed investor, the leverage could be very high for the young and poor, but the absolute amount of borrowing is low. Therefore the deviation from reality is limited. The control variables of the problem are \( \{C_t, \alpha_t\}_{t=1}^T \). The state variables are \( \{t, X_t, Y_t\}_{t=1}^T \).

The Bellman equation for this problem is given by:

\[
J_t(X_t) = \text{Max}[U(C_t) + \delta p_t E J_{t+1}(X_{t+1})], \text{ for } t \leq T
\]

where \( X_{t+1} = Y_{t+1} + (X_t - C_t)(\alpha_t R_{t+1} + (1 - \alpha_t)\overline{R}) \).

The problem cannot be solved analytically. I derive the policy functions numerically by using backward induction. In the last period the investor consumes all available wealth so the utility \( U(X_{T+1}) \) corresponds to \( J_{T+1}(X_{T+1}) \). I can now iteratively substitute this value function in the Bellman equation and compute solutions for the previous periods. In order to coincide with the Geometric Brownian Motion of the risky returns, I discretize log cash-on-hand values onto a grid structure. The grid has equal time increments as well as equal steps in \( \log(X_t) \). From each grid point, I allow a multinomial forward move to a relatively large number of subsequent grid points (e.g., 41) at the next time step. I calculate the associated probabilities by using the discrete normal distribution with a specified value for the control variables. For the values not landing on grid points, the spline estimation is used. Given the relatively large grid points, the estimation error is limited. See more details about this methodology in Appendix A and Hodder and Jackwerth (2006).
3. Parameterization

3.1 The Estimation of the Labor Income Process

Following Cocco, Gomes, and Maenhout’s (2005) work, I used the PSID data from 1970 to 1992 to estimate the deterministic labor income process as a function of age and other characteristics. One important distinction from the previous empirical work is the composition of labor income. I define the labor income as the sum of reported wage, unemployment compensation, workers compensation, social security, and child support. Supplemental social income, transfers from relatives, and other transfers are considered to be conditional on the employment status and wealth so that they are excluded in calculation of total labor income, while in Cocco, Gomes, and Maenhout’s (2005) work, all items are included. This distinction is not only required by the set-up of the model, but also is designed to rationalize the 0.5% observations of zero income under the broadest definition. All nominal values are converted into real terms by using the Consumer Price Index with 1992 as base year.

To control for education the sample was split into three groups: the observations without high school and college education, a second group with high school education but without a college degree, and finally college graduates. Formally, the following cross-sectional time series regression is estimated:

\[ \log(Y_{it}) = f(t, Z_{it}) + \varepsilon_{it}. \]  

For each education group I assume that the deterministic function \( Y_t = f(t, Z_t) \) is additively separable in \( t \) and \( Z_t \). The vector \( Z_t \) includes personal characteristics other than age, such as marital status and family size. I fit a third-order polynomial to the age dummies to obtain the profiles. Table 1 and Figure 1 report the results for the three education groups. The coefficients of the age dummies are significant. The results match intuition and mimic the work of Cocco, Gomes, and Maenhout (2005). One shortcoming of the estimation is that the coefficients of marital status and family size are so low that the labor income is dominated by the age profile and the heterogeneity of households cannot play a role in the labor income. The retirement income is modeled as a constant fraction of the labor income in the last
working-year. The probabilities of unemployment $\lambda_c$ and $\lambda_u$, however, cannot be easily and consistently estimated since the mobility of the unemployed is not directly recorded in the data. The benchmark model assumes 0.1 and 0.5 for $\lambda_c$ and $\lambda_u$, respectively. The alternative values are tested in section 6.

Table 1: Labor income process

The table shows the results of a cross-sectional estimation with fixed effects. The estimation is based on the PSID data from 1970 to 1992. The log real income is regressed on family size, marital status (dummy), and age dummies. The sample excludes households with female heads, retirees, non-respondents, students, and housewives. T-statistics are listed on the right column to the corresponding coefficients.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Non-educated Coefficient</th>
<th>t-stat</th>
<th>High school coefficient</th>
<th>t-stat</th>
<th>College Coefficient</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Real Income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family size</td>
<td>0.0147683</td>
<td>2.09</td>
<td>0.0080912</td>
<td>1.83</td>
<td>0.0316233</td>
<td>4.77</td>
</tr>
<tr>
<td>Marital status</td>
<td>0.0409316</td>
<td>1.32</td>
<td>0.0670331</td>
<td>4.25</td>
<td>0.0770813</td>
<td>3.41</td>
</tr>
<tr>
<td>Age</td>
<td>0.1222785</td>
<td>4.62</td>
<td>0.1992648</td>
<td>14.19</td>
<td>0.3561045</td>
<td>14.12</td>
</tr>
<tr>
<td>Age$^2$/10</td>
<td>-0.025327</td>
<td>-3.94</td>
<td>-0.0409482</td>
<td>-11.55</td>
<td>-0.0686501</td>
<td>-11.29</td>
</tr>
<tr>
<td>Age$^3$/100</td>
<td>0.0014546</td>
<td>2.93</td>
<td>0.0026003</td>
<td>9.12</td>
<td>0.004298</td>
<td>9.12</td>
</tr>
<tr>
<td>Constant</td>
<td>8.169711</td>
<td>23.95</td>
<td>7.135779</td>
<td>41.04</td>
<td>4.598723</td>
<td>13.97</td>
</tr>
</tbody>
</table>

|            | No. of observations     | 9445   | 26388                   | 11875  |
|            | No. of groups           | 1138   | 2898                    | 1199   |
|            | Average obs per group   | 8.3    | 9.1                     | 9.9    |
|            | R-square within         | 0.0315 | 0.0401                  | 0.1425 |
|            | F-stat                  | 53.92  | 195.98                  | 354.63 |
| Replacement rate |                          | 0.88983 | 0.68212                | 0.938873 |

3.2 Other Parameters

Other parameters used to calibrate the benchmark model are mostly taken from Cocco, Gomes, and Maenhout (2005) so that the result of this paper can be compared with their results. The minimum wealth that the social welfare insures is 15000 US dollar per year in real term. The set of parameters is displayed in Table 2.

I consider an investor who starts working from age 25 and retires at age 65 until dies with certainty at age 75. The mortality probabilities for different ages are taken from United States Life Tables (Anderson, 1998).
Figure 1: Fitted labor income process

Figure 1 shows the labor income profiles of three education groups across age. The profiles are generated based on the regression results in Table 1. The investor is assumed to be married and live in a two-member family.

![Labor Income Profiles](image)

Table 2: Benchmark parameters

The table lists the parameters used to calibrate the benchmark model. Investors with cash-on-hand less than the minimum wealth and being unemployed are qualified to receiving social welfare which amounts to the difference between the minimum wealth and the cash-on-hand.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time preference parameter ( \delta )</td>
<td>0.96</td>
</tr>
<tr>
<td>Minimum wealth ( W_{aw} )</td>
<td>15</td>
</tr>
<tr>
<td>Risk aversion coefficient ( \gamma )</td>
<td>10</td>
</tr>
<tr>
<td>Prob. of unemployment for the employed ( \lambda_e )</td>
<td>0.1</td>
</tr>
<tr>
<td>Interest rate ( r )</td>
<td>0.02</td>
</tr>
<tr>
<td>Mean ( \mu )</td>
<td>0.06</td>
</tr>
<tr>
<td>Volatility ( \sigma )</td>
<td>0.157</td>
</tr>
<tr>
<td>Prob. of unemployment for the unemployed ( \lambda_u )</td>
<td>0.5</td>
</tr>
</tbody>
</table>

4. Policy Functions

4.1 The Case of a Permanently Unemployed Investor

Before the general model is calibrated, it is worth understanding why this particular model can bring a jump decision from investing to not investing rather than a continuous decision by decreasing stock exposure gradually. In order to distill the social welfare effect only, I conduct the calibration for a permanently unemployed investor who is not affected by the labor income process. Assuming that the probability of being re-employed equal to zero \( \lambda_e = 1 \), a currently unemployed
investor will remain unemployed for the rest of his life.

Figure 2 illustrates the consumption and portfolio choices by depicting consumption-wealth ratio and portfolio share in the risky asset ($\alpha$) for a 35 year-old investor who is permanently unemployed. For wealthy investors, both policy functions are constant in wealth, reflecting Merton’s (1971) solution in a frictionless market. This is not surprising since their wealth is so far above the minimum wealth that their decisions are barely affected by the social welfare.

Figure 2: Portfolio choices and consumption-wealth ratio for a 35 year-old permanently unemployed investor

The figure illustrates the optimal portfolio share in the risky asset and consumption-wealth ratio as functions of wealth (cash-on-hand) for a 35 year-old permanently unemployed investor.

However, the consumption choices of less wealthy and poor investors deviate from Merton’s (1971) solution through two effects: the *gradual* and the *jump* effect in consumption-wealth ratio. Less wealthy investors are induced to increase consumption so that the wealth is low enough to benefit from social welfare. On the other hand, they have a demand of smoothing consumption over the life-time. Disutility arises when extremely high consumption in this period is followed by very low consumption in future periods. As a result, consumption-wealth ratio is gradually increasing when the investors become poorer, which is the *gradual* effect. A *jump* effect takes place when the investor is so poor that the benefit of social welfare
dominates the disutility of lumpy consumption over the life. Then the investor will increase consumption dramatically and save little. In Figure 2, for investors whose wealth is below 30000 US dollar (two times of the minimum wealth), consumption-wealth ratio is one. In other words, the investor will quit the risky asset market completely.

Figure 3: Life-cycle portfolio choices for a permanently unemployed investor

The figure shows the portfolio shares in the risky asset for a permanently unemployed investor over his life-cycle constructed by four represented ages. Investors whose wealth below 30000 US dollar do not participate in asset markets, therefore their portfolio shares are not documented.

Social welfare also has a significant effect on portfolio choices of less wealthy and poor investors. Figure 3 shows the portfolio choices for the same investor over his life-cycle. The risk taking is non-linearly and non-monotonically decreasing in wealth. Investors whose wealth is above 150000 have a flat and conservative portfolio, since they are too rich to be qualified to social welfare. Less wealthy investors start feeling the impact of social welfare. Although they cannot benefit from social welfare in the next period, they know that later after their own savings runs out they could be supported by social welfare. So they slowly increase their risk taking. When their wealth drops below 40000, they realize that in the next period their wealth will be very likely below 30000 and based on the consumption policy function they will consume all the wealth. Therefore, it does not pay off to hold risky portfolio.
Instead, they just choose a myopic portfolio. When investors are getting even poorer, they can take advantage of social welfare directly since downside risk of their investment is insured by social welfare. So the portfolio risk increases dramatically until investors stop saving. In terms of life-cycle portfolio risk pattern, younger investors are more likely to gamble.

4.2 The General Case

4.2.1 Consumption Policies and Non-participation

In general cases, investors are only subject to some limited probability of unemployment. Whether the two driving forces of non-participation - the gradual and the jump effect in consumption-wealth ratio – are still existent in the general case is the key to explain the non-participation puzzle. In order to identify that social welfare is the main cause of non-participation, I compare the optimal savings in five different models: riskless labor income, low zero income risk without social welfare, low zero income risk with social welfare (the benchmark case), medium zero income risk with social welfare, and high zero income risk with social welfare. I take a 35 year-old employed investor as an example. The labor income process is taken from the profile of high school graduate depicted in Figure 1.

Figure 4 shows that in the case of low zero income risk without social welfare, the optimal savings is directed to the origin, which signals a constant savings-wealth ratio. In the other four cases, savings is convex (or consumption is concave) in wealth. It implies that a positive probability of receiving zero income would significantly increase investors’ saving motives. While in the cases of deterministic labor income and zero income risk with social welfare, the gradual effect in saving-wealth ratio implies that for less wealthy and poor investors, consumption could be high enough to leave nothing to save. The optimal saving strategy shifts to the right with the probability of unemployment being low. Intuitively speaking, higher zero income risk leads to stronger saving motive. This is true except for the case with extremely high probability of unemployment, where the jump effect kicks in.
Figure 4: Optimal savings for a 35 year-old employed investor

The figure shows the amount of optimal savings for a 35 year-old employed investor as a function of wealth (cash-on-hand). The investor is assumed to be married and have no children. Optimal savings is compared among five different models: riskless labor income, low zero income risk without social welfare, low zero income risk with social welfare (the benchmark case), medium zero income risk with social welfare, and high zero income risk with social welfare. I set the probabilities of unemployment equal to 10%, 50%, and 90% associated with low risk, medium risk, and high risk, respectively.

The jump effect can only be identified clearly in highly risky labor income cases. When the probability of unemployment is high, the consumption policy is dominated by the unemployment case which is analyzed in 4.1. When the probability of unemployment is low, so is the probability of benefiting from social welfare. The benefit is then not large enough to justify a deviation from the balanced consumption path.

It can be concluded that the consumption policies of employed investors are dominated by the gradual effect while those of unemployed investors are affected by both of the gradual and the jump effect. Both effects induce investors with wealth below the critical wealth to consume most of wealth and do not participate in the asset markets.

4.2.2 Conditional Portfolio Policies
Conditional on participation in asset markets, how will social welfare affect employed investors’ conditional portfolio choices in the general case? In Figure 5, the conditional portfolio policies in the general case, as well as in the case of riskless labor income and zero income risk without social welfare, are depicted as functions of wealth. The case of zero income risk without social welfare replicates Merton’s (1971) solution. The case of deterministic labor income replicates the classical puzzle that young and poor investors endowed with riskless human capital need to hold extremely risky portfolio. The case of zero income risk with social welfare shows a non-linearly and non-monotonically declining risk taking in wealth due to welfare insures.

Figure 5: Portfolio choices of a 35-year old employed investor conditional on participation

The figure shows portfolio share in the risky asset as a function of wealth (cash-on-hand) for a 35 year-old employed investor conditional on participation in asset markets. The investor is assumed to be married and have no children. Portfolio policies are compared among three cases: low zero income risk with social welfare, riskless labor income, and low zero income risk without social welfare.
4.2.3 Empirical Evidences

Guiso, Haliassos, and Jappelli (1999) document the conditional shares of the total risky assets in total assets for households classified by total asset quartiles in various countries. Although the positive correlation between the conditional share and wealth seems to be common, it is less marked than that between the probability of participation and wealth. Evidences related to this paper are provided in the data of United States (1998 Survey of Consumer Finances), Netherlands (1997 CentER Saving Survey), and Italy (1998 Survey of Household Income and Wealth). In addition to the increasing pattern in wealth observed in the third and fourth quartile (rich households), a hump pattern is found in the first and second quartile (poor households). Another empirical observation in Campbell and Viceira (2002) reveals that young adults tend to increase their financial risk-taking by taking the decision to enter the stock market rather than a decision to increase stock holdings in a continuous fashion. These findings signal the impact of social welfare on the household portfolio choices as predicted in Figure 5.

Table 3: Conditional shares of risky assets in personal portfolios by wealth

This table documents the average shares of risky assets in personal portfolios among four cohorts of investors conditional on participation based on the PSID 1984 data. The results are calculated for each education group separately. Cohorts are classified by cash-on-hand, the sum of net wealth without main home equity and income. The quartiles listed in the first row are based on the wealth distribution of high school graduates. Risky assets include stock, real estate other than main home, private business, and other valuable securities. Personal portfolios are composed of all risky assets and check accounts.

<table>
<thead>
<tr>
<th></th>
<th>Under 24000</th>
<th>24000 — 39000</th>
<th>39000 — 80000</th>
<th>Above 80000</th>
</tr>
</thead>
<tbody>
<tr>
<td>College graduates</td>
<td>0.47</td>
<td>0.48</td>
<td>0.62</td>
<td>0.81</td>
</tr>
<tr>
<td>High school graduates</td>
<td>0.63</td>
<td>0.58</td>
<td>0.67</td>
<td>0.79</td>
</tr>
<tr>
<td>Non-educated</td>
<td>0.68</td>
<td>0.61</td>
<td>0.74</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Using the PSID 1984 data, I calculate the average shares of risky assets in personal portfolios among four cohorts of investors conditional on participation in risky asset markets. Four cohorts are classified by cash-on-hand, the sum of net
wealth without main home equity and income. The results are calculated for each education group separately. Since the education group of high school graduates has the most observations, cohorts are defined as the quartiles of this group’s wealth distribution. The results are reported in Table 3. For high school graduates and non-educated investors, the first cohort invests higher proportion of financial wealth in risky assets than the second cohort. These empirical findings, in line with the ones in Guiso, Haliassos, and Jappelli (1999), provide supports to the model’s prediction: poor households tend to either choose to quit the asset markets completely or increase the portfolio risk.

5. Calibration of the Non-participation Rate

This section uses the information about the households in 1984 PSID data to calibrate the non-participation rate, which is defined as the ratio of non-participants in any risky financial market including stock, real estate, private business, and other risky assets, over the total number of households in the sample. See the procedure of calibration in Appendix B.

Table 4: Calibrated non-participation rate

The first row in the table shows the number of observations of the sub-samples. The second and the third show the calibrated number of non-participants and non-participation rates and the corresponding realized ones for three education groups. The last row shows the probability of the right predictions for non-participants. I use the Pearson’s chi-square test to test whether the model’s prediction is better than a naïve prediction: everybody participates. The three, two, and one asters represent the significance levels under 99%, 95%, and 90%.

<table>
<thead>
<tr>
<th></th>
<th>Non-educated</th>
<th>High school</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calibrated</td>
<td>Sample</td>
<td>Calibrated</td>
</tr>
<tr>
<td>No. of observations</td>
<td>422</td>
<td>1395</td>
<td>587</td>
</tr>
<tr>
<td>No. of Non-participants</td>
<td>156</td>
<td>244</td>
<td>359</td>
</tr>
<tr>
<td>Non-participation rate (%)</td>
<td>36.97</td>
<td>57.82</td>
<td>25.73</td>
</tr>
<tr>
<td>Prob. of right predictions</td>
<td>0.89***</td>
<td>0.77***</td>
<td>0.60*</td>
</tr>
</tbody>
</table>
In order to reflect the potential impact of fixed participation cost, I impose a constraint of minimum investment. Not to complicate the model, simple minimum investments are set for investors of different age, starting with 1000 for the youngest group and growing at the riskless rate until around 2500 for the oldest group. This is a rather conservative assumption because among the saving of 1000, the demand for the risky assets might be only several hundred. Even a very small amount of fixed entry cost will deter the investor’s participation. The calibrated results and the realized non-participation rate in the sample are listed in Table 4. On average, the model predicts the non-participation rate is about 25%, while the true non-participation rate in 1984 is around 40%. However, among the predicted non-participants, over 75% are right predictions. The predictions are significantly better than the naïve prediction, i.e., everyone participates, in all three groups under 90% confidence level. Overall, the calibrated results show that the model can quantitatively capture more than half of the non-participation behavior in asset markets.

6. Comparative Static Analysis

Due to the fact that the life-time labor income process does not vary much across investors, we can simply carry out the comparative static analysis by keeping track of the critical wealth which triggers the non-participation. Different values of risk aversion parameter, unemployment probability, and the insured wealth of social welfare are replaced to calculate the boundaries and compare them with the benchmark case from Figure 6 to Figure 9.

Figure 6 first shows the critical wealth for the three groups with the benchmark parameters. Contrary to intuition, high income households are more likely to quit the asset markets in the absolute term since their critical wealth is higher. The critical wealth is not much different for the young but varies a lot for the old. However, since high income households tend to hold more disposable wealth than low income ones, low income households have much higher non-participation rates in both the calibrated and the observed results.
Figure 6: The *critical wealth* levels in the benchmark case

The figure shows the lowest wealth levels for three education groups with the parameters set in the benchmark case. Investors whose cash-on-hand is below the corresponding *critical wealth* will be non-participants.

Figure 7: Comparative static analysis for college graduates

The figure shows the *critical wealth* levels for college graduates with different parameters. In addition to the benchmark case for comparison, the ‘High zero income risk’ profile is computed for a higher probability of unemployment 0.5; the ‘High Welfare’ profile is computed for a higher minimum wealth 20; the ‘Aggressive Households’ profile is computed for a lower risk aversion parameter 3.

Figure 7 through Figure 9 illustrate the sensitivities of three parameters \((\gamma=3, \lambda_e=0.5, \text{ and } W_{sw}=20)\) for different group of investors. The sensitivities vary across three education groups and age. The low and middle income groups are hardly affected by the higher welfare level, while the participation motive of the high income
group is obviously depressed by the higher welfare level. Since high income households are the main participants in the asset markets, the analysis implies that the higher welfare might hurt the financial markets more severely than expected. The higher zero income risk has a positive effect on the participation rate of high income households. It has a slightly negative effect on middle income households, while it has a significant negative effect on low income households, especially on the young ones. Since the young low income households are typically associated with a higher probability of being unemployed, this result might explain the empirical fact that the poor and young are more likely to be outside of the asset markets. The taste of the risk has a quite similar effect on the participation rate across education groups but different effect across age. The more aggressive investors are more likely to invest when they are young and less likely to invest when they are old. This result goes against the empirical observation that most young households select themselves to be non-participants. It implies that the model favors assumption of more conservative households, which corresponds to the equity premium literature. Overall, the non-participation behavior is consistently predicted with the variety of choices of parameters.

Figure 8: Comparative static analysis for high school graduates

The figure shows the critical wealth levels for high school graduates with different parameters.
Figure 9: Comparative static analysis for non-educated investors

The figure shows the critical wealth levels for investors who neither graduate from college nor from high school with different parameters.

7. Concluding remarks

The paper combines the consumption theory, portfolio theory and social insurance literature into the life-cycle model and enriches the fixed participation cost theory in explaining non-participation of risky asset markets. The paper highlights that the non-participation is mainly caused by the convex saving function in wealth. However, the lower boundary of labor income determines the extent of convexity. Social welfare then plays an important role to provide a positive boundary for income.

The paper quantitatively calibrates the non-participation rate in 1984 by using the PSID data. The result indicates that the non-participation in asset markets can be rationalized if social welfare is taken into account. Although the prevailing household portfolio literature has universally recognized the fact of limited participation and often carried out analyses on portfolio choices conditional on participation or non-participation, it is worthy of considering an integrated model which explains the participation as well. It would be interesting to revisit the portfolio choice models under this paper’s setup.

This paper explicitly models one of the channels through which social insurance policy affects the financial markets. It enriches the social insurance literature with the
detailed quantified approach to obtain the negative effect of social insurance on households’ saving and the positive effect on risk tolerance conditional on participation. The latter calls for further econometric studies on the wealth effect on conditional household portfolios. Given the availability and high quality of micro panel data in more and more countries, an individual-by-individual or class-by-class analysis of the consequences of social insurance reforms is possible and the heterogeneity of households can be fully taken account.

Appendix A: Numerical Solution

The basic structure of the model uses the methodology described in Hodder and Jackwerth (2006). Following their notations, I use grids of cash-on-hand values \( X \) and time \( t \), with \( \Delta(\log X) \) constant as well as time steps \( \Delta t \) of equal length. I use 120 log value steps with the upper boundary equal to 300000 and lower boundary equal to 1000. Hence, \( \Delta(\log X) \) is 0.047532. Time to maturity is 50 years. The number of time steps is 10. Hence, \( \Delta t \) is 5 years. To calculate the probabilities of moving from one cash-on-hand value at time \( t \) to all possible values that can be reached at \( t + \Delta t \), the range of 41 grids are used.

Each period, the state space is constructed by 121 grids in cash-on-hand and two states of unemployment and employment. In addition, two special indirect utilities are required for the state of zero cash-on-hand in order to allow zero saving to be admissible. For the unemployed households, the grids below the minimum wealth are adjusted to be the same as the minimum wealth. For the employed households, the cash-on-hand is adjusted by the income. This adjustment will generally cause the cash-on-hand not to locate on the grids. In this case, the indirect utilities are computed by using spline estimation based on the indirect utilities on the grids. Given the choices of the consumption and the proportion of risky assets, the expected utility of being unemployed or employed in the next period can be separately computed. Then, the expected utility associated to the given choices can be computed by multiplying corresponding probabilities of employment and unemployment conditional on current
status of employment. The optimal choices are the results of maximizing over the expected utilities of all admissible choices.

One distinction of the model from the previous studies which also assume zero income risk is to allow the household to consume all his cash-on-hand because in the next period his cash-on-hand will either be adjusted to the minimum wealth in case of unemployment or be adjusted by the labor income in case of employment. Both cases will prevent the household from having nothing to eat. However, in other studies (e.g., Carroll 1997) the choice of zero savings is implicitly excluded as an optimal choice since in case of receiving zero income in the next period the power utility will be minus infinity.

When implementing the backward sweep through the grids, one difficulty is to deal with behavior at the boundaries. The terminal step is trivial in that I calculate the terminal utility from the terminal wealth. To calculate the indirect utilities of the grids close to the upper and lower boundaries, I use buffers of cash-on-hand values above the upper boundary and below the lower boundary. The lower buffer for the unemployment case is trivial since the indirect utilities associated to grids which are below the minimum wealth are all equal to the indirect utility of grid equal to the minimum wealth. For the indirect utilities associated to other buffer grids, I use an approximation based on the indirect utilities on the boundaries. Assuming that the optimal consumption policies (consumption-wealth ratio) and the expected future utilities for buffers are the same as for the boundaries, the indirect utilities for buffers then are approximated by the sum of the current utility of consuming corresponding proportion of wealth and the expected future utility on the boundaries. This approach is potentially suboptimal; however, the distortion ripples only some 20-50 steps below the upper boundary, affecting mainly the early time steps.

Appendix B: The Procedure of Calibration of the Non-participation Rate

First, a fitted labor income profile for each household is calculated given the information of education, marital status, and family size. Second, the consumption and portfolio choices for this particular household are numerically solved by the
above methodology. Third, calculate the cash-on-hand as the sum of the total net wealth except main home equity and the ad hoc labor income in 1984. Last, look up the optimal consumption and portfolio choices for this investor given the value of cash-on-hand and age. The procedure is iterated for each individual. 1984 wave was the first time that PSID asked households about their allocation of wealth. Such information is also available in 1989, 1994, 1999, 2001, and 2003.
Reference


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