Risk Tolerance and Stock-Market Participation of U.S. Households Around the Financial Crisis

Abstract: There has been a great deal of literature published in recent years related to the financial crisis. However, little research exists on the impact of the financial crisis on individual investors. In this paper, we investigate how the financial crisis has impacted U.S. investors, specifically considering whether the impact varies by age. We develop a simple theoretical model in which there is a financial shock at time one and examine how the allocation toward the risky asset varies between individuals who have more or less time post-shock. Additionally, using data from the Survey of Consumer Finances, we examine the changes in risk tolerance and stock ownership around the financial crisis. We find that middle aged cohorts increased stock ownership after the crisis whereas younger cohorts decreased stock ownership. We also find that household risk tolerance decreased during the crisis with the youngest cohort having the greatest decrease and thereby becoming more risk-averse. Finally, we find that, in addition to various demographic and financial variables, risk tolerance and the usage of financial tools impact the probability of stock ownership; however, the impact varies before and after the crisis. We also find significant differences among age cohorts.

I. Introduction

There has been a great deal of literature published in recent years that has discussed the cause of the financial crisis (e.g. Shiller, 2008; Taylor, 2009) and the impact of the crisis on businesses and the economy as a whole (e.g. Lu and Whidbee, 2013; Demirguc-Kunt, Detragiache and Merrouche, 2013). While this research is of interest to a wide audience, the financial crisis has also had a major impact on individual investors as well. In general, savings for retirement is lower than needed and the amount households have saved to support retirement is rather limited (Mitchell and Moore, 1998). In 2008, only 45 percent of retirement-age households had more than $20,000 in non-retirement financial assets and the median household had $52,000 in financial assets, including those in personal retirement accounts (Poterba et al., 2011). As such, accumulated wealth for retirement is very modest. In addition to less than adequate retirement savings, the shock of the financial crisis caused the number of individuals delaying retirement age to increase from 14 percent in 2008 to 25 percent in 2009. Reasons cited for delaying retirement included the poor economy, inadequate finances, and the need to make up stock market losses (Helman, Copeland, and VanDerhei, 2011). Though the economy has shown signs of recovery, it is unclear how this experience has impacted investors as there is little empirical evidence on the subject.

From a theoretical perspective, once markets recover, financial risk appetite should resume for investors. Yet evidence shows this may not be the case; experience impacts risk tolerance which can have a lasting effect (Malmendier and Nagel, 2011; Rizzi, 2014). In this paper, we investigate how the financial crisis has impacted U.S. investors, specifically considering whether the impact varies by age. This is an important contribution of the paper given that younger cohorts have not been studied very thoroughly, and their reaction post-crisis will

have a lasting impact given their longer time horizon to retirement. If such cohorts are hesitant to invest in the market post-crisis, they are unlikely to gain the compounding returns that many financial advisors assume they will obtain over their working lifetime. As a result, this could negatively affect the wealth they are able to accumulate at retirement. Therefore it is important to consider which factors influence this cohort's willingness to invest in the market, how this cohort reacted after the financial crisis, and the resulting implications both of these effects may have for their retirement preparedness.

II. Literature Review

Previous work has examined how asset allocations change over time and which factors impact asset allocations (Sundén and Surette, 1998; Ameriks and Zeldes, 2004). Malmendier and Nagel (2011) show that experience with market fluctuations do matter and will impact asset allocation decisions moving forward. This study looks specifically at cohorts that were impacted by the Great Depression or the inflationary shocks of the 1970s. Bertaut (1998) considers how stockholding behavior of households is impacted by the 1987 market crash. Only a few studies have provided some insight into the impact of most recent financial crisis on risk tolerance and the risk-taking behavior of individuals. These studies generally find that while there is some evidence that there was a change in risk tolerance after the financial crisis, the change is small and relatively short lived (Roszkowski and Davey, 2010; Gerrrans, 2012; Hoffmann, Post, and Pennings, 2013; Gerrans, Faff and Hartnett, 2015). However, one study spans multiple countries and the average age of the individuals within the dataset was 55. The other utilizes an entirely non-U.S. sample of individuals and also has a relatively high average age of 50.

While evidence exists that risk tolerance is inversely related to age (e.g. Hallahan, Faff, and McKenzie, 2004; Yao, Sharpe, and Wang, 2011)[1], no existing research has incorporated these potential differences in an examination of the impact of risk tolerance on asset allocation decisions. As such, the current study adds to existing literature by considering the changes in risk tolerance surrounding the financial crisis and the factors that impact investor behavior, taking into account the potential impact of age. We consider this to be a significant contribution since younger investors – those in mid-life and even younger – may have reacted differently following the financial crisis. To the extent that the risk tolerance of this cohort changed and we find that risk tolerance significantly impacts investor behavior, this could have long-term consequences.

In what follows, we put forth a simple theoretical model to demonstrate how a financial shock will impact younger cohorts relative to older cohorts. That is, a financial shock should cause those with a shorter investment horizon (older cohorts) to decrease the portion of wealth allocated to risky assets more so than individuals with a longer investment horizon (younger households and/or cohorts). Using data from the Survey of Consumer Finances (SCF), we consider which factors impact the stock-holding behavior of younger households and test how these households reacted after the 2008 financial crisis.

III. Model

To look at how agents invest, we first consider the standard consumption capital asset pricing model (CCAPM) in which each agent maximizes expected utility. The utility function is increasing, concave and additively separable with future utility discounted at the rate, β. The

[1] It should be noted that Wang and Hanna (1997) find that, when you control for other demographic and financial factors, risk tolerance actually increases with age.

individual can invest in two assets, one with a riskless rate of return, *r*, and one with a stochastic return. The maximization problem is therefore given by:

$$\max E[u_t] = \max E\left[u(c_t) + \sum_{i=1}^{T} \beta^i u(c_{t+i})\right]$$

subject to the constraints

$$c_t = A_t + y_t - s_t$$

$$A_{t+1} = s_t(1 + r) + \alpha_t s_t z_t$$

where c_t is real consumption at time *t*, y_t is exogenous real labor income at time *t*, A_t is total wealth at time *t*, s_t is total real saving in *t*, α_t is the portion saved in the risky asset at time *t*, $(1 + r)$ is the gross return on the riskless asset, and z_t is the excess return of the risky asset over the riskless rate at time *t*.

In the setting we want to consider, there is a financial shock of a limited duration which would impact the return on the risky asset. This shock should affect the investment decision, α_t, for the agent. Given that different agents have different investment horizons, we expect that those agents with shorter investment horizons (lower T), will decrease the amount invested in the risky asset more than those agents with longer investment horizons. This difference corresponds to younger generations having a longer investment horizon and therefore can bear the financial loss better than those with shorter investment horizons. Of course, the impact of the financial shock will depend on the duration of the shock relative to the agent's investment horizon.

In order to consider a financial shock of this sort in the above framework, consider the following maximization problem instead:

$$\max E[u_t] = \max E\left[\sum_{i=0}^{\tau} \beta^i u(c_{t+i}) + \sum_{i=\tau+1}^{T} \beta^i u(c_{t+i})\right]$$

subject to the constraints

$$c_t = A_t + y_t - s_t$$

$$A_{t+1} = \begin{cases} s_t(1+r) + \alpha_t s_t(z_t - \varepsilon), & if\ t \leq \tau \\ s_t(1+r) + \alpha_t s_t z_t, & if\ t > \tau \end{cases}$$

where c_t is real consumption at time *t*, y_t is exogenous real labor income at time *t*, A_t is total wealth at time *t*, s_t is total real saving in *t*, α_t is the portion saved in the risky asset at time *t*, $(1+r)$ is the gross return on the riskless asset. In this model z_t is the excess return of the risky asset over the riskless rate at time *t*, which is lowered by the value ε, for $t \leq \tau$ where τ represents the duration of the financial shock to the market.[2]

Even if the duration of the financial shock, τ, is the same for all agents, agents with a longer investment horizon (greater T) should have a different reaction to the financial shock and their allocation to the risky asset will differ than that made by an agent with a shorter investment horizon. Using the model above, we plan to investigate these theoretical predictions. That is, we will solve the above maximization problem for two types of agents: one with a higher T and one with a lower T (longer vs. shorter investment horizon). We predict that agents with a shorter investment horizon will have a smaller allocation to the risky asset relative to agents with a longer investment horizon.

For simplicity, we compare the optimal allocation toward the risky asset for a household with a time horizon of two periods (T=1: t=0, t=1) relative to one with a time horizon of three periods (T=2: t=0, t=1, t=2). For both households, there will be a financial shock in the first period at *t*=0; that is, $\tau = 0$. The agent with a longer horizon has more time to recover from the financial shock relative to the agent with the shorter investment horizon. Solving the

[2] Please note this model does not capture that after a financial shock, prices per share are lower and therefore it may be optimal for those with a longer investment horizon to increase their share in the risky asset after a financial shock to take advantage of these lowered prices. The results of this model therefore underestimate the impact of the financial shock on investment in the risky asset.

maximization problem for these two types of agents we find the optimal allocation toward the risky asset. For simplicity, let $u_0' \equiv u'(s_0(1+r) + \alpha_0 s_0(z_0 - \varepsilon) + y_1 - s_1)$ and $u_1' \equiv u'(s_1(1+r) + \alpha_1 s_1(z_0) + y_2 - s_2)$.

For an agent with T=1, we find the optimal allocation toward the risk asset at t=0 is:

$$\alpha_{0,T=1} = \frac{s_0 E[u'(A_0 + y_0 - s_0)] - s_0 cov\big(\beta u_0', (1+r) + \alpha_0(z_0 - \varepsilon)\big)}{-cov\big(\beta u_0', s_0(z_0 - \varepsilon)\big)} - \frac{1+r}{E[z_0 - \varepsilon]}$$

Please see Appendix A.1 for the derivation of this solution.

Moving forward, we will determine a closed form solution for the optimal allocations for the longer time horizon and compare the optimal allocations for the two agent types to see how the asset allocation changes with a longer versus shorter investment horizon. We can also investigate how the allocation changes with respect to the duration (τ) and impact (as measured through ε) of the financial shock.

If we assume constant relative risk aversion (CRRA) preferences, we can obtain comparisons between asset allocations among the two types of agents: those with a shorter investment horizon and those with a longer investment horizon. From Pratt (1964) we know that with CRRA preferences and no financial shock (i.e. $E[z_t] = E[z_{t+1}]$ for all t) then a rational agent will equalize their marginal utility across time periods. In order to do so, they will choose the same allocation to the risky asset for all periods; that is, $\alpha_t = \alpha_{t=1}$ for all *t*.

Suppose there is a shock to the return on the risky asset at the first time period though. If the same allocation is made toward the asset in all periods, marginal utility will not be equalized. The time period in which there is a financial shock will have a lower wealth value, thereby increasing the marginal utility for this time period since marginal utility is decreasing (utility is increasing and concave). In order to equalize marginal utilities, as desired by CRRA preferences, the allocation toward the risky asset should be decreased in the time period during

which there is a shock to risky asset's return. Therefore, when individuals have CRRA preferences and there is a shock to the return on the risky asset in the first time period we will have the following predictions:

Prediction 1: For agents with CRRA preferences, T=2, and a financial shock in the first time period, an individual will have a lower allocation toward the risky in the first period relative to the second period ($\alpha_0 < \alpha_1$).

Prediction 2: For agents with CRRA preferences, T=3, and a financial shock in the first time period, an individual will have a lower allocation toward the risky asset in the first period only ($\alpha_0 < \alpha_1 = \alpha_2$).

We are also interested in how the size of the financial shock impacts the asset allocation changes. The size of the shock, ε, determines the extent to which wealth in the first period is lowered. Therefore, it also influences how much marginal utility is higher in the time period with a financial shock relative to other time periods without a shock. In order to equalize marginal utility across all time periods, the allocation toward the risky asset in the period during which there is a shock has to be lowered. The degree to which it is lowered is dependent on the size of the financial shock though. If the financial shock is higher, wealth is even lower in that time period, and the allocation shift will need to be greater. Therefore, when individuals have CRRA preferences, we expect that the decrease in the allocation toward the risky asset for the period in which there is a financial shock is increasing in the size of the shock. That is, $\frac{\partial(\alpha_1 - \alpha_0)}{\partial \varepsilon} > 0$. As the size of the financial shock increases, the greater the difference in the

allocation toward the risky asset across the time periods with and without a financial shock.

The model posited here demonstrates rational behavior. During a time of financial crisis, individuals should reduce their allocation toward risky investments. When the economy has recovered, they can increase their allocation. Do individuals actually behave in this manner though? Evidence shows households decrease their contributions to their 401(k) plans during recessionary periods (Butrica and Smith, 2014), but how does that impact them moving forward? Do older generations act differently than younger generations because their investment horizon is different? In what follows, we investigate how households' participation in the stock market changes after the financial crisis for various age cohorts.

IV. Data

To test the theoretical predictions discussed above, we use the Survey of Consumer Finances (SCF). The SCF, sponsored by the Federal Reserve, is typically a triennial survey. The survey collects detailed demographic and financial household information of participants. Twice in the past, the current version of the survey was re-administered to prior respondents – the respondents to the 1983 survey were interviewed again in 1986 and 1989, and, most recently, the respondents to the 2007 survey were interviewed again in 2009.[3] We utilize this most recent panel dataset in our study. The availability of this panel data allows us to observe whether there were changes in financial decisions made by households as well as risk preferences and expectations around the time of the financial crisis. This is discussed in more detail in the following section.

[3] It should be noted that the full survey process can take up to 90 minutes. However, during the second interview, respondents were asked a sub-set of the questions from the 2007 survey, with the focus being on those related to finances and investments. Additionally, a few new questions were included in the 2009 survey related specifically to small business and mortgage refinancing and modification. The re-survey process was generally completed within 45 to 60 minutes.

The survey participants range considerably in age. For example, in 2009, the youngest participant is 20 and the oldest is 95. This allows us to examine multiple sub-groups and provide insight into differences in the reaction to the financial crisis across age cohorts.

V. Estimation Method

In the first part of the analysis, we determine whether the financial crisis impacted the decision of households to participate in the stock market by observing whether there was a significant change in the percentage of households that held stock in 2007 in comparison to 2009. We also consider whether there is an observable movement in or out of the stock market by households. For example, did households that held stock in 2007 also hold stock in 2009 or do we observe some households moving out of the stock market and other households moving into the stock market? Additionally, we consider whether changes in stock ownership is impacted by the extent of exposure.

Next, we examine questions on willingness to accept risk, financial planning, and investment tools in 2007 and 2009. The survey questions used in this portion of the analysis allow us to determine if there are any significant observable shifts in risk preferences, financial planning, and financial decision-making following the crisis. To explore the issue of risk tolerance, we consider households' responses to the question: "***Which of the following statements comes closest to describing the amount of financial risk that you (and your [husband/wife/partner]) are willing to take when you save or make investments?***" The possible responses are:

1. Take substantial financial risks expecting to earn substantial returns.
2. Take above average financial risks expecting to earn above average returns.

3. Take average financial risks expecting to earn average returns.
4. Not willing to take any financial risks.

Planning horizons have been used in prior studies as a measure of financial knowledge. The survey question used to capture planning horizons asks: "***In planning (your/your family's) saving and spending, which of the following is most important to [you/you and your (husband/wife/partner)]: the next few months, the next year, the next few years, the next 5 to 10 years, or longer than 10 years?***" When considering financial decision-making, we utilize survey questions that provide information on investment decisions and tools. Specifically, questions related to how much shopping around households engage in before making financial decisions, the tools used by households in making decisions expectations about future money and investment changes.

In the empirical portion of the analysis, we model stock market decisions as a function of the risk preferences, financial planning, and the financial decision-making factors discussed above as well as a variety of other financial and demographic factors to determine if there is empirical evidence that these factors impact either stock market participation and whether the impact various across age groups. The dependent variable in the models is an indicator variable equal to one if the household holds stock in that year, and zero otherwise. Given the dependent variable, we use probit regressions to capture the impact of varying factors on the probability of stock ownership. We draw upon prior literature to determine the various demographic and financial factors to include in the model, along with our variables of interest. Other factors included are age, marital status, presence of children, education level, health status, home ownership, household income, existence of a defined contribution plan at the current job, and two variables that proxy for being financially

constrained.[4]

Additionally, since the data used in this study is from a survey, it is possible that some data is missing. As discussed in detail on the Federal Reserve website and in the SCF documentation, missing data is imputed a total of five times thereby producing five implicates (five separate imputation replicates). To account for the multiple implicates, a repeated-imputation inference (RII) technique is used. This methodology is commonly employed in studies using SCF data and has some major advantages when compared to other techniques, such as using only one implicate or averaging the implicates.[5]

VI. Observed Changes Post-Financial Crisis

Existing research on risk tolerance is mixed, with some studies finding evidence that risk tolerance is relatively consistent or stable over time while others have found that it fluctuates (e.g. Yao, Hanna, Lindamood, 2004; Van de Venter, Michayluk, and Davey, 2012; Weber, Weber, and Nosic, 2013). Based on the theoretical model, it is expected that individuals will have a lower allocation toward risky assets during the time of the financial crisis and that households ownership of stock would increase after the initial shock of the financial crisis passed. As a result, we would expect the percentage of household owning stock to increase between 2007 and 2009. Our results indicate that the percentage of households owning stock was relatively consistent between 2007 and 2009 at approximately 18 percent. However, as shown in Figures 1 and 2, reactions to the financial crisis varied by age group. Specifically,

[4] The first variable is equal to one if either the household indicated it had been turned down for credit in the past five years or had thought of applying for credit during that period but did not because it expected it might be turned down, and zero otherwise. The second variable is equal to one of the household indicated its spending exceeded income in the past year, and zero otherwise.

[5] For a detailed discussion of the RII technique and a comparison to other modeling procedures, see Montalto and Yuh (1998).

while those in the 35 to 44 and 45 to 54 age groups did increase stock ownership by two percent, there was a three percent decline in stock ownership among the younger age group and no change in stock ownership with the older age group.

Next, we consider the impact of the extent of exposure. The model predicts that that the larger the exposure, the more likely a household is to change its participation in the stock market following the financial crisis. The percentage of financial assets held in stock varies widely across the sample from zero to nearly 100 percent. To determine if the changes in stock ownership between 2007 and 2009 varies based on the extent of exposure in the stock market, we define a variable called "Stock Heavy" that is equal to one if the amount of stock held is above a certain percent of all financial assets in 2007. We utilize cut-offs of 30 percent, 40 percent, and 50 percent to determine if the results observed are sensitive to the cut-off selected.

As shown in Table 1, we find that using the 30 percent cut-off, 64 percent of those heavily invested in stocks in 2007 still own stocks in 2009 while of those that are not heavily invested in stock in 2007, 70 percent still own stock in 2009. Interestingly, the higher the cut-off used, the greater the differences observed. For example, using a 50 percent cut-off, the percent of those heavily invested in stock in 2007 that still own stock in 2009 drops to approximately 57 percent while those with lower stock ownership in 2007 that still own stock in 2009 remains at around 70 percent. This suggests that the extent of exposure to the market at the time of the financial crisis may have some effect on the future stock ownership decisions of households.

Figures 3 and 4 report changes in risk tolerance and planning horizons by age groups. A negative (positive) number indicates that there was an increase (decrease) in risk tolerance

form 2007 and 2009 and the size indicates the magnitude of the movement. A similar measure is created that capture changes in planning horizons. As shown in Figure 3, we find that approximately 45 percent of households changed their risk tolerance levels between 2007 and 2009. Nearly 28 percent of households became more risk averse while about 17 percent increase their risk tolerance. Interestingly, the younger group had the largest decrease in risk tolerance while the 35 to 44 and 45 to 54 age groups showed the greatest increase in risk tolerance.

In examining Figure 4, we observed a fairly equal split among households as it relates to planning horizons. Approximately one third of the group did not change their planning horizon, one third increased their planning horizon and the remaining third decreased their planning horizon. When we consider the impact of age, we find those in the younger age group showed the greatest increase in their planning horizon while the 35 to 44 and 45 to 54 age groups showed the greatest decrease.

Finally, Table 2, there were some observable changes in investment decisions and tools between 2007 and 2009. Households were asked "when making major saving and investment decisions…. (o)n a scale from 1 to 5, where one is almost no shopping, three is moderate shopping and five is a great deal of shopping, where would (you/your family) be on the scale?" As shown in Panel A, while approximately 78 percent of households reported at least moderate shopping around, the percentage of households that indicated a great deal of shopping around increased between 2007 in 2009. Resources used in making investment decisions are summarized in Panel B. Households consistently report relying on friends, relatives, other contacts, or personal research and making decisions. This is closely followed by the use of financial professionals and financial planners. Interestingly, when comparing

the resources used in 2009 to those in 2007, only the use of the Internet increased, with the use of all of the other resources decreasing.

Households were also asked questions about changes to "the ways you arrange your money or investments." First, they were asked if they had made changes over time and then they were asked based on the result of experiences over the past two years, if they expect to make changes in the future. As shown in Panel C, more than half of the households had made changes over time, with a slightly greater percentage of the younger cohort making changes. Also, nearly 46 percent reported that they expected to make changes in the future, with more households in the 35 to 44 and 45 to 54 age groups indicating they would make changes in the future. The most frequently reported change over time made was "spend less, cut back." The most frequently reported expected change was "save more" followed by "spend less, cut back."

VII. Empirical Models

Summary statistics for the variables used in the model are reported in Table 3. The results of the empirical model containing the full sample of households are reported in Table 4. Starting with the age cohort variables, the coefficients on the 35 to 44 and 45 to 54 age groups are significant and negative in 2007 indicating that, in general, the younger age groups are less likely to own stock than the omitted group of those 55 to 64. However, none of the age variables are significant in 2009. We also find that white households and those with college degrees are more likely to own stock while those households with children are less likely to own stock. These results are evident in both 2007 and 2009. Home ownership, income, and participation in a defined contribution plan are all positively associated with stock ownership

in both years.

As it relates to our variables of interest, when we consider the resources used when making investment decisions, we find that households that utilize their own research are more likely to own stock relative to the omitted group of do not use any resources. This result is evident in both years. Additionally, households that use financial consultants are more likely to own stock in 2009. We find that willingness to accept risk is strongly associated with stock ownership relative to the category of not willing to take any financial risk in both 2007 and 2009. Longer planning horizons are also associated with higher likelihoods of stock ownership, but only in 2007. We also conduct tests of the coefficients of the risk and planning horizon variables. While we find no differences in either time period when comparing the coefficients of the planning horizon variables, we find a statistically significant difference in the risk variables for 2009. This result suggests that the coefficients on levels of risk tolerance do have a varying (significantly different) impact on stock ownership.

Collectively, these results suggest more consistency in the impact of the demographic variables but more variation in the impact of the financial variables across the two time periods. For example, prior to the financial crisis, the utilization of financial consultants had no impact on stock ownership. However, it appears that, following the financial crisis, households that utilized financial consultants were more likely to own stock than those that did not. Also, while the risk variables showed a general increase in the probability of stock ownership as risk tolerance increased in 2007, the probability of stock ownership for those willing to take on substantial risk was actually lower than those willing to take on above average risk following the financial crisis. Finally, while longer planning horizons were associated with a higher probability of stock ownership before the financial crisis, planning had not impact on the

probability of owning stock post-crisis.

When we consider the factors that influence stock ownership separately for each of the four age groups, we find some interesting results. First, as shown in Table 5, we find that education and income appear to consistently impact stock ownership, regardless of age or time period. On the other hand race and children have some impact on the probability of stock ownership, but only for particular age groups. We also find that home ownership is generally positive and significant for all but the 25 to 35 year age group. Also, participation in a defined contribution plan is only significant in the two younger age groups.

When we consider the variables of interest, we find a positive and significant impact of the use of financial consultants on stock ownership, but only for the 35 to 44 age group. Additionally, we find a positive and significant impact for use of research for all but the youngest age group. There is limited and somewhat mixed evidence past experiences, clubs, or other resources impact the probability of stock ownership, but only for the youngest and oldest age groups. Also, the credit constrained variable is significant and negative in both years for those 35 to 44.

As it relates to risk tolerance, we find that risk tolerance has an impact on stock ownership for all age groups except those 35 to 44. Specifically, for 25 to 34 year olds, those that are above average (in both years) and substantial risk takers (in 2007 only) are more likely to own stock than those that are not willing to take any risk. All three of the variables are significant and positive in both 2007 and 2009 for the 45 to 54 age group and the average and above average variables are significant and positive in both years with the oldest age group. Finally, the planning horizon variables only appeared to impact the probability of stock ownership for those in the 35 to 44 and 45 to 54 age groups, but only in 2007. We again

conduct tests of the coefficients of the risk and planning horizon variables. While we find no differences in either time period when comparing the coefficients of the planning horizon variables, we find a statistically significant difference in the risk variables for both the youngest cohorts (in both years) and the oldest cohort (in 2009 only). This result suggests that the coefficients on levels of risk tolerance vary in their impact on stock ownership, but only for particular age groups.

These results suggest that financial tools do vary in their impact on the probability of stock ownership with research being the most impact factor for those 35 and older and past experience, club, or other being more influential among the younger age group. It is also evident that risk tolerance does have a significant impact on the probability of stock ownership. However, the results indicate that the impact of the varying degrees of risk tolerance differ by age group. Finally, there is only limited evidence that planning has any influence on the decision to own stock.

VIII. Conclusion

The financial crisis and resulting recession has had a lasting impact on the U.S. economy. The crisis definitely influenced many businesses and a new environment for financial markets has emerged. How the crisis affected individual investors has not been considered as thoroughly. Reports have indicated that many of those close to retirement were forced to delay their retirement so their retirement portfolios had time to recover and rebound (Helman et al., 2011). Workers' contributions to their 401(k) accounts decreased during the crisis as well (Butrica and Smith, 2014). However, little research exists on how overall investments in the stock market and risk tolerance changed after the crisis and how this may impact asset

allocation moving forward. More importantly, little work has examined how younger cohorts were affected by the crisis. Specifically, did their risk tolerance or investment in risky assets change? If so, and this behavior continued moving forward, even as the economy continues to recover, this behavior could have a significant impact on younger generations' accumulated savings as they age. In this paper, we focus on how a financial shock impacts younger versus older cohorts who have longer versus shorter investment horizons. We investigate not only differences in risk tolerance around the crisis for various age groups but also the resulting impact on investment behavior and investment in the stock market.

We develop a simple theoretical model in which there is a financial shock at time one and look to consider how the allocation toward the risky asset varies between individuals who have more or less time post-shock. For individuals with CRRA preferences, they should optimally reduce their allocation toward the risky asset during the period when there is a financial shock. The greater the financial shock, the greater the reduction in the allocation toward risky assets during this period. Optimally, investment in the risky asset should rebound once the economy recovers. Empirically, however, this may not be the behavior observed.

Using data from the SCF, we examine changes in risk tolerance, stock ownership, and factors which influence stock ownership before and after the crisis for various age cohorts. We find that although stock ownership was relatively consistent around the crisis, middle-aged cohorts increased stock ownership after the crisis whereas younger cohorts decreased stock ownership. Our analysis also shows that the amount of exposure to the market at the time of the crisis greatly influences changes in stock ownership moving forward. In terms of risk tolerance, household risk tolerance decreased during the crisis with the youngest cohort having the greatest

decrease and thereby becoming more risk-averse.

In considering which factors influenced stock ownership, we find that those who use their own research to make investment decisions and/or use financial consultants are more likely to own stock. Those households who are more willing to take on risk are more likely to own stock, but our results show that the impact of risk tolerance and other financial variables on stock ownership have a time varying effect before versus after the crisis. Furthermore, this varying impact is only for certain age groups, so there is a significant difference among age cohorts. In summary, our results indicate that financial tools vary in their effect on stock ownership. Risk tolerance also influences stock ownership, but this effect differs by age group.

Moving forward, for the theoretical results, we will determine a closed form solution for the optimal allocations for investors with a longer time horizon and a financial shock in the first period. We will compare the optimal allocations for the two types of agents to see how the asset allocation changes with a longer versus shorter investment horizon. We will also investigate how the allocation changes with respect to the duration (τ) and impact (as measured through ε) of the financial shock.

The richness of the dataset allows for some additional testing that could shed more light on the impact of risk tolerance on investment decisions. For example, a close look at the data reveals large differences in risk tolerance level by income. For example, in 2007, more than 41 percent of households with above average income report being substantial or above average risk takers while only about 20 percent of households with below average income fall into these two categories. Though post-crisis, there were more high income and low income households that report not being risk takers, the differences among the risk categories for the two groups persist. It would be of interest to interact the risk tolerance variables with income

to determine if the differences in risk tolerance observed across income results in a varying effect on stock ownership.

Additionally, our study provides evidence of changes to risk tolerance, planning horizons, and investment decisions and tools over time. We also find that, to some extent, all of these factors impact asset allocation decisions. The next step would be to determine whether the impact of these changes explain changes in asset allocation decisions. We hope to include these and other robustness specifications in future versions of the paper.

IX. References

Ameriks, John and Stephen P. Zeldes, 2004. How Do Household Portfolio Shares Vary With Age? Working Paper.

Bateman, Hazel, Towhidul Islam, Jordan Louviere, Stephen Satchell, and Susan Thorp, 2011. Retirement Investor Risk Tolerance in Tranquil and Crisis Periods: Experimental Survey Evidence, *Journal of Behavioral Finance*, 12, 201-218.

Bergstresser, Daniel and James Poterba, 2004. Asset Allocation and Asset Location: Household Evidence From the Survey of Consumer Finances. *Journal of Public Economics*, 88, 1893-1915.

Bertaut, Carol C., 1998. Stockholding Behavior of U.S. Households: Evidence from the 1983-1989 Survey of Consumer Finances. *Review of Economics and Statistics*, 80(2), 263-275.

Butrica, Barbara A. and Karen E. Smith, 2014. 401(k) Participant Behavior in a Volatile Economy. *Journal of Pension Economics and Finance*, 15(1), 1-29.

Cox, Donald and Tullio Jappelli, 1993. The Effect of Borrowing Constraints on Consumer Liabilities, *Journal of Money, Credit, and Banking*, 25(2), 197-213.

Demirguc-Kunt, Asli, Enrica Detragiache and Ouarda Merrouche, 2013. Bank Capital: Lessons from the Financial Crisis, *Journal of Money, Credit and Banking*, 45(6), 1147-1164.

Fuss, Roland, Thomas Gehrig, and Philipp B. Rindler, 2011. Did the 2007-2008 Financial Crisis Change Risk Perception? University of St. Gallen Working Paper.

Gerrans, Paul, 2012. Retirement savings investment choices in response to the global financial crisis: Australian evidence. *Australian Journal of Management*, 37(3), 415-439.

Gerrans, Paul, Robert Faff, and Neil Hartnett, 2015. Individual financial risk tolerance and the global financial crisis. *Accounting & Finance*, 55, 165-185.

Guillemette, Michael, and Michael Finke, 2014. Do Large Swings in Equity Values Change Risk Tolerance? *Journal of Financial Planning*, 27, 44-50.

Hallahan, Terrence, Robert Faff, and Michael McKenzie, 2003. An Exploratory Investigation of the Relation Between Risk Tolerance Scores and Demographic Characteristics, *Journal of Multinational Financial Management*, 13, (4–5), 483–502.

Hallahan, Terrence, Robert Faff, and Michael McKenzie, 2004. An Empirical Investigation of Personal Financial Risk Tolerance, *Financial Services Review*, 13, 483–502.

Helman, Ruth, Craig Copeland and Jack VanDerhei, March 2011. The 2011 Retirement Confidence Survey: Confidence Drops to Record Lows, Reflecting "the New Normal."

EBRI Issue Brief No. 355, Employee Benefit Research Institute.

Hoffmann, Arvid O.I., Thomas Post, and Joost M.E. Pennings, 2013. Individual Investor Perceptions and Behavior During the Financial Crisis, *Journal of Banking and Finance*, 37(1), 60-74.

Lu, Wenling and David A. Whidbee, 2013. Bank Structure and Failure During the Financial Crisis, *Journal of Financial Economic Policy*, 5(3): 281-299.

Malmendier, Ulrike, and Stefan Nagel, 2011. Depression Babies: Do Macroeconomic Experiences Affect Risk Taking? *Quarterly Journal of Economics*, 126:1, 373-416.

Mitchell, Olivia S. and James F. Moore, 1998. Can Americans Afford to Retire? New Evidence on Retirement Savings Adequacy. *Journal of Risk and Insurance*, 65(3), 371-400.

Montalto, Catherine Phillips and Yoonkyung Yuh, 1998. Estimating Nonlinear Models with Multiply Imputed Data, *Financial Counseling and Planning*, 9(1): 97-101.

Necker, Sarah and Michael Zigelmeyer, 2013. Risk Taking After the Financial Crisis, Working Paper.

Pennings, Joost M. E. and Ale Smidts, 2000. Assessing the Construct Validity of Risk Attitude. *Management Science*, 46(10), 1337-1348.

Pennings, Joost M.E. and Brian Wansink, 2004. Channel Contract Behavior: The Role of Risk Attitudes, Risk Perceptions, and Channel Members' Market Structures, *Journal of Business*, 77(4), 697-724.

Poterba, James, Steven Venti, and David Wise, 2011. The Composition and Drawdown of Wealth in Retirement, *Journal of Economic Perspectives*, 25(4): 95-118.

Pratt, J.W., 1964. Risk Aversion in the Small and in the Large. *Econometrica*, 32, 122-136.

Rizzi, Joseph V., 2014. Post-Crisis Investor Behavior: Experience Matters. In Investor Behavior: The Psychology of Financial Planning and Investing. Ed. H. Kent Baker and Victor Ricciardi. John Wiley and Sons, pgs. 439-455.

Roszkowski, Michael J. and Geoff Davey, 2010. Risk Perception and Risk Tolerance Changes Attributable to the 2008 Economic Crisis: A Subtle but Critical Difference. *Journal of Financial Service Professionals*, 42-53.

Shiller, Robert J., 2008. The Subprime Solution: How Today's Global Financial Crisis Happened, and What to Do about It. Princeton University Press.

Sunden, Annika E. and Brian J. Surette, 1998. Gender Differences in the Allocation of Assets in Retirement Savings Plans, *American Economic Review*, 88(2), 207-211.

Taylor, John B., 2009. Getting Off Track: How Government Actions and Interventions Cause, Prolonged, and Worsened the Financial Crisis. Hoover Institution Press.

Van de Venter, Gerhard, David Michayluk, and Geoff Davey, 2012. A longitudinal study of financial risk tolerance. *Journal of Economic Psychology*, 33, 794-800.

Vrieling, M. R., 2013. The Impact of the Financial Crisis on the Individual Risk Tolerance Level of an Investor: How is the Relationship Between Risk Tolerance and Demographic Factors Affected by the Crisis? Master Thesis, Finance Department, Tillburg University.

Wang, Hui and Sherman Hanna, Does Risk Tolerance Decrease with Age?, *Financial Counseling and Planning*, 8(2), 27-31.

Weber, Martin, Elke U. Weber, and Alen Nosic, 2013. Who takes Risks When and Why: Determinants of Changes in Investor Risk Taking, *Review of Finance*, 17, 847-883.

Weber, Elke U., 2004. Who's Afraid of a Poor Old Age? Risk Perception in Risk Management Decisions. In Pension Design and Structure: New Lessons from Behavioral Finance, Ed. Olivia S. Mitchell and Stephen P. Utkus, Oxford UP, pgs 53-66.

Yao, Rui, Deanna L. Sharpe, and Feifei Wang, 2011. Decomposing the age effect on risk tolerance, *Journal of Socio-Economics*, 40, 879-887.

Yao, Rui, Sherman D. Hanna, and Suzanne Lindamood, 2004. Changes in Financial Risk Tolerance, 1983-2001. *Financial Services Review*, 13, 249-266.

Figure 1: Stock Ownership by Age Groups

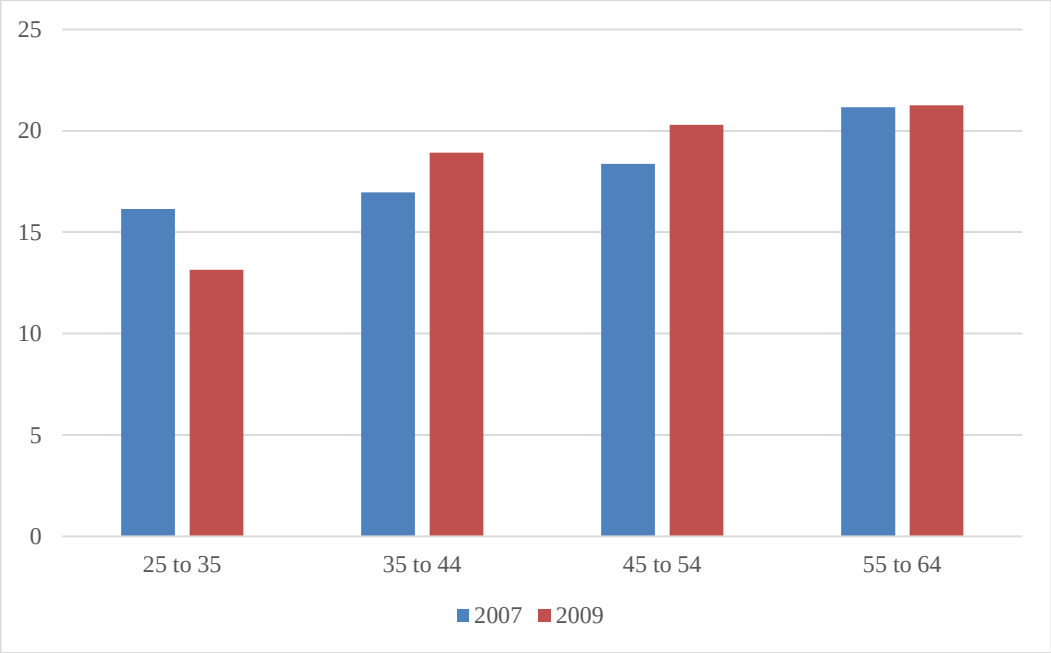


Figure 2: Change in Stock Ownership by Age Groups

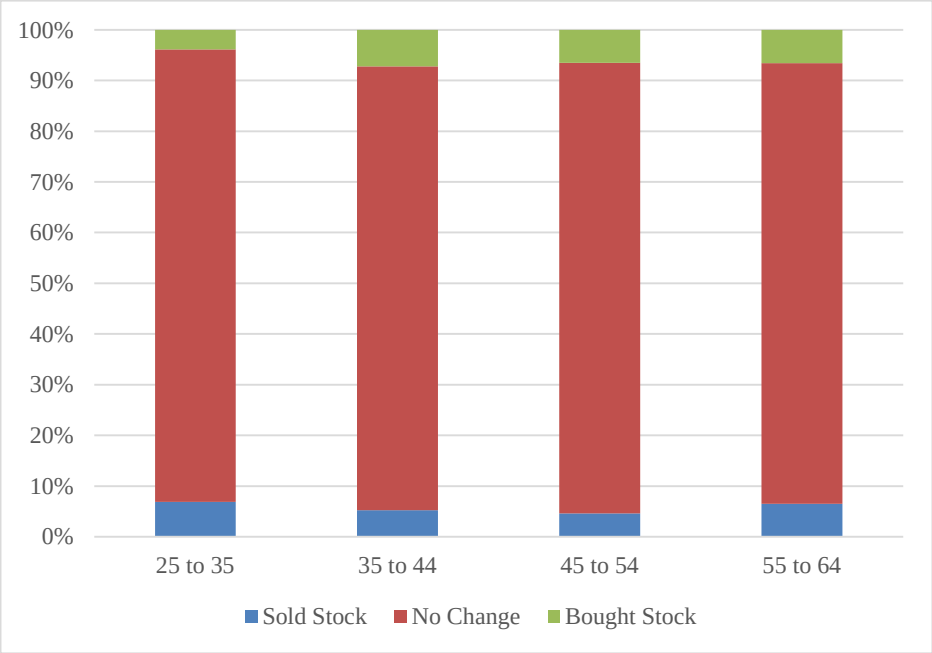
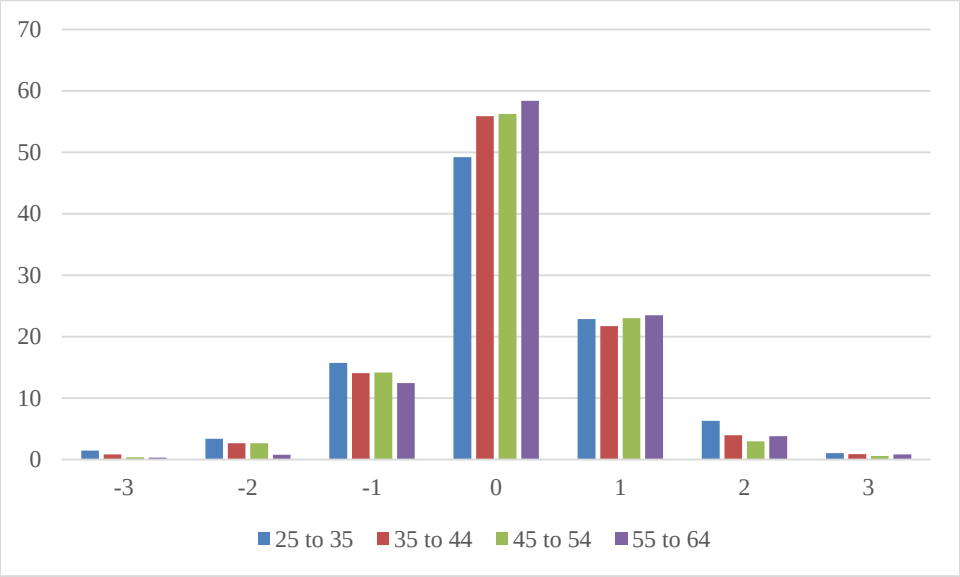
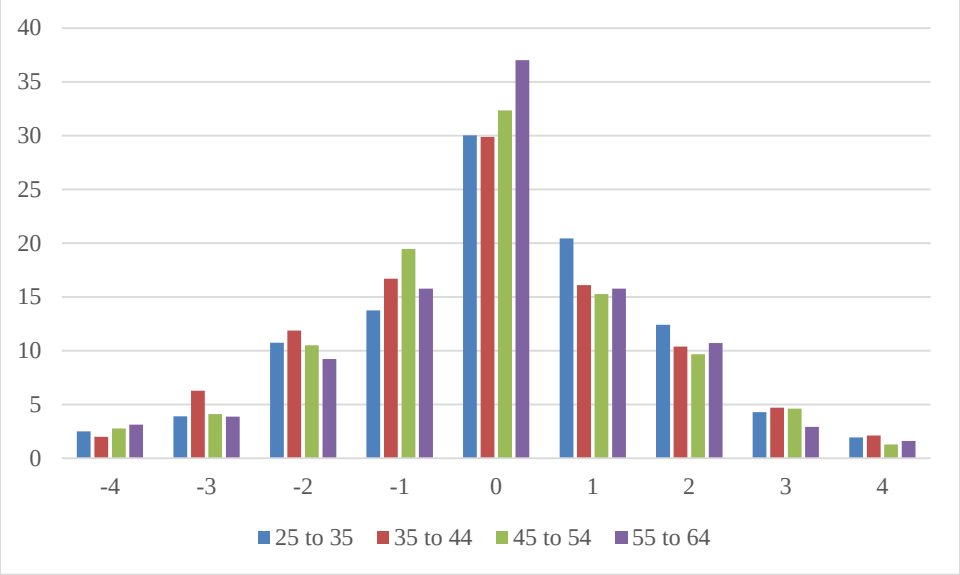


Figure 3: Change in Risk Tolerance by Age Groups



Note: Negative number indicates increase in risk tolerance and a positive number represents a decrease in risk tolerance.

Figure 4: Change in Planning Horizon by Age Groups



Note: Negative number indicates a decrease in planning and a positive number represents an increase in planning.

Table 1: Exposure and Stock Ownership in 2009

	30% Cut-Off	40% Cut-Off	40% Cut-Off
Stock Heavy	64%	59%	57%
Not Stock Heavy	70%	70%	70%

Note: The percentages reported for the 'Stock Heavy' and 'Not Stock Heavy' variable represents the percentage of households that were invested in stock in 2007 that still own stock in 2009.

Table 2: Investment Decisions and Tools

Panel A: Shopping Around

	Moderate Shopping Around			Great Deal of Shopping Around		
Age Cat.	**2007**	**2009**	**Change**	**2007**	**2009**	**Change**
25 to 35	57.79	55.59	-2.2	19.86	20.72	0.9
35 to 44	58.11	54.2	-3.9	21.66	24.05	2.4
45 to 54	61.42	55.88	-5.5	17.78	23	5.2
55 to 64	52.8	53.31	0.5	22.73	24.2	1.5
Total	57.77	54.78		20.41	23.05	

Panel B: Tools*

Tools	**2007**	**2009**	**Change**
Call Around	20%	19%	-0.01
Magazines/Newspapers	19%	15%	-0.03
Mail, TV, Radio, or Telemarketer	24%	19%	-0.05
Internet	35%	37%	0.02
Friend, Relative, Contact or Personal Research	45%	40%	-0.06
Financial Professional	38%	35%	-0.03
Financial Planner	31%	27%	-0.04
Self, Spouse, Shop Around or Other	7%	7%	0.00
Don't Shop Around/Don't Invest	8%	11%	0.02

Panel C: Changes to Money and Investments

Age Cat.	**Have Made**	**Will Make**
25 to 35	54.30	45.49
35 to 44	51.97	47.80
45 to 54	49.49	47.75
55 to 64	50.80	41.98
Total	51.52	45.95

* The survey defines financial professionals are bankers, brokers, real estate brokers/builders, stores/dealers, or insurance agents and financial planners are lawyers, accountants, and financial planners. All of the changes to the use of investment resources are significantly different at the 5 percent or better level.

Table 3: Summary Statistics by Age Categories (2009)

Variable	25 TO 34	35 TO 44	45 TO 54	55 TO 64	Total
Have Stock	0.1315	0.1892	0.2029	0.2125	0.1856
Age	32.1430	41.8428	51.3611	61.0699	46.7667
Married	0.5723	0.5825	0.5911	0.5548	0.5764
White	0.6558	0.6903	0.7373	0.7914	0.7190
Have Kids	0.6693	0.6935	0.5370	0.2403	0.5406
College Degree	0.3953	0.4164	0.4147	0.4025	0.4081
Healthy	0.8312	0.8217	0.7657	0.7202	0.7848
Own Home	0.5333	0.6889	0.7805	0.8195	0.7101
Log of Household Income	10.8117	10.9440	11.0229	10.9125	10.9298
Have DC Plan	0.4293	0.4861	0.5201	0.3379	0.4494
Use Financial Consultant	0.4110	0.5130	0.5044	0.5621	0.4991
Use Research	0.9298	0.9008	0.9493	0.8289	0.9046
Use Personal or Business Contacts	0.4537	0.4028	0.3880	0.3426	0.3962
Use Past Experience, Club or Other	0.0481	0.0696	0.0795	0.0655	0.0667
Don't Shop Around/Don't Invest	0.1358	0.1024	0.0910	0.1104	0.1084
Credit Constrained	0.3637	0.3524	0.2143	0.1465	0.2689
Not A Risk-Taker	0.4190	0.3747	0.3889	0.4675	0.4097
Average Risk-Taker	0.3740	0.4149	0.4346	0.4241	0.4135
Above Average Risk Taker	0.1504	0.1688	0.1395	0.0921	0.1390
Substantial Risk-Taker	0.0565	0.0416	0.0369	0.0163	0.0378
Plan < One Yr Ahead	0.1937	0.1847	0.1685	0.1663	0.1779
Plan One Year Ahead	0.1741	0.1554	0.1170	0.1143	0.1393
Plan Few Yrs Ahead	0.3015	0.2882	0.2943	0.3252	0.3013
Plan 5 to 10 Yrs Ahead	0.2289	0.2131	0.2835	0.2790	0.2515
Plan > 10 Yrs Ahead	0.1019	0.1586	0.1367	0.1153	0.1299
Spend More	0.1704	0.2221	0.2145	0.1572	0.1937

For the purposes of analysis, the investment resources categories are condensed. Use Financial Consultant include financial planners and financial professionals; Use Research includes calling around, using magazines, television, radio, advertisements, telemarketers, internet/online service; and calling around; Use Personal or Business Contacts includes information from friends and/or relatives, material from work or business contacts, and other personal research; and Use of Past Experience, Club or Other includes relying on self, spouse, past experiences, shopping around, or using other institutional resources. The omitted category in the models is Don't Shop Around/Don't Invest. Also, more detailed information regarding the investment resources is provided in Table 2.

Table 4: Model Results – Full Sample

Variable	2007	2009
Age < 35	0.0406	-0.1740
	[0.104]	[0.116]
Age 35-44	-0.1510*	-0.0119
	[0.087]	[0.086]
Age 45-54	-0.1605**	-0.0472
	[0.076]	[0.074]
Married	-0.0398	-0.0263
	[0.076]	[0.072]
White	0.2292***	0.2412***
	[0.084]	[0.083]
Have Kids	-0.1119*	-0.1670***
	[0.064]	[0.064]
College Degree	0.4396***	0.4498***
	[0.064]	[0.064]
Healthy	0.0031	0.1305
	[0.090]	[0.091]
Own Home	0.3428***	0.3604***
	[0.094]	[0.097]
Log of Household Income	0.2770***	0.2987***
	[0.024]	[0.027]
DC Plan at Current Job	0.1586***	0.1236**
	[0.060]	[0.060]
Use Financial Consultant	0.1048	0.1282**
	[0.065]	[0.063]
Use Research	0.0970***	0.1072***
	[0.026]	[0.027]
Use Personal or Business Contacts	-0.0455	-0.0336
	[0.061]	[0.061]
Use Past Experience, Club or Other	0.0965	0.1492
	[0.097]	[0.098]
Credit Constrained	-0.1338	-0.1184
	[0.087]	[0.082]
Average Risk Taker	0.3364***	0.2661***
	[0.090]	[0.078]
Above Average Risk Taker	0.4461***	0.5138***
	[0.098]	[0.092]
Substantial Risk Taker	0.5274***	0.3868***
	[0.139]	[0.144]
Plan One Year Ahead	0.2024	-0.0799
	[0.141]	[0.131]
Plan Few Yrs Ahead	0.2659**	-0.0049
	[0.112]	[0.104]
Plan 5 to 10 Yrs Ahead	0.3217***	-0.0071
	[0.109]	[0.105]
Plan > 10 Yrs Ahead	0.2470**	-0.0469
	[0.115]	[0.113]

Spending Exceeds Income	0.0604	0.1221
	[0.081]	[0.076]
Constant	-5.1966***	-5.2692***
	[0.288]	[0.308]
Observations	14,480	14,275

Standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Table 5: Model Results by Age Groups

	AGE 25 TO 34		AGE 35 TO 44		AGE 45 TO 54		AGE 55 TO 64	
Variable	**2007**	**2009**	**2007**	**2009**	**2007**	**2009**	**2007**	**2009**
Married	-0.221	0.105	0.0525	-0.1297	-0.1103	0.0085	0.0589	0.0006
	[0.216]	[0.232]	[0.178]	[0.157]	[0.136]	[0.127]	[0.136]	[0.134]
White	0.1521	0.1864	0.2367	0.1186	0.3987**	0.5827***	0.1015	0.041
	[0.197]	[0.201]	[0.176]	[0.153]	[0.163]	[0.165]	[0.172]	[0.174]
Have Kids	-0.3738**	-0.3028	-0.1776	-0.3133**	0.049	0.0331	-0.1035	-0.2085*
	[0.186]	[0.212]	[0.159]	[0.144]	[0.115]	[0.109]	[0.113]	[0.124]
College Degree	0.3163*	0.4059**	0.6363***	0.4637***	0.3906***	0.4142***	0.3971***	0.4972***
	[0.186]	[0.200]	[0.143]	[0.134]	[0.118]	[0.116]	[0.116]	[0.119]
Healthy	0.1382	-0.0397	0.0087	-0.0579	0.1067	0.2776*	-0.0443	0.2495
	[0.254]	[0.281]	[0.224]	[0.189]	[0.167]	[0.167]	[0.152]	[0.159]
Own Home	0.0219	0.1483	0.4720**	0.3558*	0.4286**	0.6473***	0.5132**	0.2216
	[0.202]	[0.214]	[0.214]	[0.189]	[0.196]	[0.213]	[0.216]	[0.209]
Log of Household Income	0.3580***	0.3712***	0.2985***	0.4056***	0.2763***	0.2926***	0.2649***	0.2377***
	[0.133]	[0.134]	[0.063]	[0.068]	[0.039]	[0.046]	[0.038]	[0.043]
DC Plan at Current Job	0.5712***	0.5810***	0.2357*	0.2011	0.0959	0.0016	-0.0381	0.0603
	[0.175]	[0.190]	[0.141]	[0.128]	[0.111]	[0.110]	[0.103]	[0.109]
Use Financial Consultant	-0.0331	-0.0172	0.2464*	0.3022**	0.0868	0.0538	0.1069	0.1034
	[0.167]	[0.177]	[0.142]	[0.129]	[0.121]	[0.112]	[0.123]	[0.120]
Use Research	0.0112	0.0089	0.1921***	0.1200*	0.0854*	0.1479***	0.0817*	0.0952**
	[0.079]	[0.099]	[0.056]	[0.062]	[0.044]	[0.049]	[0.048]	[0.047]
Use Personal or Business Contacts	0.1889	0.0722	0.0787	-0.1124	-0.0902	-0.0812	-0.1708	0.019
	[0.176]	[0.187]	[0.131]	[0.123]	[0.109]	[0.109]	[0.111]	[0.114]
Use Past Experience, Club or Other	0.7208**	-0.2622	0.2023	-0.0724	-0.1087	0.0825	0.114	0.3815**
	[0.332]	[0.373]	[0.243]	[0.236]	[0.167]	[0.170]	[0.161]	[0.163]
Credit Constrained	0.0831	-0.2153	-0.3027*	-0.2850*	-0.1252	0.052	-0.0304	0.0299
	[0.196]	[0.212]	[0.174]	[0.152]	[0.168]	[0.160]	[0.194]	[0.173]
Average Risk Taker	-0.0066	0.1864	0.1883	0.219	0.5482***	0.2758*	0.3109**	0.2883**
	[0.225]	[0.239]	[0.216]	[0.167]	[0.177]	[0.143]	[0.157]	[0.136]
Above Average Risk Taker	0.6216***	0.7092***	0.3746	0.2526	0.4177**	0.5195***	0.4597***	0.6985***
	[0.237]	[0.261]	[0.229]	[0.198]	[0.191]	[0.166]	[0.176]	[0.172]

Substantial Risk Taker	0.6060*	0.561	0.1979	0.2126	0.7244***	0.4638*	0.4379	0.3237
	[0.329]	[0.347]	[0.314]	[0.286]	[0.254]	[0.264]	[0.282]	[0.311]
Plan One Year Ahead	-0.2376	0.1446	0.5879*	-0.1077	0.302	-0.385	0.1525	0.1488
	[0.327]	[0.328]	[0.322]	[0.250]	[0.288]	[0.274]	[0.269]	[0.256]
Plan Few Yrs Ahead	-0.0292	0.1724	0.353	0.0309	0.4085*	-0.0452	0.3054	-0.021
	[0.248]	[0.294]	[0.277]	[0.205]	[0.239]	[0.198]	[0.195]	[0.194]
Plan 5 to 10 Yrs Ahead	0.4087	0.1817	0.5365**	0.0952	0.4375*	-0.0936	0.2102	0.0035
	[0.272]	[0.310]	[0.269]	[0.213]	[0.230]	[0.194]	[0.188]	[0.196]
Plan > 10 Yrs Ahead	0.3308	-0.1625	0.4192	-0.1154	0.2896	-0.2945	0.1848	0.2619
	[0.287]	[0.350]	[0.277]	[0.224]	[0.237]	[0.212]	[0.204]	[0.211]
Spending Exceeds Income	0.2305	0.3762	0.0164	0.0592	0.0026	0.1583	0.0044	0.049
	[0.216]	[0.247]	[0.178]	[0.157]	[0.145]	[0.128]	[0.156]	[0.149]
Constant	-5.7437***	-6.2152***	-6.2222***	-6.0640***	-5.7702***	-5.8922***	-4.8975***	-4.4505***
	[1.385]	[1.453]	[0.728]	[0.737]	[0.514]	[0.536]	[0.465]	[0.491]
Observations	2,315	2,310	3,550	3,535	4,460	4,400	4,155	4,030

Standard errors in brackets

*** p<0.01, ** p<0.05, * p<0.1

Appendix A

Let $u_0' \equiv u'(s_0(1+r) + \alpha_0 s_0(z_0 - \varepsilon) + y_1 - s_1)$ and $u_1' \equiv u'(s_1(1+r) + \alpha_1 s_1(z_0) + y_2 - s_2)$.

A.1 Derivation of Optimal Asset Allocations

For T=1, with a shock in the first period, the agent's maximization problem at time 1 is given by

$$\max_{c_0,c_1} E[u(c_0) + \beta u(c_1)]$$

which can be rewritten as

$$\max_{s_0,\alpha_0} E[u(A_0 + y_0 - s_0) + \beta u(s_0(1+r) + \alpha_0 s_0(z_0 - \varepsilon) + y_1 - s_1)]$$

Taking the first order condition with respect to s_0 we find:

$$-E[u'(A_0 + y_0 - s_0)] + \beta E\big[\big((1+r) + \alpha_0(z_0 - \varepsilon)\big)u_0'\big] = 0 \qquad (1.1)$$

Taking the first order condition with respect to α_0 we find:

$$\beta E[u_0' s_0(z_0 - \varepsilon)] = 0 \qquad (1.2)$$

Equation (1.1) can be rewritten as:

$$E[u'(A_0 + y_0 - s_0)] = E[\beta u_0']E[(1+r) + \alpha_0(z_0 - \varepsilon)] + cov\big(\beta u_0', (1+r) + \alpha_0(z_0 - \varepsilon)\big)$$

which implies

$$E[\beta u_0'] = \frac{E[u'(A_0 + y_0 - s_0)] - cov\big(\beta u_0', (1+r) + \alpha_0(z_0 - \varepsilon)\big)}{E[(1+r) + \alpha_0(z_0 - \varepsilon)]} \qquad (1.3)$$

Equation (1.2) can be rewritten as:

$$E[\beta u_0']E\big[s_0\big(z_0 - \varepsilon - (1+r)\big)\big] + cov\left(\beta u_0', s_0\big(z_0 - \varepsilon - (1+r)\big)\right) = 0$$

And can be reduced to

$$E[\beta u_0'] = \frac{-cov\big(\beta u_0', s_0(z_0 - \varepsilon)\big)}{E[s_0(z_0 - \varepsilon)]} \qquad (1.4)$$

Substituting (1.4) into (1.3) we find:

$$\frac{E[u'(A_0 + y_0 - s_0)] - cov\big(\beta u_0', (1+r) + \alpha_0(z_0 - \varepsilon)\big)}{E[(1+r) + \alpha_0(z_0 - \varepsilon)]} = \frac{-cov\big(\beta u_0', s_0(z_0 - \varepsilon)\big)}{E[s_0(z_0 - \varepsilon)]}$$

which can be rearranged as

$$(1+r) + \alpha_0 E[z_0 - \varepsilon] = \frac{E[s_0(z_0 - \varepsilon)]\big\{E[u'(A_0 + y_0 - s_0)] - cov\big(\beta u_0', (1+r) + \alpha_0(z_0 - \varepsilon)\big)\big\}}{-cov\big(\beta u_0', s_0(z_0 - \varepsilon)\big)}$$

Further reducing we find

$$\alpha_{0,} = \frac{s_0 E[u'(A_0 + y_0 - s_0)] - s_0 cov\big(\beta u_0', (1+r) + \alpha_0(z_0 - \varepsilon)\big)}{-cov\big(\beta u_0', s_0(z_0 - \varepsilon)\big)} - \frac{1+r}{E[z_0 - \varepsilon]} \qquad (1.5)$$

Substituting the value for (1.5) into (1.1) we can solve for s_0.