Systemic Risk in the Insurance, Banking, Brokerage and 
Non-Financial Sectors: Time-Lags and Persistence

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Abstract

Common systemic risk measures focus on the instantaneous occurrence of triggering and systemic events. However, systemic events may occur with a time-lag to the triggering event. To study this contagion period and the resulting persistence of institutions’ systemic risk, we develop and employ the Conditional Shortfall Probability (CoSP), which is the likelihood that a systemic market event occurs with a specific time-lag to the triggering event. Based on CoSP we propose two aggregate systemic risk measures, namely the Aggregate Excess CoSP and the CoSP-weighted time-lag that reflect the systemic persistence and contagion period of an institution’s triggering event.

Our empirical results show a significant systemic time-lag for various institutions. The size of this time-lag and the related systemic risk (measured by CoSP) depend on the respective institution and market. In general, we find that brokers exhibit the most persistent systemic impact on all considered markets, whereas non-financial companies have the smallest systemic persistence. Insurance companies are exposed to the largest systemic persistence among financial institutions, particularly if systemic risk is triggered by non-financial companies. In contrast, the systemic persistence of insurance companies is similar to that of banks but substantially smaller than the systemic persistence of brokers.
1 Introduction

According to the The Group of Ten (2001, p. 126) "systemic financial risk is the risk that an event will trigger a loss of economic value or confidence in, and attendant increases in uncertainty about, a substantial portion of the financial system that is serious enough to quite probably have significant adverse effects on the real economy". This definition was also adopted by the Financial Stability Board (2011, p. 5) in the sense, that an institution or market is considered as being systemic if its "failure or malfunction causes widespread distress, either as a direct impact or as a trigger for broader contagion". Systemic events may be caused by (over-)reactions of market participants, for example by fire sales of securities, bank runs or insurance runs (in particular through surrenders in life insurance). Such reactions typically evolve over time and are often accompanied by strong herding behavior (see Ouarda et al. (2013) or Kehoe and Cole (2000)). A vital element of contagion is causality: Systemic distress is a result of triggering events that cause (the lack of) cash-flows - or the information on (the lack of) cash-flows - to migrate from institution to institution through their interconnectedness. This effect is often referred to as domino effect (see Smaga (2014)).

Systemic domino effects have an inherent time dimension. They start with one or multiple triggering events and result in the contagion of capital markets and even the whole economy. For this reason, one might expect that systemic risk measures incorporate the timing of triggering and subsequent systemic events. Interestingly, all recently proposed risk measures that play an important role in the discussion about the measurement of systemic risk do not take such time-lags into consideration. This observation motivates our article, in which we develop measures for systemic risk that explicitly take the time-lag between triggering and systemic events into account. Hereby, we aim to describe the timing dimension of systemic risk.

Still, information efficiency can be the rationale for measuring systemic risk without incorporating time-lags between triggering events and resulting systemic effects, since informationally efficient capital markets in the semi-strong or strong form (see Fama (1969)) would immediately reflect the effects of interconnectedness and contagion in the market prices. Thus, systemic risk would solely exist as simultaneous distress of the market and an institution, i.e., the market would react instantaneously to a triggering event.

However, the idealization of an informationally efficient market in the semi-strong or strong form is unlikely to hold in practice. For example, Billio et al. (2010) find that stock returns exhibit large autocor-
relations with a lag of one month particularly during the recent crisis 2007-2009. Market frictions prevent arbitrage opportunities from being completely exploited (see for example Farmer and Lo (1999) or Lo (2004)). These frictions may be caused by transaction costs or borrowing constraints, but also by the high complexity and opaqueness of interconnected markets and companies, particularly in the financial services sector (see Moghadam and Viñals (2010) or Adrian et al. (2014)). Complex products like hybrid debt (e.g. CoCo-Bonds), reinsurance, specific forms of parent-subsidiary relationships, captives and other forms of capital transfer mechanisms contribute to this opaqueness.

In our approach we build on recent literature aiming to measure systemic risk by using stock returns (see, e.g. Acharya et al. (2012), Adrian and Brunnermeier (2014) or Black et al. (2016)), which reflect market participants’ expectations on the future firm policy, on policy-decisions (like e.g. bailouts), but also, e.g., herding behavior. However, the major part of the literature focuses on simultaneous movements of returns (for example see Bisias et al. (2015)). In other words, the authors (implicitly) assume a semi-strong or strong form of information efficiency.

The most prominent method that, in contrast, considers a time-lag between return series is the Granger causality test (cf. Granger (1969)): In the sense of Granger causality, a time-series \( X \) Granger-causes a time-series \( Y \) if lagged information on \( X \) provides statistically significant information about \( Y \). By employing a nonlinear Granger causality test for CDS spreads Chen et al. (2013) find that the influence of banks on insurers is more persistent than vice versa. Billio et al. (2010) conclude that the impact of banks on hedge funds, insurers and brokers is more significant than vice versa. Moreover, Giglio et al. (2015) examine the predictive power of common risk measures and lower quantiles of the distribution of macroeconomic shocks by means of Granger causality tests and a factor model. Nonetheless, Granger causality tests only examine if the coefficient of a specific lagged return is statistically significantly different to zero for a chosen significance level. Moreover, they do not focus on systemic events but on a model for the whole return series and, thus, are only valid within the underlying distributional and modeling assumptions.

To sum up, a thorough analysis of the contagion period between triggering and systemic events is still missing. We address this issue by proposing and employing the Conditional Shortfall Probability (CoSP),

Our understanding of a time-lag between trigger and systemic events is different from forward-measures as e.g. the Forward-ΔCoVaR: Forward-measures focus on a forecast of future values of the systemic risk measure (in the case of Forward-ΔCoVaR the future value of ΔCoVaR is forecasted, cf. Adrian and Brunnermeier (2014)), whereas the risk measure itself still focuses on simultaneous movements of market’s and institution’s returns.
which is the likelihood of a systemic event occurring with a specific time-lag to a triggering event. Motivated by its properties, we use CoSP to define measures of the systemic persistence and contagion period, namely the Aggregate Excess CoSP and the CoSP-weighted time-lag.

Furthermore, we conduct an empirical analysis of the proposed risk measures that focuses on the structural differences between the systemic risk of banks, brokers, insurance and non-financial companies. The systemic relevance of (life) insurance companies, in particular, has recently caused massive disputes. Due to the long-term investments on the one hand, and the insurers’ core business activities being fundamentally different from the banking business on the other hand, many authors argue that (life) insurers are less systemically relevant than banks (see Thimann (2014) or Haefeli and Liedtke (2012)). This view is also supported by the econometric analysis of Acharya et al. (2010), but it is not generally agreed on. In particular, Billio et al. (2010) and Adrian and Brunnermeier (2014) find that the systemic risk in the insurance sector is generally not smaller than in the banking sector, whereas Weiß and Mühlnickel (2014) conclude that the systemic risk of insurance companies is only driven by the insurer’s size. To conclude, the specific systemic role of insurance companies is still not clear and is causing controversial discussions between academics, regulators and insurance companies.²

By considering the timing dimension of systemic risk, our article provides a broader perspective on the systemic riskiness of insurance companies. First empirical results show that systemic risks are often spread among institutions and markets with a significant time-lag. In particular, the systemic risk of insurance companies is similar to banks, whereas their exposure to systemic risk is larger than the exposure of banks and brokers.

The remainder of the paper is organized as follows. Section 2 gives an overview of existing systemic risk measures. In Section 3 we introduce new systemic risk measures that incorporate time-lags. Section 4 describes the data sample which we use for our empirical analysis in Section 5. The final Section 6 concludes and gives an outlook to future research directions.

2 Traditional Measures for Systemic Risk

Bisias et al. (2015) report four main cross-sectional systemic risk measures: distress insurance premium (see Huang et al. (2009)), marginal expected shortfall (see Acharya et al. (2010)), ∆CoVaR (see Adrian

and Brunnermeier (2014) and co-risk (see Chan-Lau et al. (2009)). The distress insurance premium (DIP) is a hypothetical insurance premium against catastrophic losses in the financial/banking system and is estimated by using CDS spreads (cf. Huang et al. (2009)). As Black et al. (2016) show in an empirical analysis, the DIP is mainly driven by risk-neutral probabilities of default. The idea of Marginal Expected Shortfall (MES) is, that a financial institution is systemically risky if it performs poorly at the same time that a benchmark market performs poorly. Thus, MES reflects the exposure of a financial institution to the market’s systemic risk. Benoit et al. (2013) show that MES is proportional to an institution’s systematic risk, as measured by its time-varying beta.

In the following, we will shortly review $\Delta$ CoVaR to motivate our own approach. $\Delta$ CoVaR measures the change in the market’s risk conditional on a financial institution being in distress. Hereby, the market’s risk conditional on a specific event $E$, $\text{CoVaR}_E(q)$, is defined as the Value-at-Risk (VaR) conditional on an event $E$, i.e.

$$\mathbb{P}(r^M \leq \text{CoVaR}_E(q) \mid E) = q,$$

where $r^M$ is the return of a market index. Then, $\Delta$ CoVaR is the difference between the market’s VaR conditional on the institution’s return, $r^I$, being at the benchmark state $BM^I$ and the market’s VaR conditional on an institution’s triggering event $TE^I$, i.e.

$$\Delta \text{CoVaR} = \text{CoVaR}_{TE^I}(q) - \text{CoVaR}_{BM^I}(q).$$

Adrian and Brunnermeier (2014) define the triggering event as the institution’s return being at the $VaR(q)$, i.e. $TE^I = \{r^I = VaR^I(q)\}$, and the benchmark event as the institution’s return being at the median state, i.e. $BM^I = \{r^I = VaR^I(0.5)\}$, which yields

$$\Delta \text{CoVaR}^{=}(q) = \text{CoVaR}_{r^I=VaR^I(q)}(q) - \text{CoVaR}_{r^I=VaR^I(0.5)}(q).$$

However, more severe losses than $VaR^I(q)$ are not considered in the triggering event. Hence, it may be appropriate to examine a more general version of the risk measure, instead. For this purpose, Ergün and Girardi (2013) employ

$$\Delta \text{CoVaR}^{\leq}(q) = \text{CoVaR}_{r^I \leq VaR^I(q)}(q) - \text{CoVaR}_{r^I \in [\mu^I \pm \sigma^I]}(q),$$

$^{3}$Co-Risk is very similar to CoVaR, except that it examines CDS spreads instead of returns.
where $\mu^I$ and $\sigma^I$ are, respectively, the mean and standard deviation for the return of institution $I$. The change in the triggering event definition from being exactly at the VaR to being at or below the VaR also effects the consistency of CoVaR: Mainik and Schaanning (2014) show that $\text{CoVaR}_{r^I \leq \text{VaR}^I(q)}(q)$ is a continuous and increasing function of the dependence parameter between $r^I$ and $r^M$, while $\text{CoVaR}_{r^I = \text{VaR}^I(q)}(q)$ is not.

The reported traditional systemic risk measures have in common that their calculation is based on a cross-sectional analysis of returns or CDS spreads. In terms of $\Delta\text{CoVaR}$, the systemic risk measure reflects a simultaneous event of exceptionally high losses of both the market index and an institution. Any contagion effects in the sense that a high loss of an institution can trigger losses of a capital market index with a time-lag are, therefore, not captured. This is our motivation to propose alternative systemic risk measures that take contagion periods into account.

3 Measuring Lagged Systemic Risk

3.1 The Conditional Shortfall Probability

The Conditional Shortfall Probability (CoSP) is a systemic risk measure that explicitly accounts for time-lags between trigger and systemic events. Consistent with $\Delta\text{CoVaR}^\leq$, we interpret the occurrence of one of the $q \cdot 100\%$ smallest returns as a proxy for a trigger or systemic event. To assess the time-lagged influence of an institution’s triggering event on the market, we define the Conditional Shortfall Probability (CoSP)

$$\psi_\tau(q) = \mathbb{P}(SE^M_\tau \mid TE^I) = \mathbb{P}(r^M_\tau \leq \text{VaR}^M(q) \mid r^I \leq \text{VaR}^I(q)).$$

(5)

In this definition $r^M_\tau$ is the market return $\tau$ days after the institution’s return $r^I$. $SE^M_\tau$ and $TE^I$ are the systemic and triggering event, respectively. Thus, $\psi_\tau$ is the likelihood of an exceptionally small market return occurring $\tau$ days after an exceptionally small return of institution $I$. CoSP is very similar to $\Delta\text{CoVaR}^\leq$. In fact, we can also define a time-lagged $\text{CoVaR}^E_\tau$ by

$$\mathbb{P}(r^M_\tau \leq \text{CoVaR}^E(q) \mid E) = q,$$

(6)
and compute the time-lagged \( \Delta \text{CoVaR} \leq \) by

\[
\Delta \text{CoVaR}^{\leq}_{\tau}(q) = \text{CoVaR}^{\leq}_{\tau r \leq \text{VaR}(q)}(q) - \text{CoVaR}^{\tau}_{r \in [\mu \pm \sigma]}(q).
\]  

(7)

Both measures, CoSP and \( \Delta \text{CoVaR}^{\leq}_{\tau}(q) \), generate the same order of institutions according to systemic risk if the conditional market returns stochastically dominate each other.\(^4\) However, CoSP is generally easier to estimate and the standard error of CoSP is often smaller.\(^5\) In other words, less data is needed to estimate CoSP. In Appendix A we discuss several other properties of CoSP.

It is worthwhile to take a second look at the interpretation of CoSP, since \( \psi_{\tau} \), at first sight, seems to be the probability that a triggering event takes exactly \( \tau \) days to affect the market return. However, one specific triggering event may contribute to several lags:

Figure 1: Exemplary sequence of trigger and systemic events.

Figure 1 depicts an exemplary sequence of events: \( E \) and \( A \) as well as \( F \) and \( B \) occur simultaneously, thus, with lag \( \tau = 0 \) (straight arrows). But \( E \) and \( F \) also contribute to lags \( \tau = 1 \) (dashed arrows) and \( \tau = 2 \) (dotted arrows) due to the events \( B, C \) and \( D \). Should a triggering event \( TE^I \), instead, contribute utmost to one lag \( \tau^* \)? The selection of such a lag would require a lot more knowledge about the returns than is actually available: For example, the "true" reaction to event \( E \) may be \( A (\tau^* = 0), B (\tau^* = 1) \) or \( C (\tau^* = 2) \) or \( D (\tau^* = 3) \). However, we can not gauge the direct causal relationship between the market’s and institution’s events and, thus, cannot determine the correct lag \( \tau^* \). Still, it seems reasonable to assume that \( E \) can trigger and/or amplify all simultaneous and subsequent systemic events \( SE^M \).\(^6\)

Consequently, for most lags we will find events with different time-lags \( \tau \) and, thus, have \( \psi_\tau(q) > 0 \).

\(^4\)We show this in Appendix A.3
\(^5\)In Section B we compare the standard errors of CoSP and \( \Delta \text{CoVaR} \).
\(^6\)Note that according to the Financial Stability Board (2011, p. 5), an institution is not only considered as systemically important if its malfunction directly triggers a systemic event but also if it triggers broader contagion.
Nevertheless, not all observed lags directly correspond to systemic risk. A natural reference level for CoSP is given by the VaR-level $q$, since $TE^I$ and $SEM^M_\tau$ are independent if, and only if

$$
\psi_\tau(q) = q.
$$

(8)

In Appendix A.2 we derive this property and give an example with independent student-distributed returns. Hence, lags between randomly occurring events (i.e. noise) are captured by the reference level $q$. In contrast, we focus on lags that occur unusually often, i.e. are not caused by independent events. Therefore, we identify an institution as systemically risky at lag $\tau$, if $\psi_\tau(q) > q$. Furthermore, for very large lags one would intuitively presume that the influence of the institution’s return $r^I$ on the market return $r^M_\tau$ diminishes, i.e. $TE^I$ and $SEM^M_\tau$ are independent.\footnote{This behavior may result from arbitrage opportunities being exploited in the long-run.} Hence, we conjecture (and find this confirmed in the empirical analysis) that

$$
\lim_{\tau \to \infty} \psi_\tau(q) = q.
$$

(9)

We interpret CoSP as a measure for the systemic influence which an exceptionally small return of the institution has on the market: If $\psi_\tau(q)$ declines slowly, more events occur at larger lags, thus, the influence of the institution’s events lasts longer. However, not only the speed of decline but also the size of $\psi_\tau(q)$ is important to consider, since larger values for $\psi_\tau(q)$ indicate a larger risk. Therefore, we suggest to compute the size of the area between the reference level and $\psi_\tau(q)$ as a measure for the systemic persistence, i.e.

$$
\bar{\psi}(q) = \int_1^{\infty} (\psi_\tau(q) - q) \, d\tau.
$$

(10)

We call $\bar{\psi}(q)$ the **Aggregate Excess CoSP**. Note that we do not include co-movements at lag $\tau = 0$, since they do not reflect systemic persistence and may cause substantial noise. A second measure is the **CoSP-weighted time-lag**, i.e.

$$
\bar{\tau} = \frac{1}{\psi(q)} \int_1^{\infty} \tau \, (\psi_\tau(q) - q) \, d\tau.
$$

(11)

This measure is an average of all time-lags, which are weighted with their contribution to the Aggregate Excess CoSP.\footnote{Note that $\bar{\tau}$ exhibits analogies to the duration concept.} Hence, it is essentially a measure for the (weighted) time-lag between triggering and systemic event. A major advantage of $\bar{\tau}$ is, that it is measured in time units (e.g. days).
3.2 Estimation of CoSP

To estimate $\psi_\tau(q)$ we employ historical simulation (HS)\footnote{There exist several studies discussing and improving the statistical properties of HS and other estimation approaches for return series, for example [Hendricks (1996), Hull and White (1998), Kuester et al. (2006) or Pritsker (2001)]. However, they focus on quantile or moment estimation and do not consider a time-lag and, thus, cannot be applied for the estimation of CoSP.} The use of this simplified approach is motivated by two reasons: Firstly, it is little known about the inter-dependence of lagged tail-returns. Thus, it is unreasonable to start the analysis by imposing distributional or modeling assumptions. Secondly, standard approaches to handle return series data like GARCH-filtering (examples can be found e.g. in Ergün and Girardi (2013) or Chen et al. (2013)) cannot be employed since volatility updates affect $\psi_\tau$, as we show in Appendix C.

The estimation procedure is similar to Acharya et al. (2010): For $n$ observations an estimate for \( \text{VaR}^I(q) \) is given by the \( q \cdot 100\% \)-th smallest value. Then, $\hat{\psi}_\tau(q)$ is the number of days with the $q \cdot 100\%$ smallest market and institution return outcomes with lag $\tau$ relative to the number of days with the $q \cdot 100\%$ smallest institution return outcomes. Since the latter is given by $q(n - \tau)$, we have

\[
\hat{\psi}_\tau(q) = \frac{1}{q(n - \tau)} \sum_{t=1}^{n-\tau} \mathbb{1}\{r_I^t \leq \hat{\text{VaR}}^I(q), r_M^{t+\tau} \leq \hat{\text{VaR}}^M(q)\}.
\]  

(12)

The larger the lag $\tau$ is, the less observations we can use and, thus, the estimation precision declines. To isolate significantly large values from noise, in Section refSubsectionAppendixLowerBoundSignificance we compute a lower bound of significance, which is given by

\[
k_{\tau}^*(q) = \frac{1}{q(n - \tau)} \left( F_{\text{Bin}(n - \tau, q^2)}^{-1}(1 - \alpha) + 1 \right)
\]  

(13)

for a significance level $\alpha \in (0, 1)$. Thus, if $\hat{\psi}_\tau(q) \geq k_{\tau}^*(q)$, we can reject the hypothesis that $SE^M_\tau$ and $TE^I_\tau$ are independent on the significance level $\alpha$. Therefore, we call an institution significantly systemically risky at lag $\tau$, if $\hat{\psi}_\tau(q) \geq k_{\tau}^*(q)$.

Nonetheless, the estimation of $\hat{\psi}_\tau(q)$ may lead to large outliers that affect the Aggregate Excess CoSP and, particularly, the CoSP-weighted time-lag. Thus, we suggest to fit $\hat{\psi}_\tau(q)$ for $\tau = 1, 2, ..., \tau$ to the function

\[
H(\tau) = q + e^{-a\tau^2 + br + c}, \quad \text{with } a > 0, \ b, c \in \mathbb{R}.
\]  

(14)
This choice has several advantages: Since \( H \) is always larger than \( q \) but converges to \( q \), we only capture systemically risky lags, i.e. fit the part of \( \hat{\psi}_\tau(q) \) that lies above \( q \). However, if \( \hat{\psi}_\tau(q) < q \), we have \( H \approx q \), indicating that the systemic risk is zero. Furthermore, we can use \( H \) to estimate \( \bar{\psi} \) and \( \bar{\tau} \) by

\[
\bar{\psi} \approx \int_1^\infty e^{-a\tau^2 + b\tau + c} d\tau = e^{c + \frac{\tau^2}{4a}} \left( 1 - \text{erf} \left( \frac{2a - b}{2\sqrt{a}} \right) \right) \tag{15}
\]

and

\[
\bar{\tau} \approx \frac{1}{\bar{\psi}} \int_1^\infty \tau e^{-a\tau^2 + b\tau + c} d\tau = \frac{1}{4a^{3/2}\bar{\psi}} \left( \sqrt{\pi} \frac{e^{\frac{b^2}{4a}} + c}{\sqrt{\pi}} \left( 1 - \text{erf} \left( \frac{2a - b}{2\sqrt{a}} \right) \right) \right) + 2\sqrt{a}e^{b-a+c} \tag{16}
\]

### 3.3 The Exact Time-Lag

Another interesting question is, how large the specific time-lag \( \tau^* \) that lies between a triggering event and the first systemic market reaction. To answer this question, we study a discrete approximation of the probability distribution of the exact time-lag \( \tau^* \), which is

\[
F_{\tau^*}(x) = \mathbb{P}(\tau^* = x) = \mathbb{P} \left( r^M_2 \leq VaR^M(q) \text{ and } r^M_0, ..., r^M_{x-1} > VaR^M(q) \mid r^I_0 \leq VaR^I(q) \right) \tag{18}
\]

To examine the properties of \( F_{\tau^*} \) we can apply the whole toolbox of statistics, for example the mean, standard deviation, quantiles, etc. However, the estimation may lead to substantial outliers for \( F_{\tau^*} \), thus, we do not employ the mean or standard deviation. Instead, we study the median value \( \tau_{0.5}^* \), i.e. the median exact time-lag between trigger and systemic event.

### 4 Data and Descriptive Statistics

Our data sample includes all historical daily returns of 917 publicly traded financial institutions that are classified as banks, brokers or insurers in Datastream. Moreover, we examine the returns of 35 non-financial companies that are selected according to market capitalization. All returns are daily for the

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10 We derive these results in Appendix A.4 and A.5.
11 \( F_{\tau^*} \) is a probability distribution if almost surely sometime a systemic market event occurs, i.e.

\[
1 = \mathbb{P} \left( \bigcup_x r^M_2 \leq VaR^M(q) \text{ and } r^M_0, ..., r^M_{x-1} > VaR^M(q) \mid r^I_0 \leq VaR^I(q) \right) = \sum_x F_{\tau^*}(x), \tag{17}
\]

which seems reasonable.
12 The names of the 10 largest companies in each subsector are reported in Table 2.
13 The company names are reported in Table 1.
period from November 21, 1995 to November 20, 2015. To avoid endogeneity, we compute our own indices as described in Appendix E.1 that exclude the currently considered institution. In Figure 11 (a) we show the resulting indices for banks (BAN), brokers (BRO), insurers (INS) and the whole financial market (FIN) if no institution is excluded from the index. Furthermore, we consider three continent-specific indices for non-financial companies, namely indices for Americas (AMC), Asia (ASIA) and Europe (EU), which are shown in Figure 11 (b).

In Appendix E.2 we report the descriptive statistics for the returns of all companies included in the data sample. For this purpose, we estimate the mean and standard deviation of returns for all single companies and report the distribution of these estimates among the subsectors banks (BAN), brokers (BRO), insurers (INS) and non-financial companies (NoFIN) in Table 3 and Figure 12. Moreover, Table 4 shows the mean, standard deviation and quantiles for different indices. Hereby, all financial indices are computed as described in Section E.1 without excluding any companies.

All tables and figures indicate a very homogeneous data sample: The mean returns are approximately zero for all companies, whereas 95% of the companies exhibit a standard deviation of their returns between 1% and 4.7%. Finally, in Figure 12(a) we show the distribution of empirical correlation coefficients between single companies and the financial market index.

5 Empirical Findings

We apply the methodology described in Section 3 and examine the persistence and delay of systemic risk for the data sample described in Section 4. For the reference level of CoSP we use $q = 1\%$, which seems reasonable to capture exceptionally small returns.

In Figure 2 we show the CoSP for several institutions that exhibit a typical pattern. As presumed in Section 3.1, the fitted CoSP is declining and converges to the reference level $q$ for $\tau \to \infty$. Furthermore, CoSP is significantly larger than the reference level for several lags $\tau > 0$, thus, there is statistically significant influence between triggering events and systemic events with a time-lag. Moreover, there are clear differences between the subsectors: For non-financials the CoSP declines relatively fast and is rather small (for Apple and Amazon.com the CoSP is for no lag significantly larger than the reference level), whereas the CoSP for financial institutions seems to be larger and of slowly declining shape.

\textsuperscript{14}More examples can be found in Appendix E and can be provided by the authors on request.
This observation is supported by the distribution of the Aggregate Excess CoSP: In Figure 3, we show the distribution of the Aggregate Excess CoSP for the 10% most persistent companies, i.e. the 10% companies with the largest CoSP in the specific subsector. Clearly, the persistence measure is larger than zero. Thus, most companies are systemically important beyond lag $\tau = 0$. Moreover, for most indices brokers have the largest systemic persistence, whereas non-financials have the smallest. Interestingly, the systemic persistence of insurance companies is not substantially smaller than the persistence of banks but sometimes even larger. One might suspect that this effect is caused by one or two single companies. However, the boxplots consist of 25 insurance companies. Thus, single companies with exceptionally large $\bar{\psi}$ are outliers and do not lie between the 25% and 75% quantiles.

To study the exposure to different markets, Figure 4 also depicts the distribution of the Aggregate

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15 In Figure 16 in the appendix we show the distribution of the Aggregate Excess CoSP for the total data sample. However, there seem to be no substantial differences between the subsectors. This observation indicates that our data sample is very broad and, in particular, includes companies with very large and very small systemic importance. Moreover, in Section F we show the boxplots for the 25% most persistent institution, which, still, do not show the differences as clearly as for the 10% most persistent institutions.

16 This corresponds to the results of Billio et al. (2010) and Adrian and Brunnermeier (2014).
(a) Aggregate Excess CoSP w.r.t. the FIN index.  (b) Aggregate Excess CoSP w.r.t. the American NoFIN index.

Figure 3: Aggregate Excess CoSP w.r.t. the FIN and American NoFIN indices triggered by the 10% most systemically persistent institutions of the subsectors BAN, INS, BRO and NoFIN. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$.

Excess CoSP among the 10% most persistent institutions while comparing different indices for one subsector, respectively. Interestingly, the persistence on the Asian non-financial market is the smallest for all subsectors. In contrast, the systemic risk exposure of brokers, insurers, financial companies in general and the American non-financial market is approximately equal, whereas the exposure of banks is the smallest for the financial sector. Furthermore, the European non-financial market is very persistently exposed to the systemic risk of all financial subsectors.

In Figure 5 we show the distributions of the CoSP-weighted time-lag. The large value for the systemic influence of non-financial companies on insurance companies is particularly striking: It indicates that non-financial companies systemically affect the insurance sector with a large time-lag. Moreover, insurers are in general affected with a large time-lag by all subsectors, while the lag between insurers’ triggering events and systemic events is relatively small, as Figure 6 shows.
Figure 4: Aggregate Excess CoSP exposure of different financial indices BAN, BRO, INS, FIN and non-financial indices AMC, ASIA, EU, triggered by the 10% most systemically persistent banks, brokers, insurers and non-financial companies. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$. 

(a) Aggregate Excess CoSP triggered by banks.  
(b) Aggregate Excess CoSP triggered by brokers. 
(c) Aggregate Excess CoSP triggered by insurance companies. 
(d) Aggregate Excess CoSP triggered by non-financial companies.
(a) CoSP-weighted time-lag triggered by banks. (b) CoSP-weighted time-lag triggered by brokers.

(c) CoSP-weighted time-lag triggered by insurers. (d) CoSP-weighted time-lag triggered by non-financials.

Figure 5: CoSP-weighted time-lag exposure of different financial indices BAN, BRO, INS, FIN and non-financial indices AMC, ASIA, EU, triggered by the 10% most systemically persistent banks, brokers, insurers and non-financial companies. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$.

(a) CoSP-weighted time-lag for FIN index. (b) CoSP-weighted time-lag for Americas NoFIN index.

Figure 6: CoSP-weighted time-lag w.r.t. the FIN and American NoFIN indices triggered by the 10% most systemically persistent institutions of the subsectors BAN, INS, BRO and NoFIN. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$.
6 Conclusion

Since cash-flows, information and market participants’ reactions may take time to spread within and across (financial) markets, systemic risk is not only the risk of simultaneously occurring events, but it is also the risk of systemic market reactions that occur with a time-lag to the triggering event. Thus, in addition to the common systemic risk measures that examine the instantaneous systemic risk, we propose to also consider the time-lag between triggering and systemic events and the persistence of systemic risk. In particular, we define the Conditional Shortfall Probability (CoSP), $\psi_\tau(q)$, which is the likelihood that a systemic market event occurs $\tau$ days after a triggering event of the institution. The reference level $q$ corresponds to independence between triggering and systemic event. In typical cases, $\psi_\tau(q)$ is exponentially declining and converges to the reference level.

Motivated by the properties of CoSP, we define two aggregate risk measures: The Aggregate Excess CoSP, $\bar{\psi}$, and the CoSP-weighted time-lag, $\bar{\tau}$. Aggregate Excess CoSP is the area between $\psi_\tau(q)$ and the reference level $q$. Thus, it reflects the overall influence of a company on the market, i.e. its systemic persistence. The CoSP-weighted time-lag is the weighted average of all time-lags. The weighting factors are the relative contributions of the time-lags to the Aggregate Excess CoSP. Both measures are constructed from CoSP but represent different dimensions of systemic risk, namely the systemic persistence and systemic contagion period.

In our empirical analysis we study the 10% most systemically persistent companies in the subsectors banks, brokers, insurance and non-financial companies. The results show that brokers have the largest systemic persistence and time-lag among all institutions, whereas non-financial companies exhibit the smallest persistence and time-lag. Regarding the exposure to systemic persistence, we observe the largest exposure on the European non-financial market and the smallest on the Asian non-financial market. Interestingly, banks have the smallest exposure to systemic persistence among the financial institutions, whereas brokers, insurers and the American non-financial market’s exposure is rather large.

We find that the systemic persistence of insurance companies is not smaller than the persistence of banks.\textsuperscript{17} Moreover, we observe that both banks and brokers have a more persistent systemic influence on insurance companies than vice versa.\textsuperscript{18} Interestingly, the insurance industry’s exposure to systemic

\textsuperscript{17} This result is similar to Adrian and Brunnermeier (2014) and Billio et al. (2010) but is contrary to Berdin and Sottocornola (2015).

\textsuperscript{18} This result is in line with the findings of Chen et al. (2013).
persistence and the contagion period are the largest among the financial sector. Moreover, the contagion period is particularly large for systemic events in the insurance industry triggered by non-financial companies, but, in contrast, is substantially smaller for systemic events for banks and brokers triggered by non-financials. This result can be seen as a first hint that shocks from the non-financial sector take their time to migrate through the books of banks and brokers until they affect the insurance industry.

In conclusion, we find significant evidence that triggering events systemically affect the market with different time-lags and persistence. Since this is, to our knowledge, the first article studying systemic contagion periods, further research on this topic can take various forms. For example, the interdependence between measures for lagged systemic risk and measures for instantaneous systemic risk (e.g. $\psi_0$, MES or $\Delta \text{CoVaR}$) may shed light on the relationship between systemic contagion and (the tail of) co-movements of returns. Moreover, the drivers of systemic persistence and contagion periods may be fundamentals like leverage ratios, asset duration or firm size that need to be identified in order to understand the underlying rationale of the timing dimension of systemic risk.
Appendix

A Properties of CoSP

The conditional shortfall probability (CoSP) is given as

$$\psi_\tau(q) = \Pr(r^M_\tau \leq \text{VaR}^M(q) \mid r^I \leq \text{VaR}^I(q)).$$  \hspace{1cm} (19)$$

Thus, $\psi_\tau(q)$ is very similar to the coefficient of lower tail dependence. In particular, the latter is the limit of $\psi_\tau(q)$ as $q$ approaches 0, i.e.

$$\lambda_\tau = \lim_{q \to 0^+} \psi_\tau(q),$$  \hspace{1cm} (20)

where $\lambda_\tau$ is the coefficient of lower tail dependence between $r^I_0$ and $r^M_\tau$ (cf. McNeil et al. (2015, p.247)).

A.1 Symmetry

In general, $\psi_\tau(q)$ is symmetric, i.e.

$$\psi_\tau(q) = \Pr(r^M_\tau \leq \text{VaR}^M(q) \mid r^I_0 \leq \text{VaR}^I(q))$$

$$= \frac{\Pr(r^M_\tau \leq \text{VaR}^M(q), r^I \leq \text{VaR}^I(q))}{q} = \Pr(r^I \leq \text{VaR}^I(q) \mid r^M_\tau \leq \text{VaR}^M(q)).$$  \hspace{1cm} (21)

However, considering the latter probability, $\Pr(r^I \leq \text{VaR}^I(q) \mid r^M_\tau \leq \text{VaR}^M(q))$, for $\tau > 0$ does not make sense, since the conditioning event would occur after the main event. Therefore, $\psi_\tau$ incorporates a natural sense of causality for $\tau > 0$, whereas $\psi_0$ is symmetric, i.e.

$$\psi_0(q) = \Pr(r^M \leq \text{VaR}^M(q) \mid r^I \leq \text{VaR}^I(q)) = \Pr(r^I \leq \text{VaR}^I(q) \mid r^M \leq \text{VaR}^M(q)).$$  \hspace{1cm} (23)

This is also very reasonable, since $\psi_0(q)$ is the result of co-movement between $r^M$ and $r^I$.

A.2 Independence

By definition, two trigger and systemic event, i.e. $TE^I = \{r^I \leq \text{VaR}^I(q)\}$ and $SE^M_\tau = \{r^M_\tau \leq \text{VaR}^M(q)\}$, are stochastically independent if, and only if,

$$\Pr(SE^M_\tau, TE^I) = \Pr(SE^M_\tau) \cdot \Pr(TE^I),$$  \hspace{1cm} (24)
which is equivalent to (assuming $\mathbb{P}(TE^I_0) > 0$)

$$\psi_\tau(q) = \frac{\mathbb{P}(SE^M_\tau, TE^I)}{\mathbb{P}(TE^I)} = \mathbb{P}(SE^M_\tau) = q.$$ (25)

Hence, we have $\psi_\tau(q) = q$ if, and only if, trigger and systemic event $SE^M_\tau$ and $TE^I$ are stochastically independent.

In Figure 7 we show the estimate for CoSP, $\hat{\psi}_\tau(0.01)$, from 10,000 independent observations for $r^M$ and $r^I$ that were drawn from a student-t(696) distribution.\footnote{The degrees of freedom ($\nu = 696$) correspond to the 1\% smallest estimated degrees from the data sample, cf. Section B.} Clearly, there exist numerous observations of events $SE^M_\tau$ and $TE^I$ for all lags $\tau$. However, $\hat{\psi}_\tau$ fluctuates around the reference level $q$ and is almost always below the confidence bound. In this case one would reason that $r^I$ is not systemically relevant for $r^M$ at any lag. This conclusion is also supported by the fitted CoSP, which stays constant at level $q = 1\%$.

![Figure 7: Estimated CoSP, $\hat{\psi}_\tau(0.01)$, for independent student-t(696) distributed returns.](image)

**A.3 Relation to CoVaR**

We examine two different return distributions $r^{I1}$ and $r^{I2}$ and a market return $r^M$, and assume that $r^M_\tau \mid r^{I1} \leq VaR^{I1}(q)$ first-order stochastically dominates $r^M_\tau \mid r^{I2} \leq VaR^{I2}(q)$, i.e. for all $x \in \mathbb{R}$

$$\mathbb{P}(r^M_\tau \leq x \mid r^{I1} \leq VaR^{I1}(q)) \leq \mathbb{P}(r^M_\tau \leq x \mid r^{I2} \leq VaR^{I2}(q)).$$ (26)

Then, $\psi^{I1}_\tau(q) \leq \psi^{I2}_\tau(q)$. Moreover, for CoVaR$^\tau$ we have

$$\text{CoVaR}_{r^{I1}_\tau \leq VaR^{I1}(q)}(q) \geq \text{CoVaR}_{r^{I2}_\tau \leq VaR^{I2}(q)}(q).$$ (27)
Hence, with respect to both risk measures $I_2$ is more systemically risky than $I_1$. Also, if the market risk conditional on the benchmark events is approximately equal, i.e. CoVaR$^{(BM)}_{I_1}(q) \approx$ CoVaR$^{(BM)}_{I_2}(q)$, for $\Delta \text{CoVaR}$ we have

$$\Delta \text{CoVaR}^{(I_1)}_{S}(q) \geq \Delta \text{CoVaR}^{(I_2)}_{S}(q).$$

The condition that $r^M_\tau | r^{I_1}_\tau \leq \text{VaR}^{I_1}_q$ first-order stochastically dominates $r^M_\tau | r^{I_2}_\tau \leq \text{VaR}^{I_2}_q$ is very common among financial return series. In Figure 8 we show two exemplary empirical cumulative density function (ecdf) for lag $\tau = 0$ for the unconditional and conditional returns of the financial index. Particularly in the lower tail the stochastic dominance is very clear.\footnote{Note that stochastic dominance in the lower tail is sufficient to obtain the same order of the institutions.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8}
\caption{Ecdf of returns from the financial index conditional on institutions’ financial distress.}
\end{figure}

\subsection*{A.4 Estimation of Aggregate Excess CoSP}

Firstly, note that

$$\int (H(\tau) - q) \, d\tau = \int e^{\text{erf} \left( \frac{2a \tau - b}{\sqrt{4a}} \right)} \, d\tau = e^{\frac{\pi^2}{16a}} \text{erf} \left( \frac{2a \tau - b}{\sqrt{4a}} \right).$$

Therefore, for $a > 0$,

$$\tilde{\psi} = \int_1^\infty e^{-x^2 + bx + c} \, dx = e^{\frac{\pi^2}{16a}} \lim_{\tau \to \infty} \left( \text{erf} \left( \frac{2aL - b}{2\sqrt{a}} \right) - \text{erf} \left( \frac{2a - b}{2\sqrt{a}} \right) \right) \quad (30)$$

$$= e^{\frac{\pi^2}{16a}} \left( \text{erf} \left( \frac{2a - b}{2\sqrt{a}} \right) \right).$$

\footnote{Note that stochastic dominance in the lower tail is sufficient to obtain the same order of the institutions.}
since \( \lim_{x \to \infty} \text{erf}(x) = 1 \). However, if \( a = 0 \), we have

\[
\int (H(\tau) - q) \, d\tau = \int e^{b\tau + c} \, d\tau = \frac{1}{b} e^{bx + c},
\]

thus, if \( b < 0 \),

\[
\bar{\psi} = \int_{1}^{\infty} e^{b\tau + c} \, d\tau = \lim_{L \to \infty} \frac{1}{b} \left( e^{bL + c} - e^{b+c} \right) = \frac{1}{b} \left( -e^{b+c} \right).
\]

### A.5 Estimation of the CoSP-weighted lag

Firstly, note that

\[
\int \tau (H(\tau) - q) \, d\tau = \int \tau e^{-\tau^2 + b\tau + c} \, d\tau = \frac{1}{4a^{3/2}} \left( \sqrt{\pi be^{b^2/4}} \text{erf} \left( \frac{2a\tau - b}{2\sqrt{a}} \right) - 2\sqrt{ae^{(b-a\tau)+c}} \right).
\]

Therefore,

\[
\tau = \frac{1}{\bar{\psi}} \lim_{L \to \infty} \frac{1}{4a^{3/2}} \left( \sqrt{\pi be^{b^2/4}} \text{erf} \left( \frac{2aL - b}{2\sqrt{a}} \right) - 2\sqrt{ae^{(b-aL)+c}} \right)
\]

\[
- \frac{1}{\bar{\psi}4a^{3/2}} \left( \sqrt{\pi be^{b^2/4}} \text{erf} \left( \frac{2a - b}{2\sqrt{a}} \right) - 2\sqrt{ae^{b-a+c}} \right)
\]

\[
= \frac{1}{4a^{3/2} \bar{\psi}} \left( \sqrt{\pi be^{b^2/4}} \left( 1 - \text{erf} \left( \frac{2a - b}{2\sqrt{a}} \right) \right) + 2\sqrt{ae^{b-a+c}} \right).
\]

However, if \( a = 0 \), we have

\[
\int \tau (H(\tau) - q) \, d\tau = \int \tau e^{b\tau + c} \, d\tau = \left( \frac{\tau}{b} - \frac{1}{b^2} \right) e^{b\tau + c},
\]

thus, if \( b < 0 \),

\[
\bar{\psi} = \int_{1}^{\infty} \tau e^{b\tau + c} \, d\tau = \frac{1}{\bar{\psi}} \lim_{L \to \infty} \left( \frac{L}{b} - \frac{1}{b^2} \right) e^{bL + c} - \left( \frac{1}{b} - \frac{1}{b^2} \right) e^{b+c}
\]

\[
= \frac{1}{\bar{\psi}} \left( \frac{1}{b^2} - \frac{1}{b} \right) e^{b+c}.
\]
B Standard errors for HS estimators of $\Delta\text{CoVaR}$ and $\psi_r$

In this Section we examine the standard errors of HS estimators for $\Delta\text{CoVaR} \leq$ and $\psi_r$. For simplicity, we focus on lag $\tau = 0$ (i.e. co-movements), since the computation and results for all other lags are equivalent. Firstly, we will derive approximations for the standard errors. However, these provide a good approximation only under specific conditions and with enough observations. Therefore, we, secondly, perform a Monte-Carlo analysis.

Let $X_1, ..., X_n$ be sorted iid realizations from a continuous distribution $F$. Then, an estimate for the $q$-quantile, $\text{VaR}(q)$, is the $\left\lfloor qn \right\rfloor$-th value, $\hat{\text{VaR}}(q) = X_{\left\lfloor qn \right\rfloor}$. Although, the exact standard error for $\hat{\text{VaR}}(q)$ is generally not available in closed form, asymptotically we have (cf. Serfling [1980])

$$\hat{\text{VaR}}(q) \overset{d}{\sim} \mathcal{N}\left( \text{VaR}(q), \frac{q(1-q)}{n f(\text{VaR}(q))^2} \right),$$

where $f = F'$ is the density function of $F$. Hence, the asymptotic standard error of $\hat{\text{VaR}}(q)$ is given by

$$\sqrt{\frac{q(1-q)}{n f(\text{VaR}(q))^2}},$$

where $f = F'$ is the density function of $F$.

To estimate $\text{CoVaR}^M_{r^I \leq \text{VaR}^I(q)}(q)$ one, firstly, sorts the observed institution returns, i.e. $r^I_{(1)} \leq r^I_{(2)} \leq ... \leq r^I_{(n)}$, and, secondly, computes the $q$-th smallest market return of the subsample $(r^M_{(k)})_{k=1,...,[nq]}$, i.e. $\hat{\text{CoVaR}}^M_{r^I \leq \text{VaR}^I(q)}(q) = r^M_{(\left\lfloor qn \right\rfloor)}$, where $r^M_{(1)} \leq r^M_{(2)} \leq ... \leq r^M_{(nq)}$. Thus, $\hat{\text{CoVaR}}^M_{r^I \leq \text{VaR}^I(q)}(q)$ is essentially a $q^2$-quantile of the market return $r^M$ with approximate asymptotic standard error

$$\tilde{s}e\left(\hat{\text{CoVaR}}^M_{r^I \leq \text{VaR}^I(q)}\right) = \frac{1}{n f(\text{CoVaR}^M_{r^I \leq \text{VaR}^I(q)}(q))^2} \sqrt{\frac{q^2(1-q^2)}{n}},$$

where $f = f_{r^M}$ is the density of market returns $r^M$.

Denote by $q_{BM} = \mathbb{P}(BM)$ the benchmark state $BM$. Then, the asymptotic standard error of $\hat{\Delta}\text{CoVaR}^\leq$ is given by

$$\sqrt{\frac{q^2(1-q^2)}{n f(\text{CoVaR}^M_{r^I \leq \text{VaR}^I(q)}(q))^2} + \frac{qq_{BM}(1-qq_{BM})}{n f(\text{CoVaR}^M_{r^I \leq \text{VaR}^I(q)}(q))^2} + \frac{2\rho\sqrt{q^4(1-q^2)q_{BM}(1-qq_{BM})}}{n f(\text{CoVaR}^M_{r^I \leq \text{VaR}^I(q)}(q)) f(\text{CoVaR}^M_{r^I \leq \text{VaR}^I(q)}(q))}},$$

where $\rho$ is the correlation between the CoVaR-estimators. Moreover, if $q_{BM} \approx q$ and $\rho \geq 0$ as well as
\( f(\text{CoVaR}_{r^1}^{M(r^1 \leq \text{VaR}(q))}) < f(\text{CoVaR}_{BM}^{M(r^1 \leq \text{VaR}(q))}) \), we have the following upper bound for the standard error:

\[
\hat{\text{se}} \left( \hat{\Delta \text{CoVaR}} \right) \leq \sqrt{\frac{2q^2(1-q^2)}{nf(\text{CoVaR}_{r^1 \leq \text{VaR}(q)}^{M(r^1 \leq \text{VaR}(q))})^2}} = \sqrt{\frac{2q^2(1-q^2)}{n}} \frac{1}{f(\text{CoVaR}_{r^1 \leq \text{VaR}(q)}^{M(r^1 \leq \text{VaR}(q))})}.
\] (44)

The HS estimator for CoSP, \( \psi_0 \), is given by

\[
\hat{\psi}_0(q) = \frac{\sum_{t=1}^{n} 1 \{ r^M_t \leq \text{VaR}^M(q), r^I_t \leq \text{VaR}^I(q) \}}{qn}.
\] (45)

Thus, the standard error, \( \hat{\psi}_0(q) \), is

\[
\hat{\text{se}} \left( \hat{\psi}_0(q) \right) = \sqrt{\text{var} \left( \frac{\sum_{t=1}^{n} 1 \{ r^M_t \leq \text{VaR}^M(q), r^I_t \leq \text{VaR}^I(q) \}}{qn} \right)} = \frac{1}{qn} \sqrt{naq\psi_0(1 - q\psi_0)} = \sqrt{\frac{\psi_0(1 - q\psi_0)}{qn}}.
\] (46)

However, we do not know the density function \( f \) of the market returns and cannot verify the assumptions for obtaining the approximations. Therefore, in the following we perform a Monte-Carlo analysis, which is twofold: On the one side, we show the mean relative standard errors of the risk measures for returns, that are student t-distributed. For this purpose, we estimate the covariance matrix, means and degrees of freedom of the returns and financial index from our data sample by means of the method of moments (cf. Section E.2) and use the estimates to draw iid samples from the student distribution.\(^{21}\) For a fixed number of observations \( n \) the mean relative standard error is given as

\[
\text{MRSE} = \sqrt{\frac{1}{T-1} \sum_{\tau=1}^{T} \left( \hat{\vartheta}^{(n)}_{\tau} - \bar{\vartheta}^{(n)} \right)^2}.
\] (47)

where \( \hat{\vartheta}^{(n)}_{\tau} \) is the \( \tau \)-th realization of the estimator (either \( \hat{\Delta \text{CoVaR}} \) or \( \hat{\psi} \)) and \( \bar{\vartheta}^{(n)} \) the average value of the estimator. The MRSE can be interpreted as the average standard deviation relative to the true value of \( \vartheta \). Since the latter is not known, we approximate this true value by \( \bar{\vartheta}^{(n)} \). For the analysis we define the benchmark event for \( \Delta \text{CoVaR}^\leq \) as \( BM = \{ r^I \in [\text{VaR}(0.5 \pm q/2)] \} \), such that \( P(BM) = q \) and

\[
\Delta \text{CoVaR}^\leq = \text{CoVaR}_{r^1 \leq \text{VaR}^I(q)}^I(q) - \text{CoVaR}_{r^1 \in [\text{VaR}(0.5 \pm q/2)]}.
\] (48)

The results are shown in Figure 9. Clearly, only in case of a large correlation \( \rho \) between the institu-

\(^{21}\) For the degrees of freedom we, first, estimate the parameter for each return series individually and, second, take the 1% smallest value (which is 696) to incorporate the heaviest observed tails for the Monte-Carlo analysis.
tion’s and market’s returns with a small significance level (1%) and few observations the standard error for \( \hat{\psi} \) is larger than for \( \Delta \text{CoVaR} \). In all other cases \( \psi \) can be estimated more precisely. Moreover, the degrees of freedom \( \nu \) (i.e. the tail-shape) does not substantially influence the estimation precision. The comparable large values for \( \nu \) might be a reason for this fact. Furthermore, the analytical standard error for \( \hat{\psi}_\tau(q) \) exactly matches the simulation results, whereas the approximation for \( \Delta \text{CoVaR}, se\left(\hat{\Delta \text{CoVaR}}\right) \), provides a very bad fit.

On the other hand, we apply a nonparametric bootstrap algorithm to draw samples from the historical returns of a specific institution and the financial index. Figure 10 depicts the resulting standard errors for three companies that show a typical pattern: Similar to student-distributed returns, the standard error for a 5%-significance level are slightly larger for \( \Delta \text{CoVaR} \), whereas for a 1%-significance level the standard error for \( \hat{\psi} \) is slightly larger for a small number of observations, while the standard errors are approximately equal for a larger number of observations.

To conclude, in most cases, the estimation precision for the Conditional Shortfall Probability is better than for \( \Delta \text{CoVaR} \). For a further discussion of the estimation precision of systemic risk measures (in particular MES and \( \Delta \text{CoVaR} \)) we refer to Danielsson et al. (2015) and Danielsson and Zhou (2015).
Figure 9: Mean relative standard errors for student distributed returns.
Figure 10: Mean relative standard errors from Bootstrap analysis.
**C Heteroscedasticity and Historical Simulation**

A common way to handle non-stationarity of a time-series is to apply a GARCH-filter to the series and work with the residuals (see e.g. Chen et al. (2013) or Bisias et al. (2015)). The rationale of a GARCH-model is a stochastic volatility process, such that the return series is given as

\[ r_t^x = \sigma_t^x \varepsilon_t^x, \tag{49} \]

where \( \sigma_t^x \) is a predictable stochastic process (i.e., \( \sigma_{t+1}^x \) is known at time \( t \)) and \( \varepsilon_t^x \) is independent and identically distributed white noise. Since in this case the residuals \( \varepsilon_t^x \) are stationary, have all desirable properties for statistical inference. For example, as suggested by Hull and White (1998), the one-period ahead Value-at-Risk may be computed with the transformed observations

\[ \hat{\sigma} r_t^x = \hat{\sigma} \varepsilon_t^x. \tag{50} \]

This approach is justified, since the VaR of \( r_{t+1}^x \) is deterministically linked to the VaR of \( \varepsilon_{t+1}^x \), in particular

\[ q = P_t \left( r_{t+1}^x \leq VaR_{t,q} \left( r_{t+1}^x \right) \right) = P_t \left( \varepsilon_{t+1}^x \leq \frac{1}{\sigma_{t+1}^x} VaR_{t,q} \left( r_{t+1}^x \right) \right) \tag{51} \]

and \( \sigma_{t+1}^x \) is known at time \( t \).\(^{23}\) Can we derive a similar approach for CoSP? For this purpose, we need to infer \( VaR_{t,q} \left( r_{t+1+\tau}^M \right) \) from \( VaR_{t,q} \left( \varepsilon_{t+1+\tau}^M \right) \). However, at time \( t \) we do not know \( \sigma_{t+1+\tau}^M \) for \( \tau > 0 \), i.e. \( \sigma_{t+1+\tau}^M VaR_{t,q} \left( \varepsilon_{t+1+\tau}^M \right) \) is a random variable and, thus, does not equal \( VaR_{t,q} \left( r_{t+1+\tau}^M \right) \). Therefore,

\[ \psi_{t,\tau}(q) = P_t \left( r_{t+1+\tau}^M \leq VaR_{t,q} \left( r_{t+1+\tau}^M \right) \mid r_{t+1}^I \leq VaR_{t,q} \left( r_{t+1}^I \right) \right) \tag{52} \]

\[ = P_t \left( \varepsilon_{t+1+\tau}^M \leq \frac{1}{\sigma_{t+1+\tau}^M} VaR_{t,q} \left( r_{t+1+\tau}^M \right) \mid \varepsilon_{t+1}^I \leq VaR_{t,q} \left( \varepsilon_{t+1}^I \right) \right) \tag{53} \]

is not equal to

\[ P_t \left( \varepsilon_{t+1+\tau}^M \leq VaR_{t} \left( \varepsilon_{t+1+\tau}^M \right) \mid \varepsilon_{t+1}^I \leq VaR_{t} \left( \varepsilon_{t+1}^I \right) \right). \tag{54} \]

We conclude, that it is not possible to estimate \( \psi_{\tau}(q) \) with GARCH-filtered residuals.

\(^{22}\)The GARCH model is based on the ARCH model from Engle (1982) and was developed by Bollerslev (1986).

\(^{23}\)In this section we denote by \( VaR_{t,q}(X) \) the Value-at-Risk of the random variable \( X \) at time \( t \) on the significance level \( q \).
D Lower bound of significance for $\hat{\psi}_\tau$

The HS estimator for $\psi_\tau$ is given as

$$\hat{\psi}_\tau = \frac{\sum_{t=1}^{n-\tau} \mathbb{1}_{\{r^M_{t+\tau} \leq \text{VaR}^M(q), r^I_t \leq \text{VaR}^I(q)\}}}{qn}. \quad (55)$$

Denote by $n_\tau = n - \tau$ the number of available observations for lag $\tau$. Under the assumption that $\mathbb{1}_{\{r^M_{t+\tau} \leq \text{VaR}^M(q), r^I_t \leq \text{VaR}^I(q)\}}$ are iid for $t = 1, \ldots, n_\tau$, we have that

$$\sum_{t=\tau+1}^{n} \mathbb{1}_{\{r^M_{t+\tau} \leq \text{VaR}^M(q), r^I_t \leq \text{VaR}^I(q)\}} \sim \text{Bin}(n_\tau, q_\psi^\tau). \quad (56)$$

Hence, under the null hypothesis $H_0 : \psi_\tau = q$, i.e. that $r^M_{t+\tau}$ and $r^I_t$ are independent, we have

$$qn_\tau \hat{\psi}_\tau \sim \text{Bin}(n_\tau, q^2). \quad (57)$$

We would like to reject the null hypothesis if $\hat{\psi}_\tau \geq k^*$ with a significance level of $\alpha \in (0, 1)$. Therefore, an exact lower bound for the rejection area can be computed as

$$\alpha = \mathbb{P}_{H_0} (\hat{\psi}_\tau \geq k^*) = \mathbb{P}_{H_0} \left( \sum_{t=\tau+1}^{n} \mathbb{1}_{\{r^M_{t+\tau} \leq \text{VaR}^M(q), r^I_t \leq \text{VaR}^I(q)\}} \geq qn_\tau k^* \right) \quad (58)$$

$$= 1 - F_{\text{Bin}(n_\tau, q^2)}(qn_\tau k^* - 1) \quad (59)$$

$$\Leftrightarrow 1 - \alpha = F_{\text{Bin}(n_\tau, q^2)}(qn_\tau k^* - 1) \quad (60)$$

$$\Leftrightarrow qn_\tau k^* - 1 = F_{\text{Bin}(n_\tau, q^2)}^{-1}(1 - \alpha) \quad (61)$$

$$\Leftrightarrow k^* = \frac{1}{qn_\tau} \left( F_{\text{Bin}(n_\tau, q^2)}^{-1}(1 - \alpha) + 1 \right), \quad (62)$$

where $F_{\text{Bin}(n_\tau, q^2)}^{-1}$ is the (lower) inverse cumulative distribution of the Binomial distribution.
Moreover, an asymptotic bound is given as

\[ \alpha = P_{H_0}\left(\hat{\psi}_\tau \geq k^*\right) = P_{H_0}\left(\sum_{t=\tau+1}^n 1\{r_{t+\tau}^{M_t} \leq \text{VaR}^{M_t}(q), r_{t+\tau}^{I_t} \leq \text{VaR}^{I_t}(q)\} \geq qn_r k^*\right) \]  

(63)

\[ \approx 1 - \Phi\left(\frac{qn_r k^* - q^2 n_r}{\sqrt{n_r q^2 (1 - q^2)}}\right) \]  

(64)

\[ \iff 1 - \alpha \approx \Phi\left(\frac{qn_r k^* - q^2 n_r}{\sqrt{n_r q^2 (1 - q^2)}}\right) \]  

(65)

\[ \iff \frac{qn_r (k^* - q)}{\sqrt{n_r q^2 (1 - q^2)}} \approx \Phi^{-1}(1 - \alpha) \]  

(66)

\[ \iff k^* \approx q + \sqrt{\frac{1 - q^2}{n_r}} \Phi^{-1}(1 - \alpha). \]  

(67)

However, simulations suggest that the asymptotic bound is substantially too small in comparison to the exact bound.

## E  Data and Methodology

### E.1 Building an own index

To be able to use a market index without having endogeneity issues, i.e. with companies that are already incorporated in the index, we build our own index. Therefore, we denote by $MC_t^{(i)}$ the market capitalization of company $i$ at time $t$, i.e. $MC_t^{(i)} = P_t^{(i)} \cdot \text{Shares}_{t}^{(i)}$, where $P_t^{(i)}$ is the stock price and $\text{Shares}_{t}^{(i)}$ the number of shares at time $t$. Moreover, by $TR_t^{(i)}$ we denote the total return index (dividend-adjusted) of company $i$. The market is $S \subseteq \{1, ..., M\}$, i.e. the companies that are included in the market. Then, the index for market $S$ excluding company $j$ is given as the weighted average of total return index-returns:

\[ INDEX_{S, j} = \sum_{s \in S \setminus \{j\}} \frac{MC_{t-1}^{(s)}}{\sum_{s \in S \setminus \{j\}} MC_{t-1}^{(s)}} \frac{TR_t^{(s)}}{TR_{t-1}^{(s)}}. \]  

(68)

To adjust for different currencies, we calculate the market capitalization in US dollar. Therefore, the time $t$ price of company $s$ is given by

\[ P_t^{(s)} = \tilde{P}_t^{(s)}/ER_t^{(s)}, \]  

(69)

where $\tilde{P}_t^{(s)}$ is the time $t$ price in currency $\tilde{C}$ and $ER_t^{(s)}$ is the exchange rate from currency $\tilde{C}$ to US Dollar.

\footnote{The total return index reflects the evolution of the stock price assuming that dividends are re-invested to purchase additional units of equity.}
at time $t$. Finally, the market return is computed as

$$r^M_t = r^S_t = \log\left( \frac{INDEX^S_{tj}}{INDEX^S_{t-1}} \right).$$

(70)

E.2 Data and descriptive Statistics

![Financial indices](image1)

![Industrial indices](image2)

(a) Financial indices.

(b) Industrial indices.

Figure 11: Financial and non-financial indices.
<table>
<thead>
<tr>
<th>Company name</th>
<th>Industry</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALPHABET 'A'</td>
<td>Internet, Health, Technology</td>
<td>USA</td>
</tr>
<tr>
<td>AMAZON.COM</td>
<td>E-Commerce, Consumer Electronics</td>
<td>USA</td>
</tr>
<tr>
<td>APPLE</td>
<td>Consumer Electronics</td>
<td>USA</td>
</tr>
<tr>
<td>ASTRazeneca</td>
<td>Health, Pharmaceuticals</td>
<td>UK</td>
</tr>
<tr>
<td>AT&amp;T</td>
<td>Telecommunications</td>
<td>USA</td>
</tr>
<tr>
<td>BP</td>
<td>Oil &amp; Gas</td>
<td>UK</td>
</tr>
<tr>
<td>BRITISH AMERICAN TOBACCO</td>
<td>Consumer Goods</td>
<td>UK</td>
</tr>
<tr>
<td>COCA COLA</td>
<td>Consumer Goods</td>
<td>USA</td>
</tr>
<tr>
<td>DAIMLER</td>
<td>Automotive</td>
<td>Germany</td>
</tr>
<tr>
<td>DIAGEO</td>
<td>Consumer Goods</td>
<td>UK</td>
</tr>
<tr>
<td>E ON (XET)</td>
<td>Energy</td>
<td>Germany</td>
</tr>
<tr>
<td>ENI</td>
<td>Oil &amp; Gas</td>
<td>Italy</td>
</tr>
<tr>
<td>EXXON MOBIL</td>
<td>Oil &amp; Gas</td>
<td>USA</td>
</tr>
<tr>
<td>GAZPROM</td>
<td>Oil &amp; Gas</td>
<td>Russia</td>
</tr>
<tr>
<td>GENERAL ELECTRIC</td>
<td>Energy, Oil &amp; Gas</td>
<td>USA</td>
</tr>
<tr>
<td>GLAXOSMITHKLINE</td>
<td>Health, Pharmaceuticals</td>
<td>UK</td>
</tr>
<tr>
<td>L’OREAL</td>
<td>Consumer Goods</td>
<td>France</td>
</tr>
<tr>
<td>LVMH</td>
<td>Consumer Goods</td>
<td>France</td>
</tr>
<tr>
<td>MICROSOFT</td>
<td>Consumer Electronics</td>
<td>USA</td>
</tr>
<tr>
<td>NESTLE ‘R’</td>
<td>Consumer Goods</td>
<td>USA</td>
</tr>
<tr>
<td>NOKIA</td>
<td>Telecommunications</td>
<td>Finland</td>
</tr>
<tr>
<td>NOVARTIS ‘R’</td>
<td>Pharmaceuticals</td>
<td>Switzerland</td>
</tr>
<tr>
<td>PROCTER &amp; GAMBLE</td>
<td>Consumer Goods</td>
<td>USA</td>
</tr>
<tr>
<td>RIO TINTO</td>
<td>Metals, Mining</td>
<td>UK</td>
</tr>
<tr>
<td>ROCHE HOLDING</td>
<td>Pharmaceuticals</td>
<td>Switzerland</td>
</tr>
<tr>
<td>ROYAL DUTCH SHELL A</td>
<td>Oil &amp; Gas</td>
<td>UK</td>
</tr>
<tr>
<td>SAMSUNG ELECTRONICS</td>
<td>Consumer Electronics</td>
<td>South Korea</td>
</tr>
<tr>
<td>SANOFI</td>
<td>Pharmaceuticals</td>
<td>France</td>
</tr>
<tr>
<td>SIEMENS (XET)</td>
<td>Energy, Infrastructure</td>
<td>Germany</td>
</tr>
<tr>
<td>TELEFONICA</td>
<td>Telecommunications</td>
<td>Spain</td>
</tr>
<tr>
<td>TOTAL</td>
<td>Oil &amp; Gas</td>
<td>France</td>
</tr>
<tr>
<td>TOYOTA MOTOR</td>
<td>Automotive</td>
<td>Japan</td>
</tr>
<tr>
<td>VODAFONE GROUP</td>
<td>Telecommunications</td>
<td>UK</td>
</tr>
<tr>
<td>VOLKSWAGEN</td>
<td>Automotive</td>
<td>Germany</td>
</tr>
<tr>
<td>WAL MART STORES</td>
<td>Retail</td>
<td>USA</td>
</tr>
</tbody>
</table>

Table 1: Non-financial companies included in the data sample.
### Table 2: Names of the ten largest companies (by market capitalization in November 2015) in each sub-sector.

<table>
<thead>
<tr>
<th>Subsector</th>
<th>No. of companies</th>
<th>Companies</th>
<th>Banks</th>
<th>Insurance Companies</th>
<th>Brokers</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAN</td>
<td>567</td>
<td>Wells Fargo</td>
<td>Berkshire Hathaway</td>
<td>Allianz</td>
<td>Goldman Sachs</td>
</tr>
<tr>
<td>BRO</td>
<td>151</td>
<td>JPMorgan Chase</td>
<td>Berkshire Hathaway</td>
<td>American Internlgp</td>
<td>Blackrock</td>
</tr>
<tr>
<td>INS</td>
<td>199</td>
<td>Bank of America</td>
<td>AIGroup</td>
<td>AXA</td>
<td>Charles Schwab</td>
</tr>
<tr>
<td>NoFIN</td>
<td>35</td>
<td>Commonwealth Bank</td>
<td>CMEGroup</td>
<td>AXA</td>
<td>Hong Kong Exsclar</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Blackrock</td>
<td></td>
<td>Intercontinentallex</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MITSUBISHIUFJFINLGP</td>
<td></td>
<td>ZURICHINSURANCEGROUP</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>ROYAL BANKOF CANADA</td>
<td></td>
<td>FRANKLINRESOURCES</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>ACE</td>
<td></td>
<td>NOMURA HDG</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MACQUARIEGROUP</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: Mean and quantiles of the mean and and standard deviation of returns for single companies of different subsectors.

<table>
<thead>
<tr>
<th>Subsector</th>
<th>No. of companies</th>
<th>( \mu )</th>
<th>( \mu_{0.1} )</th>
<th>( \mu_{0.5} )</th>
<th>( \mu_{0.9} )</th>
<th>( \sigma )</th>
<th>( \sigma_{0.1} )</th>
<th>( \sigma_{0.5} )</th>
<th>( \sigma_{0.9} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAN</td>
<td>567</td>
<td>2.71e-04</td>
<td>-2.52e-04</td>
<td>2.97e-04</td>
<td>8.22e-04</td>
<td>0.024</td>
<td>0.015</td>
<td>0.022</td>
<td>0.033</td>
</tr>
<tr>
<td>BRO</td>
<td>151</td>
<td>3.44e-04</td>
<td>-2.00e-04</td>
<td>3.50e-04</td>
<td>7.67e-04</td>
<td>0.024</td>
<td>0.015</td>
<td>0.024</td>
<td>0.034</td>
</tr>
<tr>
<td>INS</td>
<td>199</td>
<td>3.17e-04</td>
<td>-0.98e-04</td>
<td>3.57e-04</td>
<td>6.92e-04</td>
<td>0.022</td>
<td>0.015</td>
<td>0.021</td>
<td>0.032</td>
</tr>
<tr>
<td>NoFIN</td>
<td>35</td>
<td>4.26e-04</td>
<td>2.34e-04</td>
<td>3.66e-04</td>
<td>7.32e-04</td>
<td>0.020</td>
<td>0.014</td>
<td>0.019</td>
<td>0.028</td>
</tr>
</tbody>
</table>

### Table 4: Mean, standard deviation and quantiles of different index returns.

<table>
<thead>
<tr>
<th>Index</th>
<th>No. of companies</th>
<th>( \tau )</th>
<th>( \sqrt{\text{var}(r)} )</th>
<th>( r_{0.1} )</th>
<th>( r_{0.5} )</th>
<th>( r_{0.9} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAN</td>
<td>567</td>
<td>2.29e-05</td>
<td>0.013</td>
<td>-0.015</td>
<td>0.000</td>
<td>0.014</td>
</tr>
<tr>
<td>BRO</td>
<td>151</td>
<td>2.85e-04</td>
<td>0.014</td>
<td>-0.015</td>
<td>0.001</td>
<td>0.015</td>
</tr>
<tr>
<td>INS</td>
<td>199</td>
<td>6.36e-05</td>
<td>0.015</td>
<td>-0.015</td>
<td>0.000</td>
<td>0.015</td>
</tr>
<tr>
<td>FIN</td>
<td>917</td>
<td>4.17e-05</td>
<td>0.013</td>
<td>-0.014</td>
<td>0.001</td>
<td>0.014</td>
</tr>
<tr>
<td>AMC NoFIN</td>
<td>1265</td>
<td>2.56e-04</td>
<td>0.012</td>
<td>-0.013</td>
<td>0.001</td>
<td>0.012</td>
</tr>
<tr>
<td>ASIA NoFIN</td>
<td>1514</td>
<td>9.42e-05</td>
<td>0.011</td>
<td>-0.013</td>
<td>0.000</td>
<td>0.013</td>
</tr>
<tr>
<td>EU NoFIN</td>
<td>1902</td>
<td>1.99e-04</td>
<td>0.012</td>
<td>-0.013</td>
<td>0.001</td>
<td>0.013</td>
</tr>
</tbody>
</table>
Figure 12: Distribution of mean and standard deviation of returns and the correlation between company returns and returns of the financial index.
F Additional Figures

Figure 13: CoSP triggered by exemplary banks and brokers w.r.t. the FIN index.
Figure 14: CoSP triggered by exemplary brokers and insurance companies w.r.t. the FIN index.
Figure 15: CoSP triggered by exemplary non-financial companies w.r.t. the FIN index.

Figure 16: Aggregate Excess CoSP w.r.t. the FIN and American NoFIN indices triggered by all institutions of the subsectors BAN, INS, BRO and NoFIN. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$. 

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Figure 17: Median exact time-lag for each subsector for 25% most risky institutions. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the whiskers’ maximum length is $1.5(q_3 - q_1)$.

Figure 18: Median exact time-lag w.r.t. the FIN and American NoFIN indices triggered by the 25% most systemically persistent institutions of the subsectors BAN, INS, BRO and NoFIN. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$.
References


