NOISE HEDGING AND
EXECUTIVE COMPENSATION

Neil A. Doherty, James R. Garven and Sven H. Sinclair*

June 2006

Abstract

We address two apparent paradoxes of risk management: (1) risk managers hedge in order to avoid negative earnings surprises, yet they tend to hedge risks uninformative of the value of the company; and (2) the presence of options in risk managers’ compensation distorts their incentive to hedge, inducing them to expose the company to too much risk. We show these to be rational responses of both owners and managers, consistent with shareholder value maximization. Our model is based on informational asymmetry between insiders (managers) and outsiders (investors). Investors derive information about company value from net cash flows (earnings), but the information revealed through earnings depends on the risk management strategy pursued by managers. Fully revealing earnings information is in the investors’ interest, so they design a compensation package to induce managers to make the earnings fully informative. With appropriate assumptions, we show that managers will select the efficient hedge strategy if their compensation includes stock options. Our model also provides a rational explanation for the observed vigorous response of stock prices to modest earnings surprises.

* Neil Doherty is the Frederick H Ecker Professor of Insurance and Risk Management, The Wharton School, University of Pennsylvania, James R. Garven is the Frank S. Groner Memorial Chair of Finance, Hankamer School of Business, Baylor University, Sven Sinclair is an economist with the Congressional Budget Office.
1 Introduction

We present a model of corporate hedging in which managers signal private information about the composition of earnings to uninformed investors. Earnings are subject to transient shocks or noise, which carries no information about future earnings; and persistent shocks or signal, which is informative of future earnings. Only managers know the composition of earnings and owners, investors, wish to be informed. Managers can increase the signal to noise ratio by their choice of hedging strategy and are rewarded for doing so in their compensation. Because such informative hedging increases stock price volatility, the active ingredient in their compensation is a stock option. Thus, this paper contradicts the conventional wisdom that options discourage managers from hedging. We also show why some risks are hedged and not others. Our theory also is consistent with the widely held practitioner view that managers hedge to manage earnings surprises. And finally, our model provides a theoretical explanation for why stock prices respond vigorously to earnings shocks.

Academic explanations for hedging fall into two classes: those based on creating value for shareholders, and those in which managers hedge to promote their own welfare. In the latter (Stulz 1984; Smith and Stulz 1985), managers who cannot diversify their portfolios, due to large portion of their wealth being tied to their company’s stock, have an incentive to reduce the stock’s volatility, thus potentially overhedging from the shareholders’ point of view. On the other hand, risk managers may underhedge if they are compensated with stock options, whose value increases with the stock’s volatility; Tufano (1996) has found evidence for this in the gold mining industry.

Explanations based on shareholder value creation are more common in the academic literature. Hedging can reduce expected tax liability due to convexity of tax schedules (Smith and Stulz 1985). Reducing the risk of bankruptcy can reduce expected cost to creditors (Main 1983) or enhance the price of services to risk averse stakeholders such as insurance policyholders (Doherty 1982). Risk reduction can alleviate agency problems between shareholders and creditors that cause underinvestment and asset substitution problems (Mayers and Smith 1983). Hedging can also create value through protection of
post-loss investment opportunities when outside sources of capital are costly (Doherty 1985; Froot, Scharfstein and Stein 1993). All these models suggest that even diversified shareholders of a widely held firm would find some hedging desirable. In that case it may not appear rational for shareholders to compensate risk managers with stock options and thus discourage hedging.

Regardless of the theoretical explanation, we should at least expect managers’ actions to be consistent with the rationale they believe to be correct. Managers commonly state that their goal in hedging is to avoid an earnings surprise and the resulting fall in share price (Tufano 1998). At the same time, it has been observed (Schrand and Unal 1997; Stulz 1996; Tufano 1996b) that firms tend to insure non-core risks, i.e., precisely those that carry little information about future earnings. An efficient market would not react to earnings surprises due to uninformative events, so the argument of protecting the share price appears inconsistent.

Our first goal is to reconcile the theoretical models with the managers’ explanations and show that the observed behavior can persist in the world where both investors and managers act rationally. Our model is based on informational asymmetry between managers and investors. Managers (insiders) know more about earnings composition than outside investors. They know which portion of an earnings surprise is informative of future earnings (signal), and which is uninformative (noise). Outsiders cannot generally tell signal from noise and rely on the observed stream of earnings when revising their. Hedging affects the proportion of noise in this flow of information from the firm to outside investors. Shareholders benefit when earnings announcements reveal more information about firm value; thus they design managers’ compensation to induce a hedging strategy that maximizes the informational content of earnings surprises. Equilibrium actions are consistent with observed behavior: compensation of risk managers contains options, and in turn they hedge uninformative risks and retain informative ones. Options do not discourage managers from hedging, but rather induce them to adopt an informationally efficient hedging strategy.
In equilibrium, managers hedge uninformative risks precisely to *increase* the volatility of the stock price, enhancing the value of their options. Of course, hedging reduces volatility of *cash flows*, but the removal of noise from earnings surprises increases their informational content. As investors’ beliefs coincide in equilibrium with managers’ actions, they interpret earnings as more informative and, accordingly, revise their valuation of the firm more vigorously.

Incidentally, the vigor with which market responds to earnings announcements has also puzzled researchers for some time. It has been claimed (Zarowin 1989; DeBondt and Thaler 1990) that the market overreacts to earnings surprises. A more recent study (Bartov, Givoly and Hayn 2000), however, disputes the overreaction hypothesis based on the absence of subsequent reversal of stock prices, and concludes that investors react rationally to earnings surprises, which are more informative than conventionally assumed. Our model, while aimed at solving a different puzzle, also provides an explanation for this excess informational content of earnings surprises.

There are some similarities between our model and DeMarzo and Duffie (1995) (D&D). There, too, hedging affects the informativeness of earnings and managers hedge to manipulate the signal outsiders receive. However, theirs is a signaling model and ours is essentially a screening one. In D&D, managers use hedging to signal their quality; in the present paper, owners design compensation scheme to induce a particular hedging strategy. The nature of asymmetry is different as well: in D&D, it relates to the parameters of the distribution, while here it is in the ability to observe and interpret the realizations of random variables. Finally, they assume a competitive labor market with short-term contracts and wage rate as the relevant parameter, while we admit long-term contracts and consider the nature of compensation structure.

Hedging in our model is costless and has no *direct* effect on the true (as opposed to reported) value of the company. Thus, while it is intuitively appealing, it is not completely obvious that investors have a strict preference for full information. They do indeed, but only if the value of the firm differs across informational regimes. Moreover, if it is also true that hedging affects information, firm value will
indirectly depend on hedging. We can thus see hedging as a desirable, value-creating action, but not through its essential function of controlling volatility, but through its incidental contribution to informational efficiency.

We present an overview of the model in Section 2. In Section 3 we describe the earnings generation process and the formation of expectations by insiders and outsiders, followed by the definition of earnings surprise and its informational content. In Section 4, we define the hedging strategies available to managers, and derive the expectations and variances of earnings surprises and their informational content under each strategy. We identify the informationally efficient hedging strategy. Section 5 shows how the different hedge strategies affect stock price volatility. Section 6 uses the fact that the informationally efficient hedging strategy results in the most volatile stock price to develop a compensation scheme that induces managers to pursue that strategy in equilibrium. The conclusion follows.

2 AN OVERVIEW OF THE MODEL

A. The Model Structure

The model we develop in the paper is depicted as a sequential game in which investors design compensation, $C$, to induce managers to select a desired hedge strategy. The hedge strategy desired by investors is one that fully reveals inside information relevant to valuing future earnings. Earnings are subject to shocks that can be categorized as informative and uninformative. Informative shocks, denoted by $m$, carry information about the future level of expected earnings. Uninformative shocks, denoted $u$, are transient and carry no information about future earnings. Insiders observe the level of earnings and can decompose shocks into informative and uninformative. We assume that outsiders, investors, observe the overall level of earnings but are unable to decompose shocks into their types. This creates an information asymmetry.

Managers can choose a hedge strategy characterized by the type(s) of shocks that are hedged:

- Strategy S1: hedge only uninformative shocks, $u$,
• Strategy S2: hedge only the informative shocks, \( m \),
• Strategy S3: hedge both \( u \) and \( m \)
• Strategy S4: hedge neither.

Since they cannot separate \( m \) and \( u \), investors are unable to observe which hedge strategy managers select, but may form beliefs as to which is chosen. We refer to the managers’ strategy and the investors’ beliefs by the pair \((S_i, S_j)\) (managers choose \( S_i \), but investors believe that \( S_j \) is chosen). All hedges are conducted at fair prices. However, hedging changes the volatilities of the firm’s earnings, \( \sigma_e(N) \), and stock prices \( \sigma_f(V) \). A key point is that, to measure the impact on price volatility, we need to see how the respective hedges change the information transmitted to investors.

We will show that the strategy-belief pair \((S_1, S_1)\) is fully revealing of informative shocks. We will also show that \((S_1, S_1)\) also leads to the highest volatility in stock prices\(^1\) because this strategy purges all noise from earnings so remaining earnings shocks provide pure signals of future value. However, since they cannot observe the chosen strategy, investors cannot condition manager compensation directly on the hedge strategy. A potential Nash equilibrium to this game can be designed by investors rewarding managers purely with long dated call options on the firm’s stock. Since \((S_1, S_1)\) leads to the highest volatility of the stock price on which the options are written, this strategy will apparently be selected by self interested managers. However, a simple option compensation plan is not sufficient to achieve this equilibrium. If investors believe that \( S_1 \) is chosen, then managers will have an incentive to switch to a hedge strategy, \( S_4 \), which increases earnings volatility. Investors will now (incorrectly) interpret earnings shocks under \( S_4 \) as pure signals of \( m \) risk and this will increase stock price volatility, and increase the value of the call options, even further. To prevent managers from switching from \( S_1 \) to \( S_4 \), compensation that combines both stock options and firing provisions that put the manager’s tenure at risk. The fully revealing subgame-perfect equilibrium is “Managers hedge only \( u \) – Owners choose “option-plus-tenure” compensation.”

\(^1\)Compared with other equilibrium candidates among strategy belief pairs, i.e.: \((S_2, S_2)\); \((S_3, S_3)\); \((S_4, S_4)\).
**B. Some Comments on the Information Assumptions**

There are at least two ways to show that full information represents value to shareholders. One possible reason is insider trading: managers can use their informational advantage to extract wealth from uninformed outsiders, through direct trading or by selling information to certain outside investors. We do not to rely on this explanation since it provides virtually no restrictions in modeling. Imperfect legal enforcement and, hence, limited insider trading is assumed, the exogenous assumption about the extent of insider trading can be the dominant driver of investors’ preferences and thus the optimal compensation packages. Furthermore, data on undetected insider trading are by definition unavailable, so the model would be all but impossible to test empirically. For these reasons, we assume that laws are perfectly enforced and no insider trading takes place, including trading information that gives the buyer trading advantage. If this assumption is relaxed, full information can only become more valuable to investors, further reinforcing the validity of our assumption.

Rather than rely on insider trading, we note that informational asymmetry can affect firm value by reducing the likelihood of value-enhancing takeovers. A large, informed investors can disguise as a raider in order to profit from a temporary under-valuation of the firm, without any interest in replacing the management or promoting efficiency. Shareholders lose if they sell shares to such arbitrage raiders. If they cannot distinguish between arbitrage raiders and raiders who create value, they demand higher takeover premium and a takeover becomes less profitable for a value enhancing raider, and hence less likely. This weakens the threat to inefficient managers, increasing agency costs and reducing firm value. Therefore, shareholders lose value from informational asymmetry, while it entrenches and thereby
benefits managers. Shareholders are willing to provide incentives to managers to reveal full earnings information. A simple model (based on Burkhard, Gromb and Panunzi, 1998) showing how investors gain from information is available from the authors, but, for brevity, we do not present the details here.

Whether a particular shock is informative or not depends on circumstance. An example of a transient shock would be a factory fire, which may carry little information about future profitability if there is no major disruption of production or if customer demand can be met from inventory. Liability losses can carry different information. For example, many firms having a moderate association with asbestos use have been caught as deep pockets in asbestos litigation. This often arises from activities of the distant past and, for some firms, may carry little information about future earning potential. On the other hand a highly publicized claim for injuries to customers from a current product can depress consumer demand and severely impact future cash flows. Credit losses can be random and uninformative for some firms, but for those involved in credit management, a current credit loss may reveal a fundamental business weakness. The predictive value of interest rate or currency fluctuations for future earnings will vary across firms.

The distinction between informative and uninformative is related to that between core and non-core risk. Core risk is fundamental to the firm’s activities and the firm can potentially earn rent for risk taking. Non-core risk is not central to the firm’s basic activities and is likely beyond managerial control. The centrality of core risk implies that bad realizations might well raise questions about future profitability (e.g., a downturn in demand, managerial errors, mispricing, poor credit control). Risks that are non-core are unlikely to carry much information. Risk managers often tend to retain core risk and hedge non-core risk or transfer it to a specialized risk bearer who has a comparative advantage in hedging, e.g., an insurer (Schrand and Unal 1997, Stulz, 1996, Tufano 1996b).
3 EARNINGS GENERATION PROCESS

A. Basic Relationships

In any period $t$ the cash flow of the firm is $N_t$, announced at the end of the period. The value of the cash flow comprises three components: some underlying base level, $N_t$, and two random, independent shocks $m_t$ and $u_t$. The base level $N_t$ may be viewed as some long-term average $N_0$ (which may change over time, but only in a deterministic way, known to all parties)$^2$, plus a stochastic part that reflects accumulation of past shocks.

An “m” shock is informative of future cash flows: for any year $t$, the current cash flow will reflect the accumulated effects of prior “m” shocks $m_0, m_1, m_2, …, m_{t-1}$. However, “u” shocks affect only earnings in year $t$ and carry no information about future cash flows.$^3$ We further assume that the “m” shocks decay geometrically in time at the rate $\phi \in (0, 1)$, so that at time $t+s$ the remaining effect of a shock $m_t$ is $\phi^s m_t$. The structure of the described process resembles that of many macroeconomic forecasting problems. It is thus not surprising that it naturally calls for tools that are often used in connection with time series data (the Kalman filter; see Appendix A). The body of the paper emphasizes the intuition of the informational structure of the earnings process; a more formal analysis is relegated to the Appendix.

B. Summary of Notation

\[ N_t = N_{t-1} + m_t + u_t = \text{period-}t \text{ cash flow (before hedging) realized at the end of the period;} \]
\[ Y_t = \text{period-}t \text{ cash flow after hedging;} \]

$^2$As long as all changes in $N_0$ over time are deterministic and known to all parties, they are immaterial for the analysis in the rest of the paper, which focuses on the stochastic process of earnings. Therefore, we ignore any time subscript in $N_0$.

$^3$In reality, information might not bifurcate quite so simply. The information content of a profit shock is likely to vary along a more continuous space. Nevertheless, the dichotomy is convenient for modeling and has been used by others (e.g., Core and Schrand 1999).
\( N_t \) is a base level of free cash flow; \( N_t = N_0 + \sum_{j=0}^{t-1} \phi^{t-j} m_j \),

where \( \phi \) is the rate of decay of informative shocks, assumed deterministic and known to all parties.\(^4\)

\( m_t \) is a random shock to period-\( t \) free cash flow (FCF) that carries information about future FCF; \( u_t \) is an uninformative random shock;

\( m_t \) and \( u_t \) are standardized such that \( E(m_t) = E(u_t) = 0 \), \( \text{Cov}(m_t; u_t) = 0 \);

\( E(\cdot) \) without superscript denotes expectation with full information on previous \( N_t, m_t \) and \( u_t \);

\( E^I(\cdot) \) denotes expectation held by investors with incomplete information on previous \( N_t \);

\( E_t(\cdot) \) denotes expectation at the beginning of period \( t \).

\( N_t, m_t \) and \( u_t \) are all observable by managers;

investors observe only \( Y_t \) (which is the same as \( N_t \) if no risk is hedged);\(^5\)

the distributions of \( m \) and \( u \) are stable over time.

For our purposes, distinctions between period cash flows and earnings are not important and we will use the terms interchangeably. Throughout the paper, the term “hedging strategy” will refer to the manager’s choice of hedging \( u \)-risk, \( m \)-risk, both, or neither.

We will now describe how insiders and outsiders form earnings expectations under different information assumptions. The earnings surprises—deviations of actual earnings from expectations—cause investors to revise their valuation of the company. The extent of this revision depends on the information contained (or believed to be contained) in earnings. Note that we will not introduce hedging until we have defined the information content of earnings; thus, in this section, \( N_t = Y_t \).

\(^4\)We assume that any growth is incorporated in the deterministic part \( N_0 \). It is also possible for growth to affect the persisting shocks. If we assume such growth to be geometric with rate \( g \), then \( \phi(1+g) \) always appears as a single factor that can be interpreted by a modified decay factor. In this case, there is no loss of generality from ignoring growth in the stochastic part of the earnings process.

\(^5\)This assumption has been criticized on the grounds that today’s sophisticated financial markets should see through the firm’s hedging activities and deduce the earnings before hedging. On the other hand, there is abundant evidence that firms engage in increasingly complex financial activities, aimed at keeping the public in the dark. The recent Enron case is a clear example of the market’s limited ability to decipher complex actions.
C. Earnings surprises with full information on $N$, $m$ and $u$.

We start from earnings expectations when all three components of earnings are known. This corresponds to the information available to managers. The earnings for period 0 are:

$$N_0 = N_0 + m_0 + u_0$$  

(3)

and, recalling that $E(m) = E(u) = 0$, the beginning-of-period expectation is:

$$E_0(N_0) = N_0$$  

(4)

Similarly, for any period $t$, the realized value of earnings is

$$N_t = N_0 + \sum_{j=0}^{t-1} \phi^{t-j} m_j + m_t + u_t$$  

(5)

$$= N_0 + \phi \left[ (N_{t-1} - N_0) - u_{t-1} \right] + (m_t + u_t)$$

and an agent who has full information on prior realizations of $m$ and $u$ holds the beginning-of-period expectation

$$E_t(N_t) = N_0 + \phi \left[ (N_{t-1} - N_0) - u_{t-1} \right].$$  

(6)

D. Earnings surprises with partial information

At the end of any period $t-1$, outsiders observe $N_{t-1}$, but not its composition. These uninformed investors form conditional expectations $E_t^1(m_{t-1}) = E(m_{t-1} | N_0, \ldots, N_{t-1})$ and $E_t^1(u_{t-1}) = E(u_{t-1} | N_0, \ldots, N_{t-1})$.

Thus, at the beginning of period $t$, investors hold the following conditional expectation on $N_t$:

$$E_t^1(N_t) = N_0 + \sum_{j=0}^{t-1} \phi^{t-j} E\left( m_j \mid N_0, \ldots, N_{t-1} \right)$$

$$= N_0 + \phi \left[ (N_{t-1} - N_0) - E\left( u_{t-1} \mid N_0, \ldots, N_{t-1} \right) \right].$$  

(7)

The first expression is (7), based on equation (5), incorporates revisions of conditional expectations of all past values of “$m$” shocks with each new observation of earnings $N_t$. The second
expression in (7), analogous to (6), is equivalent to the first, but represented by the conditional
expectation of last-period “u” shock. Essentially, outsiders are trying to separate the information
contained in the entire earnings history from the noise introduced by the last-period shock.

E. Informational content of earnings surprises

A key element of our analysis is the stock price response to earnings announcements. Since we
want to evaluate the merits of convex instruments (options) in managerial compensation, we are
interested in the expected volatility of stock price at a given future time, which may be interpreted as the
option expiration date. Earnings surprises should cause a more vigorous stock price response the more
information they carry.

Consider a firm that conveys information $N_{t-1}$ to investors enabling them to form conditional
expectation on $N_t$. Eventually, $N_t$ is revealed and can be represented as the sum of its expectation and an
earnings surprise $ESt$. From (5) and (7):

\[ (8) \quad ESt = N_t - E_t^1 (N_t) \]
\[ = (m_t + u_t) - \phi [u_{t-1} - \text{E}(u_{t-1} | N_0, \ldots, N_{t-1})] \]

An earnings announcement will contain some new information, coming from the “m” shock, that
is relevant to estimation of future earnings. As the value of the firm is the expected present value of its
future cash flows, we can determine how it changes with new information. Because shocks have zero
expectation and are not correlated, the two-period extension of (7) is:

\[ (9) \quad E_{t-1}^1 (N_t) = N_0 + \phi^2 \left( (N_{t-2} - N_0) - \text{E}(u_{t-2} | N_0, \ldots, N_{t-2}) \right) \]

The equity value immediately before the earnings announcement at $t$ is:

\[ (10) \quad V_t^- = \frac{N_0}{k} + \frac{E_t^1 (N_t - N_0)}{k + (1 - \phi)(1 - k)} = \frac{N_0}{k} + \frac{E_t^1 (N_t - N_0)}{\hat{k}} \]
where $k$ is the cost of equity capital (discount rate), and $\hat{k} \equiv k + (1 - \varphi)(1 - k)$. The additional term in the second denominator reflects the expected decay of excess cash flows. Immediately after the announcement the equity value will be

$$V_t^+ = N_t + (1-k) \left( \frac{N_0}{k} + \frac{E_{t+1}^1 (N_{t+1} - N_0)}{\hat{k}} \right).$$

Assuming that the current earnings ($N_t$) are fully paid out as dividends\(^6\) and no new information is revealed between announcements, the equity value immediately before the next earnings announcement (at $t+1$) will be

$$(11) \quad V_{t+1}^- = \frac{N_0}{k} + \frac{E_{t+1}^1 (N_{t+1} - N_0)}{\hat{k}}.$$

Had this value been forecasted before the announcement at time $t$, it would have involved the time-$t$ conditional expectation, $E_t^1 (V_{t+1})$. Thus, the change in share price forecast with the announcement of $N_t$ will be:

$$(12) \quad \Delta V = V_{t+1}^- - E_t^1 (V_{t+1}^-) = \frac{E_{t+1}^1 (N_{t+1}) - E_t^1 (N_{t+1})}{\hat{k}}.$$

We now define the information content of an earnings announcement, $IC_t$, as the numerator of equation (12). This is the amount of information contained in $N_t$ that affects the share value. Note that the change in share value is proportional to the informational content. Essentially, information content is the difference between the one-period-ahead forecast of stock price at time $t$ and the forecast of the same quantity made one period earlier.

---

\(^6\)The purpose of this assumption is to avoid the complication of valuing cash on hand (current earnings) as part of the equity. In a Miller-Modigliani world, we could assume any dividend policy. Our information assumptions do, however, violate the M-M assumption of market efficiency. But it still makes sense to worry only about the stream of future cash flows since it is the future price volatility that affects an option’s value.
4 HEDGING STRATEGIES

We now examine how hedging affects the information content of earnings and thereby the response of stock prices to earnings announcements. Four different strategies are considered: hedging only “u” risk, hedging only “m” risk, hedging both risk types, and hedging neither. A hedge undertaken for any period only covers the shock realized in that period. Thus, if “m” is hedged in period $t$, the counterparty will indemnify the firm for the realization of $m_t$ in that period, but the firm is still left with the carryover risk since $m_t$ affects future profits. This assumption of short maturity hedging roughly matches the reality of the derivative and insurance markets where the maturity of available contracts is fairly short, rarely exceeding a year or so. Our next step is to determine the earnings surprise to outside investors and the information content of earnings under each of the first three hedging strategies; these values have already been stated for the no-hedge strategy in equations (8) and (9). Regardless of strategy, we assume no transaction costs. Therefore, since $E(m) = E(u) = 0$, the cost of hedging is zero. In this subsection we will assume that outsiders know, or infer, the actual hedging strategy (we later show this is an equilibrium result). The key result is that one, and only one, hedging strategy is fully revealing of the composition of earnings shocks.

STRATEGY S1: Firm hedges “u” risk each year and announces the hedged cash flow $N_t – u_t$.

Because revealed earnings are purged of “u” risk, and investors are assumed to be aware of this, then earnings and prior expectations of outsiders are now:

\[
Y_t = N_t - u_t = N_0 + \varphi \left[ (N_{t-1} - N_0) - u_{t-1} \right] + m_t
\]

\[
= N_0 + \varphi (Y_{t-1} - N_0) + m_t
\]

---

7 An exception to this in practice would be the insurance of business interruption loss. For some perils, such as fire, the firm can insure both the cost of replacing the damage caused by the fire and the loss of earnings resulting from the fire. However, this business interruption insurance is available only for a limited time after the loss. Moreover, there is no ready market for business interruption caused by non-insurable perils.

8 Ignoring indirect effects related to takeovers, to be introduced later.
\[ E_t^I(Y_t) = N_0 + \varphi [ (N_{t-1} - N_0) - u_{t-1} ] \]

\[ = N_0 + \varphi (Y_{t-1} - N_0) \]

Now the earnings surprise with “u” hedged is

\[ ES_t = Y_t - E_t^I(Y_t) = m_t \]

The informational content of earnings can be derived noting that each year’s \( m_t \) can be inferred from that year’s \( N_t \), so contemporaneous earnings are fully revealing of underlying persistent shocks.

Note that the exact same relations (14) and (15) hold for the expectations and surprises of insiders. Noise is completely eliminated and the announced cash flow \( N_t - u_t \) is fully revealing of the underlying signal \( m_t \). Recalling that the definition of information content in equation (12) is based on the forecast of next-period earnings, and that the effect of current shocks will by then decay to the proportion \( \varphi \) of their current value, the information content of earnings surprise is

\[ IC_t = \varphi ES_t = \varphi m_t. \]

We can now state

**PROPOSITION 1.** If investors’ beliefs correspond with hedge strategies chosen by managers and if \( \varphi < 1 \), then the informative shock, \( m_t \), is fully revealed in announced earnings if and only if managers hedge only the uninformative risk.

The proof of the “if” part is obvious from the description of S1, and the descriptions of other three strategies will complete the “only if” part.

**STRATEGY S2: Firm hedges “m” risk each year.**

Now, the contemporaneous informative shock is hedged. Thus, current earnings will not reflect the current period realization of \( m_t \), but the current \( m_t \) will impact future earnings since the carryover effects are not hedged.

\[ Y_t = N_t - m_t = N_0 + \varphi [ (N_{t-1} - N_0) - u_{t-1} ] + u_t \]
\begin{align*}
(18) \quad E_t^I(Y_t) &= N_0 + \varphi [(Y_{t-1} - N_0) + E(m_{t-1} \mid N_0, ..., N_{t-1}) - E(u_{t-1} \mid N_0, ..., N_{t-1})] \\

Since \( m \) is fully hedged, revelations of \( N_{t-1} \) carry no information about \( m_{t-1} \) and \( E(m_{t-1} \mid N_0, ..., N_{t-1}) - m_{t-1} = 0 \). An informative signal appears in the form of the carryover effect of the prior year’s shock, \( \varphi \). This signal is mixed with the noise \( u_t \) and information is asymmetric between insiders and outsiders; this will drive a wedge between inside valuation based on \( m_t \) and outside valuation based on \( IC_t \). The earnings surprise and its informational content can be expressed as

\begin{align*}
(19) \quad ES_t &= \varphi [m_{t-1} - E(m_{t-1} \mid N_0, ..., N_{t-1}) + E(u_{t-1} \mid N_0, ..., N_{t-1})] + u_t \\
(20) \quad IC_t &= E_{t-1}^I(N_{t-1}) - E_{t-1}^I(N_{t-1}) \\
&= \varphi [ES_t - E(m_t \mid N_0, ..., N_t) + E(u_t \mid N_0, ..., N_t)].
\end{align*}

These expressions are not suitable for comparison with their counterparts \((15)\) and \((16)\) since they involve unobservable terms. Analytical results for this strategy are derived in the Appendix.

**STRATEGY S3. Firm hedges “\( m \)” and “\( u \)” risk each year.**

Using similar reasoning:

\begin{align*}
(21) \quad Y_t &= N_t - m_t - u_t = N_0 + \varphi [(N_{t-1} - N_0) - u_{t-1}] \\
(22) \quad E_t^I(Y_t) &= N_0 + \varphi [(Y_{t-1} - N_0) + E(m_{t-1} \mid N_0, ..., N_{t-1})]
\end{align*}

Since \( m \) is fully hedged, revelations of \( N_{t-1} \) carry no information about \( m_{t-1} \) and \( E(m_{t-1} \mid N_0, ..., N_{t-1}) = 0 \).

\begin{align*}
(23) \quad ES_t &= \varphi m_{t-1}
\end{align*}

The permanent “\( m \)” shock cannot be revealed instantaneously because it is hedged. However, this information is fully revealed with a one period delay since the announced cash flow is purged of the noise.
“u” components. This strategy gives insiders a temporary information advantage. By reasoning similar to that leading to equation (16),

\[
(24) \quad IC_i = \phi^2 m_{t-1}
\]

**STRATEGY S4. Firm hedges neither risk.**

This is just the case considered in Section 3D. The informational asymmetry between insiders and outsiders is quite obvious.

5. IMPACT OF Hedging SRATEGIES ON STOCK PRICES

A. Assumptions

We now consider the impact of the various hedging strategies on stock prices. To do this, suppose that outsiders know what hedging strategy is chosen by managers, not because they can observe the manager’s choice, but because they can infer that it is a rational choice for managers given the compensation system set in place. It is then necessary to derive the properties of the information content of earnings announcement in each hedging strategy. The general case, however, is not analytically tractable, so we must impose some restrictions. In particular, if we are content with the ubiquitous simplification of a mean-variance world, we can derive exact results that are also, in a well-defined sense, a good approximation for the general case.

We will use the following assumptions to derive proposition 2:

**A1(a). Investors are mean-variance optimizers (have quadratic utility).**

This implies that investors’ objective is to minimize the mean squared error of their forecast of future earnings (and hence the value of the company, which is the present value of the stream of future earnings). An alternative assumption, with exactly the same consequences, would be

**A1(b). Earnings shocks are normally distributed.**
The algorithm for extracting information about shocks and forecasting future earnings that would exactly optimize investors’ expected utility under either of these assumptions is the Kalman filter (Kalman 1960, Hamilton 1994).

Assumption A1 is somewhat restrictive; however, it is a standard assumption in finance, underlying the CAPM. Furthermore, if it is relaxed, the results are still approximately valid since, for more general shocks and utilities, the Kalman filter yields the best linear approximation of the current state and the forecast of a future state when only the first two moments of the distribution of shocks are known.

**A2. Managers use the same hedging strategy in every period and the same compensation scheme has been in place for a long time.**

This enables us to consider a steady state. The effects of managers’ ability to switch hedging strategies will be discussed later.

**A3. Informative (“m”) shocks have a non-degenerate distribution.**

It is intuitively clear (and is shown in the Appendix) that, with vanishing m-shocks, the stock price is fully deterministic.

**A4. The uninformative earnings shocks are uncorrelated with the price of the market portfolio of securities.**

---

9 We rely on A4 to analyze the stock price volatility. If, however, the differences in volatility of earnings are positively correlated with the market, increasing stock price volatility will also increase the cost of capital, attenuating (and possibly destroying) the differences coming purely from the variance of the information content of earnings surprises. It is possible to extend the model and consider how results respond to various alternatives to assumption A4. In the present paper, however, we focus on the earnings process of a single firm rather than complicate the model by taking into account correlations between the firm and the market. Furthermore, one way to justify assuming away any change in beta is to note that, if, as we argue later, hedging “u” risk is the equilibrium outcome for any single firm, then all firms in the market will hedge “u” risk in equilibrium and any connection between the market portfolio and “u” shocks that are common to many firms (such as currency exchange rate changes) will indeed be broken.
The significance of this assumption is that the variance of IC directly translates to the variance of the stock price. If A4 holds, hedging does not alter the firm’s beta and its cost of capital remains the same (i.e., the denominator of (12) is a constant independent of the hedging strategy).

**B. Hedging strategy and stock price volatility**

Denote the variance of the information content of earnings surprises under hedging strategy $S_i$ by $\text{Var}(IC_t^{(i)})$, $i = 1, 2, 3, 4$. Denote the variances of the informative and uninformative shocks, respectively, by $\sigma_u^2$ and $\sigma_m^2$.

**PROPOSITION 2.** (a) If outside investors can either observe or infer the hedging strategy, assumptions A1–A3 hold, and informative shocks decay exponentially with time ($\phi < 1$), then:

(i) $\text{Var}(IC_t^{(1)}) > \text{Var}(IC_t^{(3)})$;

(ii) $\text{Var}(IC_t^{(1)}) \geq \text{Var}(IC_t^{(4)})$ and $\text{Var}(IC_t^{(3)}) \geq \text{Var}(IC_t^{(2)})$ where, in both cases, equalities hold if the distribution of uninformative shocks is degenerate ($\sigma_u = 0$) and, keeping other things equal, $\text{Var}(IC_t^{(2)})$ and $\text{Var}(IC_t^{(4)})$ decrease monotonically as $\sigma_u$ increases;

(iii) there is a value of $\sigma_u/\sigma_m$ for which $\text{Var}(IC_t^{(4)}) = \text{Var}(IC_t^{(3)})$ and above which $\text{Var}(IC_t^{(4)}) < \text{Var}(IC_t^{(3)})$.

(b) If there is no decay of informative shocks ($\phi = 1$), then $\text{Var}(IC_t^{(0)})$ is independent of hedging strategy.

If A4 holds, the same statements can be made of stock price variances; in particular:

**COROLLARY.** If A4 holds and $\phi < 1$, hedging strategy $S_1$ results in greatest stock price volatility.

The intuition behind this Proposition is straightforward. First, hedging $m$ shocks reduces stock price volatility. Intuitively, any two strategies that treat uninformative shocks the same way, but one hedges informative shocks and the other does not, will differ in that the effects of m-shocks are delayed by one period, during which they have decayed by the factor $\phi$. This attenuates any variance by the square of the same factor; hence, hedging m-shocks reduces stock price volatility. It is less obvious that hedging $u$-shocks increases stock price volatility. On one hand, it reduces the volatility of the observed earnings; on the other hand, it purges this observed earnings of noise and increases its informativeness. So that
earnings shocks are, on average, smaller (which reduces stock price volatility) but have a higher earnings response coefficient (which increase stock price volatility). Proposition 2 shows that, under the specified conditions, the latter effect dominates. The proof is quite long and is relegated to the Appendix.

6 INFORMATIONALLY EFFICIENT HEDGING

A. The Players: Managers and Owners

Shareholders set a compensation plan for managers who then choose the firm’s hedging strategy. Once in place, the hedging strategy is fixed. We also assume that the manager’s expected tenure is long and there is no “last period” with certainty. To keep things simple, we assume that managers are risk neutral and seek only to maximize expected compensation. Compensation must be contractible in the sense that all arguments of the plan must be verifiable ex post by both parties. Information released to outsiders (such as after-hedge earnings) and market values (e.g., stock price) satisfy those criteria.

The objective of shareholders is to induce managers to select a hedging strategy that is fully revealing of shocks. When full information has sufficient value to shareholders, this maximizes shareholders’ expected wealth.

B. Hedging and Compensation Equilibrium

An equilibrium has the following characteristics. Shareholders maximize their utility by designing a compensation package that induces managers to employ the fully revealing hedging strategy S1. Managers employ that strategy because it maximizes their expected compensation. Furthermore, shareholders’ beliefs about the managers’ hedging strategy are consistent with the strategy employed.

We denote Si-Sj (i, j = 1, 2, 3, 4) a (partial) strategy-belief profile in which managers follow strategy Si and shareholders believe that strategy Sj is employed. Obviously, i = j must hold in equilibrium. Also, shareholder optimization requires j = 1, so the only profile consistent with equilibrium
is $S1-SI$.\textsuperscript{10} We will refer to earnings volatility under the strategy-belief pair $i,j$, as $\sigma_{i,j}(V)$. Using, $r_i$ as the earnings response coefficient we show in Figure 1, the relationship between earnings volatility and stock price volatility under some different strategies-belief pairs.

Figure 1 depicts potential equilibria shown in the stock volatility/earnings volatility space. The picture has been drawn to show inequalities derived elsewhere:

\[
\sigma_{1,1}(V) > \sigma_{2,2}(V); \sigma_{3,3}(V); \sigma_{4,4}(V)
\]

\[
\sigma_4(N) > \sigma_1(N); \sigma_2(N)
\]

\[
\sigma_1(V) < \sigma_1(N); \sigma_2(N)
\]

\[
r_1 > r_2; r_3; r_4
\]

where $r_i$ is the earnings response coefficient if investors believe strategy $i$ is chosen. Note the ERC is shown by the slope of the line $r_i$.

We show four possible equilibria (i.e., $i=j$) to the compensation-hedging game labeled according to the hedge strategies, “1”, “2”, “3” and “4”. Consider now a manager who is compensated with stock options and therefore has a long position in stock price volatility. If the manager selects strategy $S1$, and investors believe that $S1$ is chosen, the stock price volatility is $\sigma_{1,1}(V)$. We know from the proposition and corollary, that $\sigma_{1,1}(V)$ is the maximum volatility obtainable with any consistent strategy-belief pair (i.e., $\sigma_{1,1}(V) > \sigma_{2,2}(V); \sigma_{3,3}(V); \sigma_{4,4}(V)$). It appears that the manager will maximize the value of this option position with strategy 1. However, given his or her stock option position, the manager has the incentive to deviate to strategy $S4$ (with maximum volatility of earnings, $\sigma_4(N)$,) whenever shareholders believe $S1$ to be in place. This deviation would result in high volatility of earnings (from absence of hedging) and the

\textsuperscript{10} We assume that the shareholders’ gain from full information is sufficient not to be undone by any differential compensation to managers. This can be assured by setting the probability of a takeover high enough.
market’s strong reactions to earnings surprises (from the incorrect belief in \( S1 \) as shown by \( r_j \)). This new strategy belief pair, \( S_4-S_1 \) is shown in Figure 1 as position \( x \).\(^{11}\)

One way to prevent such switching, is to design compensation that is long in stock price volatility and short in earnings volatility. Such a compensation structure is depicted by iso-compensation lines, such as \( C-C \), with the manager’s compensation increasing to the north-west. Clearly, now the switch from the potential equilibrium \( S_1-S_1 \) to the off-equilibrium \( S_4-S_1 \) will reduce the manager’s compensation (the marginal penalty in compensation from higher earnings’ risk more than offsets the marginal gain from increasing stock volatility). As shown, the manager’s compensation will be higher under strategy belief pair, \( S_1-S_1 \), than for any other pair. The three other potential equilibria, depicted 2, 3 and 4, lead to lower manager compensation, as do other off-equilibrium pairs such as \( S_4-S_3 \) shown as \( b \); \( S_2-S_3 \), shown as \( a \) or \( S_4-S_1 \) shown as \( x \).

\(^{11}\) It is also possible that the manager may wish to deviate from \( S1 \) to \( S2 \) if the earnings volatility is higher under the latter strategy. We show such a possibility with \( \sigma_{2,1}(V) > \sigma_{1,1}(V) \).
What might a compensation structure that is long in stock price volatility and short in earnings volatility look like? Clearly stock options will satisfy the former requirement. To make the manager short in earnings volatility we might have a firing rule based on earnings realization. Moreover, firing the
manager must impose some real cost. Thus, we envision some contingent compensation that is payable only if the manager keeps her job. This could be a cash bonus or a share participation.

Compensation is a combination of $g$ shares and $h$ call options if the manager is employed at the end of the period; however, if the earnings surprise in any period falls below a level $F < 0$, set at the beginning of the period, the manager is fired.\textsuperscript{12} Other deviations can also be prevented under fairly general conditions since there are enough controls available to shareholders in designing the compensation package.

The game between owners and managers is straightforward. Owners set the proportions of cash (for punishment purposes, to generate a loss in case of firing) $\alpha$, stocks $g$, and options $h$, the trigger point for firing $F$, and the strike price $K$. The manager then chooses the variance of the earnings and stock price distribution (by choosing the strategy) to maximize his expected wealth. It may be possible to set the stock coefficient to zero and still have enough controls.

Most generally, the value of managers’ compensation, $c$, can be characterized as a sum of three expectation integrals (with $x \equiv ES$, as the integration variable): one from $-\infty$ to $F$ (trigger for firing) where the manager gets fired; one from $F$ to $+\infty$, representing the expected value of shares, and one from $\kappa$ (the earnings surprise resulting in stock price equal to $K$, the option strike price) to $+\infty$, where option value is positive:

$$
c(\Theta, i, j) = -\alpha \int_{-\infty}^{F} dG(x; i, j) + g \int_{F}^{\infty} V(x; i, j) dG(x; i, j) + h \int_{\kappa}^{\infty} [V(x; i, j) - K] dG(x; i, j)
$$

where $(\Theta, i, j) = \{\alpha, g, h, F, K\}$ is the set of shareholder controls, $G(x; i, j)$ is the distribution of earnings surprises when hedging strategy profile is $S_i - S_j$, and $V(x; i, j)$ is the stock price at the end of the period corresponding to earnings surprise $x$ when hedging strategy profile is $S_i - S_j$. The task for the shareholders

---

\textsuperscript{12} Timing is as follows: first the period $t-1$ earnings are reported, then shareholders announce the compensation parameters (including $F$), and then managers decide on the hedging strategy for the period.
is to choose the compensation parameters \( = \{c, g, h, F, K\} \) to induce the equilibrium \( S1-S1 \) as illustrated by CC in Figure 1.

**C. Limitations and Issues for Further Research**

The “no switching” assumption is important to our results, otherwise managers might make short run changes in their hedging strategy at the beginning of any period on the basis of privately revealed information. This can happen if, even if earnings surprises have zero expectation for the outsiders, they do not for the managers, who have superior information about the baseline earnings \( N_t \). This is not the case if managers are already using strategy \( S1 \), since the information is then fully revealed to the outsiders, but it could delay implementation of the owner-preferred hedging strategy until an opportune moment for the managers. The optimality for managers of using strategy \( S1 \) is, moreover, not guaranteed in a multi-period setting since occasional use of other strategies can enable them to manipulate their compensation. A similar problem also arises with options that do not expire within the next period. Large stock price changes in the first period can then distort the incentives for hedging in the following periods until the option’s expiration. Adding these multi-period features to the compensation model is likely to make it intractable, especially if one tries to quantify the loss to owners from informational asymmetry due to the use of other hedging strategies.

One modeling solution to this problem is to assume that a sufficiently high transaction cost exists to prevent such switching and that the switching cost falls in part on managers. For risks hedged through insurance, this assumption is appealing. Insurance contracting patterns tend to be based on long-term relationships (and sometimes long-term contracts). Whilst it is simple for the firm to discontinue its insurance coverage, it is often difficult and costly to reestablish coverage. Thus, it can be costly for the firm to switch its hedging strategy and, since managers will *usually* prefer to hedge uninformative risk, they will be loath to cancel coverage to exploit transient private information. However, for some firms, uninformative risk can take the form of interest volatility or commodity price risk. Hedging these risks can be accomplished with low transaction costs and switching hedging strategies will be relatively
painless. It may be noted that there is an increasing trend to bundle these financial hedges with conventional insurance products13 and this lends further support to the conjecture that transaction costs may deter switching.

CONCLUSION

This paper connects hedging strategy, shareholder welfare, and management incentives, through their respective roles in the revelation of information about a firm’s earnings and, hence, its stock price. It analyzes how various strategies in hedging different types of risks (informative and uninformative) affect the volatility of the stock price, rather than just the volatility of announced earnings. Finally, it shows that this analysis explains an apparent paradox, that shareholders want managers to hedge, yet they use stock options in compensation, making stock price volatility valuable to managers.

When information on the size and composition of earnings shocks is hidden from shareholders, the strategy that is fully revealing of private information relevant to stock valuation is the one in which uninformative risk only is hedged. The increased signal strength will enhance the sensitivity of stock prices to earnings shocks and will increase stock price volatility. This implies that compensation which is positively related to stock price volatility — i.e., including stock options — provides incentives for managers to choose the fully revealing hedging strategy. Our model yields the following predictions. Using accuracy of earnings forecasts as a proxy for S1 hedging, we would expect earnings accuracy to be positively associated with earnings response coefficients and stock price volatility. Also, option-based compensation is associated with the higher (a) accuracy of earnings forecasts, (b) earnings response coefficients, and (c) volatility of stock prices.

There is some existing evidence that speaks to our model. Our paper shares one common prediction with that of DeMarzo and Duffie (1995) and of Breeden and Viswanathan (1996), i.e., that hedging will increase the information content of earnings and so increase the earnings forecasting

13 Insurers such as Swiss Re, Ace and AIG are aggressively marketing integrated risk management products.
accuracy and the earnings response coefficient. Evidence by DaDalt, Gay and Nam (2000) supports this relationship, but of course, this has no discriminatory value between these models and ours. Further evidence by Peter Tufano (1996a) gets a little closer to our predictions. He shows that the main explanatory variable explaining why gold mines hedge using gold futures, is the compensation paid to managers. In particular, stock options are associated with lower use of such hedges. While the theory tested by Tufano is at odds with ours (hedging reduces risk which lowers the value of options and therefore discourages hedging) the empirical result is not. Tufano looks only at hedging gold risk, not other sources of risk to mines. And his analysis does not call for a distinction between informative and uninformative risk. It is likely that gold price shocks have forecasting power; i.e., it is both informative and public. If so, then Tufano’s finding that managers compensated with stock options will tend not to hedge this risk, is predicted by our model.
APPENDIX: Proof of the Proposition

A1. An Outline of the Proof

We start from the appropriate state-space representation for each hedging strategy and derive the equations for the evolution of the system in terms of observable (after-hedge) earnings $y_t$ and known quantities $\phi$, $\sigma_u$ and $\sigma_m$. From those equations, we derive the steady-state expressions for earnings surprises and their information content and, finally, take and compare the variances of the information content.

The earnings surprises are the difference between the realized and forecasted earnings, and their information content is the difference between the one-period-ahead state forecast at time $t$ and the forecast of the same quantity made one period earlier (see Section 3E). If $\phi = 1$, this is equivalent to the update of the one-period-ahead forecast. In that case, we find (see Appendix) that $\text{Var}(IC_t) = \sigma_m^2$ for every strategy $S_1$–$S_4$.

We start from the appropriate state-space representation for each hedging strategy and derive the equations for the evolution of the system in terms of observable (after-hedge) earnings $y_t$ and known quantities $\phi$, $\sigma_u$ and $\sigma_m$. From those equations, we derive the steady-state expressions for earnings surprises and their information content and, finally, take and compare the variances of the information content.

The earnings surprises are the difference between the realized and forecasted earnings, and their information content is the difference between the one-period-ahead state forecast at time $t$ and the forecast of the same quantity made one period earlier (see Section 3E). If $\phi = 1$, this is equivalent to the update of the one-period-ahead forecast. In that case, we find (see Appendix) that $\text{Var}(IC_t) = \sigma_m^2$ for every strategy $S_1$–$S_4$.

For $\phi < 1$, it is straightforward to derive the variance of the information content for strategies $S_1$ and $S_3$: 
\[ \text{(25)} \quad \text{Var}(ESt^{(1)}) = \sigma_m^2; \quad \text{Var}(IC_t^{(1)}) = \varphi^2 \sigma_m^2; \]

\[ \text{(26)} \quad \text{Var}(ESt^{(3)}) = \varphi^2 \sigma_m^2; \quad \text{Var}(IC_t^{(3)}) = \varphi^4 \sigma_m^2; \]

but for strategies S2 and S4 we obtain far more complicated expressions, for which we use Kalman filter (see A1 and A3 below). We can gain some insight by considering a few special cases. If informative shocks are vanishing ($\sigma_m = 0$) all four strategies have Var($IC$) = 0. This is just what we should expect since the market would ignore all earnings surprises knowing that they are caused entirely by uninformative shocks. In the opposite extreme case, $\sigma_u = 0$, Var($IC$) equals $\varphi^2 \sigma_m^2$ for strategies S1 and S4 and $\varphi^4 \sigma_m^2$ (which is less) for strategies S2 and S3. Of course, setting $\varphi = 1$ reproduces the result derived earlier, that Var($IC$) = $\sigma_m^2$ for every strategy S1–S4.

Finally, numerical evaluation of the S2 and S4 results for different values of $\varphi$ and $\sigma_u$, normalizing $\sigma_m$ to one, verifies that they are always less than $\varphi$, i.e., the Proposition holds.

**A2. Review of the Kalman filter**

The Kalman filter requires a state-space representation of the earnings process. In the general one-dimensional case, at time $t$ the state of the process is given by $x_t$ with the evolution given by the *state equation*

\[ \text{(A1)} \quad x_{t+1} = \varphi x_t + v_{t+1} \]

where we want $\varphi$ to be closely related to the decay factor for informative shocks, $0 < \varphi \leq 1$, while the outside observers can only observe the quantity $y_t$ which is connected with $x_t$ by the *observation equation*

\[ \text{(A2)} \quad y_t = \psi x_t + w_t \]

where $w_t$ is noise. Innovations in both equations are required to be uncorrelated:

\[ \text{(A3)} \quad E [ v_s v_t ] = E [ w_s w_t ] = 0 \text{ for } s \neq t, \]

\[ E [ v_t v_t ] = Q \]

\[ E [ w_t w_t ] = R \]

\[ E [ v_t w_t ] = 0 \text{ for all } s, t. \]
The Kalman filter forecast, at time $t$, of the state $x_{t+1}$ at time $t+1$, conditional on the observation $y_t$ and the previous period forecast, evolves according to

$$\hat{x}_{t+1|t} = \phi \hat{x}_{t|t-1} + K_t \left(y_t - \psi \hat{x}_{t|t-1}\right),$$

where $K_t$ is the Kalman gain,

$$K_t = (\psi^2 P_{t|t-1} + R)^{-1} \phi P_{t|t-1} \psi,$$

and the conditional one-period-ahead mean-square error $P_{t+1|t}$ evolves according to

$$P_{t+1|t} = \phi^2 \left[P_{t|t-1}^2 - (\psi^2 P_{t|t-1} + R)^{-1} \psi^2 P_{t|t-1}^2\right] + Q = (\phi - K_t \psi)^2 P_{t|t-1} + K_t^2 R + Q.$$

It follows that the one-period-ahead forecast of the observation $y$ is just $\hat{y}_{t+1|t} = \psi \hat{x}_{t+1|t}$, and its MSE is

$$\text{MSE}(\hat{y}_{t+1|t}) = \psi^2 P_{t|t-1} + R.$$

We will also need the expressions for $s$-period-ahead forecasts:

$$\hat{x}_{t+s|t} = \phi^s x_{t|t},$$

where subscripts $t|t$ denote time $t$ quantities estimated at time $t$,

$$P_{t+s|t} = \phi^{2s} P_{t|t} + \left[\phi^{2s-2} + ... + \phi^2 + 1\right] Q,$$

$$\hat{y}_{t+s|t} = \hat{x}_{t+s|t}.$$

### A3. Kalman filter results for strategies S1–S4

In our context, it is natural to relate $y_t$ to announced (after hedge) earnings, $w_t$ to the uninformative shocks $u_t$, and $v_t$ to the informative shocks $m_t$. The most convenient state-space representation will depend on the hedging strategy involved. With strategies S1 and S4, where m-shocks are not hedged, it will be convenient to use earnings before hedging and before uninformative shocks as the state; we will denote such defined state $a_t$:

$$a_t \equiv N_t + m_t = N_t - u_t.$$

With strategies S2 and S3, where m-shocks are hedged, it is convenient to use baseline earnings, which we will denote $b_t$, as the state:

$$b_t \equiv N_t - m_t - u_t.$$

Also note that

$$b_t = a_t - m_t.$$
With these definitions, the innovations \( v_t \) in the state equation for strategies S1 and S4 (\( m \) not hedged) are simply the informative shocks \( m_t \), but for strategies S2 and S3 (\( m \) hedged) they are the informative shocks from the previous period, and have decayed due to the passage of time: \( v_t = \phi m_{t-1} \). Similarly, in the observation equation, \( w_t = u_t \) for strategies S2 and S4, where u-shocks are not hedged, but \( w_t = 0 \) for strategies S1 and S3, where u-shocks are hedged. In all cases \( \psi = 1 \), which simplifies the equations. Since we are ultimately only interested in variances, we take the deterministic \( N_0 \) to be zero without loss of generality.

It is straightforward to show that the following results hold:

| Hedging strategy | Q    | R    | State forecast \( \hat{x}_{t+1|t} \) | MSE(\( \hat{x}_{t+1|t} \)) | MSE(\( \hat{y}_{t+1|t} \)) |
|------------------|------|------|----------------------------------|-----------------------------|-----------------------------|
| S1: hedge \( u \) | \( \sigma_m^2 \) | 0    | \( \hat{a}_{t+1|t} = \phi y_t \) | \( P_{t+1|t} = \sigma_m^2 \) | \( \sigma_m^2 \) |
| S2: hedge \( m \) | \( \phi^2 \sigma_m^2 \) | \( \sigma_u^2 \) | \( \hat{b}_{t+1|t} = \phi (P_{t|t-1} + \sigma_u^2)^{-1} (P_{t|t-1} y_t + \sigma_u^2 \hat{b}_{t-1|t-1}) \) | \( P_{t+1|t} = \phi^2 (P_{t|t-1} + \sigma_u^2)^{-1} P_{t|t-1} \) | \( \sigma_u^2 + \sigma_m^2 \) |
| S3: hedge \( u \) and \( m \) | \( \phi^2 \sigma_m^2 \) | 0    | \( \hat{b}_{t+1|t} = \phi y_t \) | \( P_{t+1|t} = \phi^2 \sigma_m^2 \) | \( \phi^2 \sigma_m^2 \) |
| S4: no hedge     | \( \sigma_m^2 \) | \( \sigma_u^2 \) | \( \hat{a}_{t+1|t} = \phi (P_{t|t-1} + \sigma_u^2)^{-1} (P_{t|t-1} y_t + \sigma_u^2 \hat{a}_{t-1|t-1}) \) | \( P_{t+1|t} = \phi^2 (P_{t|t-1} + \sigma_u^2)^{-1} P_{t|t-1} \) | \( \sigma_u^2 + \sigma_m^2 \) |

The forecasts of observations (announced earnings) are equal to the state forecasts. Earnings surprises are, by definition, the difference between the realized and forecasted earnings:

\[
ES_t \equiv y_t - \hat{y}_{t|t-1} = y_t - \hat{x}_{t|t-1}.
\]

For strategies S1 and S3 it is obvious from the table above and the observation equation that

\[
ES_t^{(1)} = m_t, \quad ES_t^{(3)} = \phi m_t,
\]

where the superscripts in parentheses indicate hedging strategies. Since the expressions for state forecasts for S2 and S4 are more complicated, we cannot derive a simple closed-form expression for the associated earnings surprises. However, by combining the definition of \( ES_t \), the expression for \( \hat{y}_{t|t-1} \), and the
evolution of \( y_t \) (from the state and observation equations), we can express \( ES_{t+1}^{(2)} \) in terms of \( y_t, \hat{b}_{t|t-1} \), and the known parameters; noting further that \( y_t = \hat{b}_{t|t-1} + ES_t \), we obtain a recursive equation for \( ES_t^{(2)} \):

\[
ES_t^{(2)} = \varphi (P_{t-1|t-2} + \sigma_u^2) + \sigma_u^2 ES_{t-1}^{(2)} + \varphi m_{t-1} - \varphi u_{t-1} + u_t
\]

and, in the same fashion,

\[
ES_t^{(4)} = \varphi (P_{t-1|t-2} + \sigma_u^2) + \sigma_u^2 ES_{t-1}^{(4)} + m_{t-1} - \varphi u_{t-1} + u_t
\]

While earnings surprises deal with the observed earnings \( y_t \), the expectations of future earnings, and hence the stock price, depend on the estimate of the unobservable state \( x_t \). An important point to note is that, if the m-shocks decay (i.e., if \( \varphi < 1 \)), then the stock price in our model is expected to decrease with time and \( \lim_{t \to \infty} E[V_{t+\tau}] = 0 \). Since the information content of earnings surprises is meant to capture the "abnormal return" only, and not the expected drift toward zero, the appropriate definition (apart from the discount factor) is the difference between the one-period-ahead forecast at time \( t \) and the forecast of the same quantity made one period earlier, i.e.,

\[
IC_t = \hat{x}_{t+|t} - \hat{x}_{t+|t-1}.
\]

If \( \varphi = 1 \), this is equivalent to \( \hat{x}_{t+|t} - \hat{x}_{t+|t-1} \), the update of the one-period-ahead forecast. In that case, it is obvious that

\[
IC_t^{(1)} = m_t \quad \text{and} \quad IC_t^{(3)} = m_{t-1},
\]

while for strategies S2 and S4 we obtain expressions

\[
IC_t^{(i)} = \frac{P_{t-1|t-2} \left( \sigma_u^2 + \sigma_m^2 \right)}{P_{t-1|t-2} \left( 2 \sigma_u^2 + \sigma_m^2 \right)} ES_t^{(i)}
\]

where \( i = 2, 4 \). In steady state \( P \) is a constant and the (unconditional) distributions of \( IC_t \) and \( ES_t \) do not depend on time, so the recursions can be recast in a closed form. After some tedious algebra we find that \( \text{Var}(IC_t) = \sigma_m^2 \) for every strategy S1–S4.

For \( \varphi < 1 \), using the expressions for two-period-ahead forecasts, it is straightforward to derive the information content for strategies S1 and S3:

\[
IC_t^{(1)} = \hat{a}_{t+|t} - \hat{a}_{t+|t-1} = \varphi y_t - \varphi^2 y_{t-1} = \varphi (y_{t-1} + m_t) - \varphi^2 y_{t-1} = \varphi m_t
\]

\[
IC_t^{(1)} = \hat{b}_{t+|t} - \hat{b}_{t+|t-1} = \varphi y_t - \varphi^2 y_{t-1} = \varphi (y_{t-1} + \varphi m_t) - \varphi^2 y_{t-1} = \varphi^2 m_t
\]
and it is easy to see that for $\phi \to 1$ these expressions converge to those derived for $\phi = 1$.

For strategies S2 and S4 we get (derivation available from the authors):

(A23)  \begin{align*}
IC_t^{(2)} &= \frac{p_{t-1}^{(2)} \left( \frac{\sigma_u^2 + \sigma_m^2}{\sigma_u^2 + \frac{1}{\phi^2} \left( p_{t-1}^{(2)} \sigma_u^2 + \sigma_m^2 \right)} \right) \sigma_u^2}{p_{t-1}^{(2)} \left( \sigma_u^2 + \sigma_m^2 \right) + \frac{1}{\phi^2} \left( p_{t-1}^{(2)} \sigma_u^2 + \sigma_m^2 \right)} ES_t^{(2)}
\end{align*}

(A24)  \begin{align*}
IC_t^{(4)} &= \frac{p_{t-1}^{(4)} \left( \frac{\phi^2 \sigma_u^2 + \sigma_m^2}{\phi^2 \sigma_u^2 + \sigma_m^2 + \sigma_u^2 \sigma_m^2 + \sigma_u^2} \right) \sigma_u^2}{p_{t-1}^{(4)} \left( \phi^2 \sigma_u^2 + \sigma_m^2 + \sigma_u^2 \sigma_m^2 + \sigma_u^2 \right) + \sigma_u^2 \sigma_m^2 + \sigma_u^2} ES_t^{(4)}
\end{align*}

**A3. Expectations and variances of $IC_t$**

Ultimately, what we are interested in are the variances of the informational content of earnings surprises for various strategies. By definition, earnings surprise $ES_t$ and its informational content $IC_t$ must have zero expectations. This is obviously true for strategies S1 and S3 and it is not hard to show that it holds for strategies S2 and S4 as well. Finally we are ready to calculate the variances. Again, the calculation is trivial for strategies S1 and S3:

(A25)  \begin{align*}
Var(ES_t^{(1)}) &= \sigma_m^2; \quad Var(IC_t^{(1)}) = \phi^2 \sigma_m^2;
\end{align*}

(A26)  \begin{align*}
Var(ES_t^{(3)}) &= \phi^2 \sigma_m^2; \quad Var(IC_t^{(3)}) = \phi^4 \sigma_m^2.
\end{align*}

To get the results for strategy S2, consider the recursion (A16) with time advanced by one period. Since $m$-shocks are hedged, $m_t$ and $ES_t$ are independent; however, $ES_t$ and $u_t$ are not and

(A27)  \begin{align*}
Cov(ES_t^{(2)}, u_t) &= \sigma_u^2.
\end{align*}

Hence,

(A28)  \begin{align*}
Var(ES_t^{(2)}) &= \phi^2 \sigma_u^2 \frac{\sigma_u^2}{P_{t-1}^{SS} + \sigma_u^2} \left[ Var(ES_t^{(2)}) + \phi^2 \sigma_m^2 + \left( 1 + \phi^2 - 2 \phi^2 \frac{\sigma_u^2 \sigma_m^2}{P_{t-1}^{SS} + \sigma_u^2} \right) \sigma_u^2 \right].
\end{align*}

In steady state, $Var(ES_t^{(2)}) = Var(ES_{t+1}^{(2)}) = Var(ES_{SS}^{(2)})$ and (A31) can be recast in closed form:

(A29)  \begin{align*}
Var(ES_{SS}^{(2)}) &= \frac{\phi^2 \sigma_m^2 + \left( 1 + \phi^2 \frac{\sigma_u^2 \sigma_m^2}{P_{SS}^{(2)} + \sigma_u^2} \right) \sigma_u^2}{1 - \phi^2 \frac{\sigma_u^2 \sigma_m^2}{P_{SS}^{(2)} + \sigma_u^2}},
\end{align*}

where
(A30) \[ P_{SS}^{(2)} = \frac{1}{2} \left\{ \phi^2 \sigma_m^2 - \left(1 - \phi^2\right) \sigma_u^2 \right\} + \sqrt{\left(1 - \phi^2\right) \sigma_m^2 - \left(1 - \phi^2\right) \sigma_u^2} + 4 \phi^2 \sigma_m^2 \sigma_u^2 \}, \]

the steady-state solution to the recursion \[ P_{t+\ell}^{(2)} = \phi^2 \left( \frac{P_{t\ell} - \sigma_u^2}{P_{t\ell - 1} + \sigma_u^2} + \sigma_m^2 \right). \]

Similarly, for strategy S4 we get

\[
\var{ES}_{SS}^{(4)} = \frac{\sigma_m^2 + \left(1 + \phi^2 \frac{P_{SS}^{(4)} - \sigma_u^2}{P_{SS}^{(4)} + \sigma_u^2}\right) \sigma_u^2}{1 - \phi^2 \left(\frac{\sigma_u^2}{P_{SS}^{(4)} + \sigma_u^2}\right)^2},
\]

where

\[
P_{SS}^{(4)} = \frac{1}{2} \left\{ \sigma_m^2 - \left(1 - \phi^2\right) \sigma_u^2 \right\} + \sqrt{\left(1 - \phi^2\right) \sigma_m^2 - \left(1 - \phi^2\right) \sigma_u^2} + 4 \sigma_m^2 \sigma_u^2 \}.
\]

Thanks to equations (A23) and (A24) we can now state the main result of this section, the variance of the information content of earnings surprises:

\[
IC_{t}^{(2)} = \phi^2 \left( \frac{P_{SS}^{(2)} \left( \sigma_u^2 + \sigma_m^2 \right) + \sigma_u^2 \sigma_m^2}{P_{SS}^{(2)} \left( \sigma_u^2 + \sigma_m^2 \right) + \sigma_u^2 \sigma_m^2 + \frac{1}{\phi^2} \left( P_{SS}^{(2)} \sigma_u^2 + \sigma_u^2 + \sigma_u^2 + \sigma_m^2 \right) \right)^2 \left( \frac{1 + \phi^2 \frac{P_{t\ell}^{(2)} - \sigma_u^2}{P_{t\ell - 1}^{(2)} + \sigma_u^2} \sigma_u^2}{1 - \phi^2 \left(\frac{\sigma_u^2}{P_{t\ell}^{(2)} + \sigma_u^2}\right)^2} \right)
\]

and

\[
IC_{t}^{(4)} = \phi^2 \left( \frac{P_{SS}^{(4)} \left( \phi^2 \sigma_u^2 + \sigma_m^2 \right) + \sigma_u^2 \sigma_m^2}{P_{SS}^{(4)} \left( \phi^2 \sigma_u^2 + \sigma_m^2 \right) + \sigma_u^2 \sigma_m^2 + \phi^2 \sigma_u^2 + \phi^2 \sigma_u^2 + \sigma_m^2 + \sigma_u^2 + \sigma_m^2} \right)^2 \left( \frac{1 + \phi^2 \frac{P_{t\ell}^{(4)} - \sigma_u^2}{P_{t\ell - 1}^{(4)} + \sigma_u^2} \sigma_u^2}{1 - \phi^2 \left(\frac{\sigma_u^2}{P_{t\ell}^{(4)} + \sigma_u^2}\right)^2} \right).
\]

These expressions are very messy, especially when (A30) and (A32) are substituted for \( P_{SS} \). The best we can do analytically is gain some insight by considering a few special cases. Then we can numerically evaluate and plot (A30) and (A32) for different values of \( \phi \) and \( \sigma_u/\sigma_m \). (We can always normalize one of the \( \sigma \)'s to one.) This will yield sufficient results for our purposes.
In the special case of vanishing informative shocks \((\sigma_m = 0)\) all four strategies have \(\text{Var}(IC) = 0\). This is just what we should expect since the market would ignore all earnings surprises knowing that they are caused entirely by uninformative shocks. In the opposite extreme case, \(\sigma_u = 0\), \(\text{Var}(IC) = \varphi^2 \sigma_m^2\) for strategies S1 and S4 and \(\varphi^4 \sigma_m^2\) (which is less) for strategies S2 and S3. Of course, setting \(\varphi = 1\) reproduces the result derived earlier from (A20), that \(\text{Var}(IC) = \sigma_m^2\) for every strategy S1–S4.

It is now clear that in numerical evaluation we should set \(\sigma_m = 1\) and vary \(\sigma_u\) from 0 up. We find that \(\text{Var}(IC_t^{(2)})\) and \(\text{Var}(IC_t^{(4)})\) are bounded above by their values for \(\sigma_u = 0\), i. e., by \(\text{Var}(IC_t^{(3)})\) and \(\text{Var}(IC_t^{(1)})\), respectively. Furthermore, there is a value of \(\sigma_u/\sigma_m\) for which \(\text{Var}(IC_t^{(4)}) = \text{Var}(IC_t^{(3)})\) and above which \(\text{Var}(IC_t^{(4)}) < \text{Var}(IC_t^{(3)})\).
REFERENCES


