OPTIMAL DEMAND FOR LIFE INSURANCE IN CHINA UNDER CULTURE BARRIERS AND INVESTMENT UNCERTAINTIES

ABSTRACT. We propose a dynamic optimization model to explore the optimal demand for life insurance of a Chinese household with two breadwinners. The lifecycle welfare is measured as the total utility of consumption over the lifetime of the household. We solve a household's portfolio problem by explicitly recognizing the existence of culture barriers, social security after retirement, loadings on insurance premiums, and uncertainty of mortality. Our results show that life insurance purchase decision is not independent but correlated with a consumer's wealth, social security, investment risk, bequest incentives, and human capital. Moreover, the presence of incompatible shared values and ideas that place cultural barriers may impede life insurance purchase, which causes a serious uninsurance/underinsurance problem in China.

Keywords: Life insurance, Utility, Human capital, Cultural barriers, Bellman equation.

1. INTRODUCTION

China's life insurance industry has experienced rapid expansion since it reinvigorated its insurance industry in the early 1980s. The annual life insurance premiums grew from $10 billion in 1999 to $214.6 billion in 2010, accounting for 4.9% of global collections. Over the first 10 years of the 2000s, China had led the world in insurance premium growth with a compound annual growth rate of 26.5%. Despite recent tightened regulations [1], according to a report by Fitch Ratings in 2014, the insurance industry in China still achieved a premium growth rate in excess of 10% each year since 2010. In 2013, the Chinese insurance market became the fourth largest market in the world [2]. Yet, due to poor awareness of insurance, the insurance sector has not played a crucial role in the Chinese economy. Analysis of the penetration indicates that the China's life insurance market is still in its infancy. In 2012, the life insurance premiums as proportion of GDP (i.e. insurance penetration) in China is only 2.5%, compared with 7.7% in the U.S. and 10.5% in the U.K. (Jeantet, 2012).

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[1] For example, in April 2014, the China Insurance Regulatory Commission (CIRC) required that low-risk and long-duration insurance products account for at least 20% of total premium sales.

[2] http://www.iii.org/publications/online-financial-services-fact-book-2013/world-rankings

To study how to stimulate and expand consumption in life insurance, the vast majority of the studies focus on the Chinese insurance market but ignore the sources of consumer uncertainty related to social, economic, and demographic structures that directly affect life insurance demand. For example, Hwang and Gao (2003) identify the key determinants that explain the rapid growth of the life insurance industry in China since its economic reform in 1978. Hwang and Greenford (2005) study the key factors affecting aggregate life insurance consumption in mainland China, Hong Kong, and Taiwan using cross-sectional data. Leverty et al. (2009) examine the Chinese insurance industry surrounding China's accession to the World Trade Organization (WTO) and advocate further deregulation to maximize the welfare gains from the entry of foreign firms. As a study at the household level, Shi et al. (2015) provide micro-econometric evidence on the determinants of life insurance demand in China using the China Household Income Project (CHIP) data of 2002. However, the timeliness of their work is limited since only the data more than a decade ago are analyzed. While these studies have improved our understanding of the insurance market in China, there is a dearth of research analyzing life insurance as part of a Chinese household's portfolio. Understanding how life insurance can affect a household's life-cycle protection is important because it will increase insurance consciousness and thus promote a stronger insurance presence in China's economy.

To fill this gap, in this paper, we propose a recursive model to assess the impact of life insurance consumption on lifecycle welfare of a Chinese household measured through a CRRA utility function with stochastic labor income and investment returns. Specifically, we explore the optimal life insurance demand for the household by explicitly recognizing the existence of culture barriers, social security after retirement, loadings on insurance premiums, and uncertainty of mortality. Our results show that life insurance purchase decision is not independent but correlated with a consumer's wealth, social security, investment risk, bequest incentives, and human capital. Moreover, the presence of incompatible shared values and ideas that place cultural barriers may impede life insurance purchase (Chan, 2009). Our results indicate that cultural barriers cause a serious uninsurance/underinsurance problem in China.

Therefore, educating Chinese consumers would be an effective way to expand their insurance awareness and help them achieve an optimal life-cycle protection with life insurance. This will also help with the growth and development of the Chinese insurance industry since such development needs not only the government policy support but also well developed theories from both professionals and academics.

2. LITERATURE REVIEW

Although the life insurance study of China remains very limited, over the last 60 years, scholars have contributed to enriched findings that focus on general issues relating to insurance demands in western countries. Zietz (2003) provides a comprehensive literature review of both theoretical and empirical studies that examine the demographic and economic factors influencing the demand for life insurance. A spectrum of factors recognized as traits determining the life insurance demand include risk aversion, deductible levels and loading of insurance products, inflation, and wealth and bequest motives. Generally speaking, there are two streams of models in theoretical studies to explain demand for life insurance, depending on (1) how the welfare is measured; and (2) how the human capital is recognized.

The first type of models maximize lifecycle utility of consumption. In such model, human capital should not be considered separately [3]. Since human capital is the present value of future labor incomes, it is annualized as part of the future consumption. Therefore, if the accumulated lifecycle utility of consumption is set as the objective function, there is no need to explicitly consider human capital since it is reflected in the recursive function of the wealth (a function of labor income). Depending on whether a continuous distribution or discrete distribution assumption is made, the optimization function is either an integral with Hamilton-Jacob-Bellman equation or a Bellman equation.

For example, with a continuous-time model, Gollier (1994) analyzes the optimal dynamic strategy of a risk-averse agent to determine whether precautionary saving is superior to insurance in

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[3] If one wants to model the lifecycle human capital in a model that maximizes the lifecycle utility, a single whole life insurance (instead of one-year renewable life insurance) should be considered. In this case, a one-time decision of life insurance is made at a specified age, saying 35 or 40. This whole life insurance is used to hedge the impact of loss of human capital due to premature death of the breadwinner.

the long run. As an early trial of dynamic programming, this paper, however, does not consider the labor income of the agent, nor does it provide numerical solutions to the optimization problem. Brown and Poterba (2000) explore the potential gain that married couples receive from annuities. Since the focus of their study is how an initial stock of wealth is annuitized through life annuities, it does not model the demand for life insurance. After 2000's, researchers propose that the demand for the insurance and the assets should be jointly determined in an asset allocation problem. Huang et al. (2008) and Purcal and Piggott (2008), for instance, incorporate the asset investment and life insurance purchase in a portfolio choice problem. However, both studies assume continuous consumption of life insurance, which makes the outcomes less realistic. In addition, although Huang et al. (2008) analyze a "family" unit to recognize the bequest motive, it does not model the family members in a household individually. While Purcal and Piggott (2008) explicitly recognize the existence of social security in retirement, it is arguable that the paper considers the bequest as an argument in the individual's preference function since if the individual dies, bequest becomes meaningless. Assuming discrete consumption of life insurance, Maurer et al. (2013) solve for optimal life insurance demand, variable investment-linked deferred annuities investment, and asset allocation between risk-free bonds and risky stocks over the life of the household. However, as Huang et al. (2008), the mortalities of members in a household are not separately modeled.

The second type of models, instead, investigate the optimal demand for life insurance to achieve the maximal utility of wealth. In such one-stage or multi-stage models, human capital should be modeled explicitly. As a representative study in this direction, Campbell (1980) proposes a state-dependent model to describe households' optimal responses to human capital uncertainty through Taylor Series expansion. In his model, the human capital is defined as the present values of household incomes before retirement and life insurance is recognized as a hedging tool of human capital. To extend the single-period model in a multi-period world, Campbell (1980) claims the household is viewed as making sequential intertemporal consumption plans where future revisions may be necessary. Economides (1982) points out that Campbell (1980)'s approximate solution dictates non-optimal holdings of life insurance for all policies whose load factors are positive. The setup of Chen et al. (2006) is similar to Campbell (1980) except they incorporate the decision of asset allocation in the optimization model. There are some models in between these two types (utility of consumption vs. utility of wealth) such as Duarte et al. (2011), which arbitrarily defines a time horizon T and maximizes the utility from consumption, from the legacy if the individual dies before time T, and from the terminal wealth if the individual is alive at time T. But selecting an arbitrary time period T for life insurance demand analysis does not make a lot sense.

Are these two types of models equivalent? Theoretically speaking, all state-dependent wealths will be gradually consumed by a household. So maximizing the lifecycle utility of consumption is comparable to maximizing a sequential utilities of periodic terminal wealths if as explained by Campbell (1980), "the one-period objective function incorporates all the relevant choice variables contained in the multi-period case". The former is formulated as a backwards optimization dynamic programming problem while the later makes the t-th period decision based on the "realized" optimal decisions in the first $t-1$ periods. The difference lies in that the first type of models consider all decision as a coherent whole while the second type makes decisions sequentially. One would agree that the first type of models that maximize the lifecycle utility of consumption are superior to the second type since the former makes all decisions systematically. Therefore in our paper, a household's lifecycle welfare is measured by the total utility of consumption over the lifetime of the household. The decisions with respect to the periodic consumption, the life insurance demand, and asset allocation are solved through dynamic programming.

3. BASIC FRAMEWORK

In order to provide a more comprehensive study of the optimal demand for life insurance, we focus on a "family unit" (*i.e.*, a household) instead of an "individual". It is because in an individual-focused model, if the individual dies, both bequest and asset investment become meaningless. Chinese economic reform has brought drastic changes to Chinese families in the past thirty years. Different from traditional stereotype in most western countries where men are the main breadwinners of the households, in modern China, usually there are two breadwinners in a family. Therefore, in this paper, we explore the needs of life insurance of a married couple, recognizing the joint consumption when both members are alive and the survivor's benefits after one spouse has died but the other is still alive.

3.1. **Life Insurance.** In this paper, we model life insurance as a pure fatal event risk management tool, which manages the premature death risk of breadwinners in a household. Following Chen et al. (2006), the annual life insurance demand for a household is measured by the amount of one-year, renewable term life insurance purchased by a household. This assumption is consistent with those in the seminal studies of Campbell (1980) and Economides (1982). Although there are a number of life insurance products such as term life, whole life, universal life insurance policies, in reality, it's not practical to assume an individual continuously purchases life insurance over time as in Huang et al. (2008) and Duarte et al. (2011). We believe all other types of life insurance policies are financial combinations and can be replicated by the one-year renewable term life insurance.

3.2. **Social Security.** Social security benefits are a big impact factor of a retiree's life quality in China. In the paper, we focus on the Chinese government social pensions.[4] China's current pension system is planned at the province level and sponsored by the central government budgets. As a typical example, the firm pays 20% of an employee's wage income as the monthly premium that goes into the social pension fund, which runs as a pay-as-you-go system. The employee pays another 8% that goes into his/her personal account (Fang, 2013).

There are also corporate pension schemes available in China, known as enterprise annuities (equivalent to the 401(k) plan in U.S.). As a form of voluntary supplemental retirement savings program, enterprise annuities are not part of China's social security pension. Since the participate rate of enterprise annuity plans is low in China, this type of retirement plans is not considered in our model.[5]

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[4] Chinese social insurance system consists of five basic insurances: pension, unemployment insurance, health insurance, work injury insurance, and maternity insurance. For simplicity, we only consider the pension since it provides the largest influence on an individual's lifecycle welfare.

[5] As of 2014, less than 20 million employees in China are participating in enterprise annuity plans, out of an urban working population of 158 million.

3.3. **Human Capital.** Human capital is generally considered as the collective skills, knowledge, or other intangible assets of individuals that can be used to create economic value for the individuals and the society. In the second type of models that maximize the utility of wealth at certain time t, human capital is defined as the present value of an individual's future labor income before retirement and the store of labor income paid as government social security benefits after the individual retires. While in the first type of models that maximizes the lifecycle utility of consumption, the labor income is added towards the total wealth of the household which will support the period consumption, life insurance purchase, as well as asset investment. In such way, labor incomes are "annualized" in form of the household's periodic consumption.

As explained at the beginning of the section, we model a "family unit" that contains two breadwinners. For simplicity, suppose the labor incomes of the two breadwinners in a family (*i.e.*, the husband and the wife) are independently distributed. Each individual breadwinner is assumed to receive his/her labor income at the beginning of the period and the individual labor income follows a lognormal distribution with the same set of parameters. Specifically, during the working life, each individual of a family earns an exogenously determined level of risky (after tax and social security contribution) labor income

$$h_t^i = h_{t-1}^i e^{\mu_h + \sigma_h Z_t^h}, \quad t \leq T;\ i = x, y,$$

where T is the individual's retirement time and Z_t^h is a standard Brownian motion. x and y stand for the husband and the wife in a household, respectively. The labor income of individual i at time 0, h_0^i, is pre-specified in the numerical analysis in Section 4. After retirement, we assume for the sake of simplicity that the individual receives a constant fraction, c, of the labor income upon retirement as lifetime pension benefits as long as he/she survives. Following the 2014 requirements on the government pension in China, an individual's retirement pension benefit is calculated as follows:

$$\begin{aligned} &\text{basic retirement pension benefit} = \\ &(\text{local average wage for the previous year} \times (1 + \text{personal contribution rate})) \qquad (1) \\ &\times \text{number of contribution years} \times 0.5\%. \end{aligned}$$

In China, the earnings used to calculate contributions are usually 60%-300% of the local average wage in the previous year [6]. If an individual's average working period (i.e. the number of pension contribution years) is 45 years and the average contribution rate is 175% of the local average wage in the previous year, his/her social security benefit replacement rate c will be around 60%. For our numerical illustrations in Section 4, we set $c = 60\%$. With this setup, the social security payment after retirement for the individual i equals

$$h_t^i = c \cdot h_T^i, \quad t > T;\ i = x, y,$$

where h_T^i is the labor income at the retirement time T for individual i $(i = x, y)$.

3.4. **Life Insurance Culture Barriers in China.**

Death and fatal misfortunes are taboo subjects in China. Chinese tend to avoid the topics of unexpected, accidental, or premature death because they believe such topics will precipitate deaths. Although death has been a central theme of western philosophy, it is rarely mentioned in the teachings of Confucianism. Chan (2009) find that "Death for common people is already a mystery, and the avoidance of the topic further mystifies and intensifies a general sense of fear regarding death". Whether a death is terrifying depends on when and how the death occurs: a good death results from a long and full life while an unexpected premature death is considered as a miserable one. Therefore, premature death is a deeply ingrained taboo topic in China. Durkheim (1965) points out that this taboo today has an independent power in shaping human action because "when something has customarily been a taboo subject, a violation of the taboo is not only socially offensive but out of the conceivable 'normality' of being a human being".

Such culture barriers hinder the life insurance market from development in the Chinese society. To create and develop the life insurance demand in China, the traditional logics of avoiding talking and thinking about misfortunes must be overcome. If there is no incompatibility between the local culture logics and the profit-oriented institutional logics of life insurance, the life insurance business in China should have developed even faster. In order to recognize the culture resistance

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[6] The information is provided by the U.S. Social Security Administration - Office of Retirement and Disability Policy at http://www.ssa.gov/policy/docs/progdesc/ssptw/2012-2013/asia/china.html.

from the Chinese concepts of life and death, in this paper, we add a factor to reflect this unique feature of the Chinese life insurance market. Using a "terror" utility function, we quantify the efforts an individual has to make to overcome the cultural barrier by thinking of and plan for the death with life insurance. Specifically, the "terror" utility is calculated as:

$$\text{terror utility} = -\eta \cdot U_{\text{terror}}(\mathbb{I}_{LP}),$$

where η is a positive scaling parameter and $\mathbb{I}_{LP}$ is the indicator function with value 1 if the life insurance premium LP is positive and 0 otherwise. U_{terror} is strictly concave with $U'_{\text{terror}} > 0$ and $U''_{\text{terror}} < 0$.

3.5. **Model Description.** As pointed by Lin and Grace (2007), life insurance demand is jointly determined as part of a household's portfolio. We assume the household can allocate financial wealth in equities, bonds, and risk-free assets. Without losing generality, suppose one period is a year. The investment strategy is supposed to be adjusted annually. Let S_t denote the market value of stock at time t, which follows:

$$dS_t = \mu_S S_t dt + \sigma_S S_t dW_t^S,$$

where μ_S and σ_S are the constant drift and volatility, respectively. Similarly, the market value of the bond B_t and risk-free asset RF_t at time t are determined as:

$$dB_t = \mu_B B_t dt + \sigma_B B_t dW_t^B,$$

and

$$dRF_t = \mu_{RF} RF_t dt + \sigma_{RF} RF_t dW_t^{RF}.$$

The Brownian motion W_t^S, W_t^B, and W_t^{RF} are assumed to be pairwise correlated. Shi et al. (2015) point out that the correlations between the returns of human capital and other financial assets will affect the demand for life insurance as well as the financial market performance. So we assume the Brownian motions driving the financial assets are pairwisely correlated with Z_t^h, the Brownian motion driving the wage process. For simplicity, the correlations between the shocks of the labor income and returns of different financial assets are set at the same level ρ.

To merge asset allocation and human capital with demand for life insurance, in the paper, we consider a family unit with two breadwinners. For the married couple of interest, the husband and the wife of age x_0 and y_0 at $t = 0$ will make joint consumption, joint investment, and individual life insurance purchase decisions in every period t at ages x and y. As explained in Section 3.1, we assume each individual invests in the one-year renewable term life insurance. Suppose the husband has the same mortality experience as the Chinese male population who will retire at time T_x at age 60 and the wife has the same mortality experience as the Chinese female population who will retire at time T_y at age 55. The maximal possible age for both male and female is set at $T^x_{\text{max}} = T^y_{\text{max}} = 105$. Denote $p_{x,t}$ and $p_{y,t}$ the one-year survival rates of the husband and wife at time t at age x and y, respectively. That is, $p_{x,t}$ ($p_{y,t}$) is the probability of survival at the end of year $t + 1$ for the husband (the wife) conditional on being alive at age x (y) at time t. Define the following Bernoulli processes for the husband and wife respectively:

$$X_{t+1} = \begin{cases} 1 & \text{with probability } p_{x,t} \\ 0 & \text{with probability } 1 - p_{x,t} \end{cases}, \tag{2}$$

and

$$Y_{t+1} = \begin{cases} 1 & \text{with probability } p_{y,t} \\ 0 & \text{with probability } 1 - p_{y,t} \end{cases}. \tag{3}$$

With $\mathbb{I}^x_0 = 1$ and $\mathbb{I}^y_0 = 1$, the survival indicators for the husband and wife are

$$\mathbb{I}^x_{t+1} = X_{t+1} \cdot \mathbb{I}^x_t, \quad t = 0, 1, \cdots$$

$$\mathbb{I}^y_{t+1} = Y_{t+1} \cdot \mathbb{I}^y_t, \quad t = 0, 1, \cdots$$

At time t, the couple may be in one of the following four states S_t:

$$S_t = \begin{cases} 1 & \mathbb{I}_t^x = 1, \mathbb{I}_t^y = 1 \quad \text{both alive} \\ 2 & \mathbb{I}_t^x = 1, \mathbb{I}_t^y = 0 \quad \text{widower} \\ 3 & \mathbb{I}_t^x = 0, \mathbb{I}_t^y = 1 \quad \text{widow} \\ 4 & \mathbb{I}_t^x = 0, \mathbb{I}_t^y = 0 \quad \text{both deceased.} \end{cases} . \tag{4}$$

So the time-dependent transition matrix $\Pi_{ij,t} = \Pr(S_{t+1} = j | S_t = i)$ for this Markov chain is determined by the husband's and wife's one-year survival rates as follows:

$$\Pi_{ij,t} = \begin{bmatrix} p_{x,t} \cdot p_{y,t} & p_{x,t}(1 - p_{y,t}) & (1 - p_{x,t})p_{y,t} & (1 - p_{x,t})(1 - p_{y,t}) \\ 0 & p_{x,t} & 0 & 1 - p_{x,t} \\ 0 & 0 & p_{y,t} & 1 - p_{y,t} \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4} . \tag{5}$$

Denote the curtate lifetimes for x and y as K_x and K_y, both of which are random at time 0. So the household lifetime $K_{\text{max}} = \max(K_x, K_y)$. That is,

$$K_{\text{max}} = \min\{K \in \mathbb{N} | S_K = S_4\},$$

where $\mathbb{N}$ stands for the set of natural number.

The household is to make joint consumption, joint asset allocation, and individual life insurance purchase decision every period. For simplicity, suppose the insurance premiums are collected at the beginning of the year and the death benefits are paid at the end of the year if the insured dies. Assuming the husband and wife each receives his/her individual labor income at the beginning of the year, our optimization problem is to maximize the lifecycle utility of the joint consumption by determining the optimal amount of life insurance measured by the premiums of one-year life insurance at time t ($LP_t = [LP_t^x, LP_t^y]$), the periodic joint consumption (C_t), and the investment strategy $w_t = [w_{S_t}, w_{B_t}, w_{RF_t}]$, where w_{S_t}, w_{B_t}, and w_{RF_t} are the proportions of wealth invested in equities, bonds, and risk-free assets at time t, respectively. That is,

$$\max_{\{LP_t, C_t, w_t\}_{t=0}^{K_{\max}}} \sum_{t=0}^{K_{\max}-1} \mathrm{E}_t[U_t] + U_B(\frac{W_{K_{\max}}}{B}), \tag{6}$$

where $\mathrm{E}_t[\cdot] = \mathrm{E}[\cdot|\mathscr{F}_t]$ and $\mathscr{F}_t$ is the information till time t. B is the strength of the couple's bequest motive with $B \geq 1$. The utility of the household at time t equals $U_t = U(C_t) + (-\eta \cdot U_{\text{terror}}(\mathbb{I}_{LP_t}))$. Both $U(\cdot)$ and $U_B(\cdot)$ are strictly concave, which implies that the household prefers more consumption/wealth to less and is risk averse. At least three continuous derivatives are assumed to exist for both utility functions. In our paper, we assume the household has the constant relative risk aversion (CRRA) utility functions as

$$U_t = \frac{C_t^{1-\gamma_c}}{1-\gamma_c} + (-\eta \cdot \frac{\mathbb{I}_{LP_t}^{1-\gamma_{\text{terror}}}}{1-\gamma_{\text{terror}}}), \tag{7}$$

where γ_c and γ_{terror} are the levels of relative risk aversion for the periodic consumption and the terror utility function, respectively. For a household with two breadwinners, the indicator $\mathbb{I}_{LP_t}$ equals 1 if either LP_t^x or LP_t^y is positive and 0 if both LP_t^x and LP_t^y are zero. In other words, as long as one of the breadwinners of the family is alive and purchases life insurance, the terror disutility will be recognized.

Expressed recursively through a Bellman equation, the optimization problem becomes

$$V_t(W_t, S_t) = \max_{LP_t, C_t, w_t} \{U_t + \beta \cdot \mathrm{E}_t[V_{t+1}(W_{t+1}, S_{t+1})]\}, \tag{8}$$

where W_t represents the household's wealth at time t and β is the discount factor. The state variable W_t evolves according to:

$$\begin{aligned} W_{t+1} = &[W_t - C_t + \mathbb{I}_t^x(h_t^x - LP_t^x) + \mathbb{I}_t^y(h_t^y - LP_t^y)] \cdot (w_{S_t}(1 + r_{S_t}) + w_{B_t}(1 + r_{B_t}) + w_{RF_t}(1 + r_{RF_t})) \\ &+ (1 - \mathbb{I}_{t+1}^x)LI_{t+1}^x + (1 - \mathbb{I}_{t+1}^y)LI_{t+1}^y \end{aligned} \tag{9}$$

where r_{S_t}, r_{B_t}, r_{RF_t} are the log rates of returns of the equities, bonds, risk-free assets in period t, respectively. LI_{t+1}^i is the amount of life insurance measured by the face value of one-year life insurance at time $t+1$ for individual i ($i = x, y$). The life insurance premium paid at time t for insurance policy of individual i ($i = x, y$), LP_t^i is determined as

$$LP_t^i = \beta(1+\lambda)q_{i,t} \cdot LI_{t+1}^i, \quad i = x, y,$$

where $q_{i,t}$ is the one-year death rate for individual i at time t. The parameter λ is the life insurance loading per dollar premium that covers risk premiums charged by an insurer and the insurer's administrative expenses and transaction costs.

The evolution of the wealth follows a Markov process, which means the current state is a sufficient statistic for forecasting future values of the state. Specifically, W_{t+1} is dependent on S_t as follows:

$$W_{ij,t+1} = \begin{bmatrix} (W_{11,t+1}|S_t = 1, S_{t+1} = 1) & (W_{12,t+1}|S_t = 1, S_{t+1} = 2) & (W_{13,t+1}|S_t = 1, S_{t+1} = 3) & (W_{14,t+1}|S_t = 1, S_{t+1} = 4) \\ (W_{21,t+1}|S_t = 2, S_{t+1} = 1) & (W_{22,t+1}|S_t = 2, S_{t+1} = 2) & (W_{23,t+1}|S_t = 2, S_{t+1} = 3) & (W_{24,t+1}|S_t = 2, S_{t+1} = 4) \\ (W_{31,t+1}|S_t = 3, S_{t+1} = 1) & (W_{32,t+1}|S_t = 3, S_{t+1} = 2) & (W_{33,t+1}|S_t = 3, S_{t+1} = 3) & (W_{34,t+1}|S_t = 3, S_{t+1} = 4) \\ (W_{41,t+1}|S_t = 4, S_{t+1} = 1) & (W_{42,t+1}|S_t = 4, S_{t+1} = 2) & (W_{43,t+1}|S_t = 4, S_{t+1} = 3) & (W_{44,t+1}|S_t = 4, S_{t+1} = 4) \end{bmatrix}. \tag{10}$$

Therefore, the expected value of the value function given the current state S_t is calculated as

$$\mathrm{E}_t[V_{t+1}(W_{t+1}, S_{t+1})|S_t = i] = \sum_{j=1}^{4} W_{ij,t+1}\Pi_{ij,t}.$$

The optimization problem (6) has to satisfy the following constraints:

$$\begin{aligned} & LP_t^x, LP_t^y, C_t \geq 0 \\ & 0 \leq w_{S_t}, w_{B_t}, w_{f_t} \leq 1 \\ & w_{S_t} + w_{B_t} + w_{f_t} = 1 \\ & C_t + LP_t^x + LP_t^y \leq W_t + h_t^x + h_t^y. \end{aligned} \tag{11}$$

4. NUMERICAL ANALYSIS

4.1. **Mortality Rates.**

While there are decades of annual mortality data available in many developed countries, the lack of mortality data is the major obstacle of capturing the morality dynamics in China. There were only two nation-wide mortality investigations in mainland China since the recreation of the life insurance market. The first one lasted nearly four years (1992-1995) and ended up with the first standard life table of mainland China—the China Life Table (1990-1993). This table was dictated by China's insurance regulatory authority as the basis of the pricing of life insurance products and the evaluation of their statutory reserves. Completed in 2005, the second mortality investigation helped to construct the China Life Table (2000-2003), which is dictated for the statutory reserve evaluation but not for the insurance product pricing. The Chinese insurance industry makes it a rule to conduct a mortality investigation every ten years.[7]

Due to the sparse of the available mortality data, our mortality projection is based on the China Life Table (1990-1993) and the China Life Table (2000-2003). Following Brown and Poterba (2000), we assume the mortality experiences between the husband and wife in a family are independent [8]. Rather than use the advanced mortality forecasting models that require enriched mortality data such as Lee and Carter (1992) and Cairns et al. (2006) for single population or Li and Lee (2005) for a group of population, we adopt a simple model that quantifies mortality reduction factors for different ages. Specifically, the one-year death rate for the husband $q_{x,t+k}$ at age x $(x = 0, 1, 2, \cdots, T_{\text{max}})$ in year $t + k$ is modeled as:

$$q_{x,t+k} = q_{x,t} \cdot R_{x,t+k}, \tag{12}$$

where $q_{x,t}$ is the husband's one-year death rate of age x in year t and $R_{x,t+k}$ is the husband's mortality reduction factor for age x at time $t+k$. Following Mao et al. (2008), $R_{x,t+k}$ is determined through the extrapolation of a log-linear trend of the death rates. That is,

$$\ln q_{x,t+k} = a_x + b_x(t + k). \tag{13}$$

So

$$R_{x,t+k} = \frac{q_{x,t+k}}{q_{x,t}} = \frac{e^{a_x+b_x(t+k)}}{e^{a_x+b_x(t)}} = e^{b_x(k)}. \tag{14}$$

The one-year death rate for the wife $q_{y,t+k}$ at age y $(y = 0, 1, 2, \cdots, T_{\text{max}})$ in year $t + k$ is similarly modeled except the subscript x in (12), (13), and (14) is replaced with y.

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[7] The investigations were all organized by China's insurance regulatory authority before year 2007. Then China Association of Actuaries (CAA) took over the responsibility after it was created in 2007.

[8] Brown and Poterba (2000) point out that their numerical results based on dependent mortality data of married and widowed individuals are not significantly different from these obtained based on independent mortality assumption.

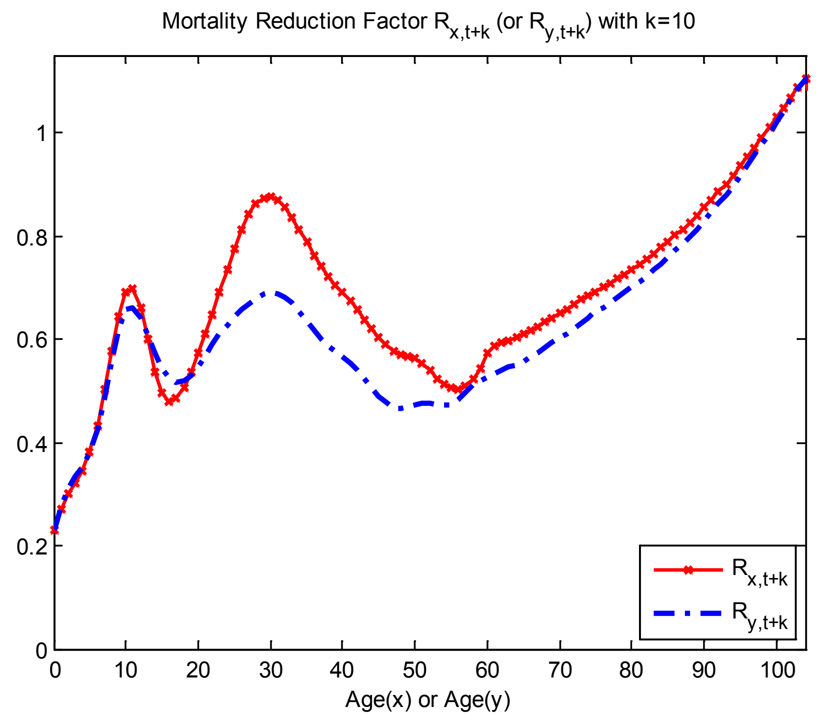

FIGURE 1. Estimated mortality reduction factor $R_{x,t+k}$ and $R_{y,t+k}$ with $k = 10$.

We assume the husband (the wife) has the same mortality experience as the Chinese male (female) population with pension plans. The parameter estimates of a_x and b_x for the husband (a_y and b_y for the wife in parentheses) are reported in Table 1. Based on $k = 10$, in Figure 1 the mortality reduction factor $R_{x,t+k}$ and $R_{y,t+k}$ are illustrated with −x− and − · −, respectively. Figure 1 shows that for both the husband and the wife, the mortality reduction factors are less than 1 (i.e., representing mortality improvement) when the age is younger than 99. The husband's reduction factor $R_{x,t+k}$ increases from 0.23 at age 0 to its first local maximum 0.70 at age 11 and then reduces. It reaches the second local maximum 0.88 at age 30 and then reduces again. After age 56, $R_{x,t+k}$ keeps increasing. The wife's mortality reduction factor $R_{y,t+k}$ varies with y in a similar pattern except all the changes are more modest compared with its counterpart $R_{x,t+k}$ at age $x = y$.

4.2. **Financial Markets.** China is recognized as the country that has one of the highest savings rates in the world. Chinese companies and individuals save over 50% of the country's GDP each year. Therefore, managing the saved money through effective asset investment is very important.

TABLE 1. The model (12) fits the China's Male-Pension (Female-Pension) data from 1993 to 2003.

Age	$a_x(a_y)$	$b_x(b_y)$	Age	$a_x(a_y)$	$b_x(b_y)$	Age	$a_x(a_y)$	$b_x(b_y)$
0	-5.76(-5.85)	-0.15(-0.15)	35	-6.71(-7.23)	-0.02(-0.05)	70	-3.43(-3.76)	-0.04(-0.05)
1	-6.11(-6.27)	-0.13(-0.13)	36	-6.62(-7.14)	-0.03(-0.05)	71	-3.34(-3.66)	-0.04(-0.05)
2	-6.42(-6.62)	-0.12(-0.12)	37	-6.53(-7.05)	-0.03(-0.05)	72	-3.25(-3.56)	-0.04(-0.05)
3	-6.68(-6.94)	-0.11(-0.11)	38	-6.44(-6.96)	-0.03(-0.05)	73	-3.16(-3.47)	-0.04(-0.05)
4	-6.91(-7.22)	-0.11(-0.1)	39	-6.35(-6.86)	-0.03(-0.06)	74	-3.07(-3.37)	-0.04(-0.04)
5	-7.11(-7.47)	-0.1(-0.1)	40	-6.26(-6.77)	-0.04(-0.06)	75	-2.98(-3.27)	-0.04(-0.04)
6	-7.30(-7.71)	-0.08(-0.09)	41	-6.16(-6.67)	-0.04(-0.06)	76	-2.89(-3.18)	-0.04(-0.04)
7	-7.47(-7.92)	-0.07(-0.07)	42	-6.07(-6.57)	-0.04(-0.06)	77	-2.8(-3.08)	-0.03(-0.04)
8	-7.61(-8.11)	-0.06(-0.06)	43	-5.97(-6.47)	-0.04(-0.06)	78	-2.71(-2.99)	-0.03(-0.04)
9	-7.73(-8.26)	-0.04(-0.05)	44	-5.87(-6.36)	-0.05(-0.07)	79	-2.63(-2.89)	-0.03(-0.04)
10	-7.80(-8.36)	-0.04(-0.04)	45	-5.78(-6.26)	-0.05(-0.07)	80	-2.54(-2.8)	-0.03(-0.04)
11	-7.82(-8.4)	-0.04(-0.04)	46	-5.68(-6.16)	-0.05(-0.07)	81	-2.45(-2.7)	-0.03(-0.03)
12	-7.75(-8.36)	-0.04(-0.04)	47	-5.58(-6.05)	-0.06(-0.08)	82	-2.37(-2.61)	-0.03(-0.03)
13	-7.62(-8.28)	-0.05(-0.05)	48	-5.49(-5.95)	-0.06(-0.08)	83	-2.28(-2.52)	-0.03(-0.03)
14	-7.46(-8.15)	-0.06(-0.06)	49	-5.39(-5.85)	-0.06(-0.08)	84	-2.20(-2.43)	-0.03(-0.03)
15	-7.29(-8.03)	-0.07(-0.06)	50	-5.30(-5.75)	-0.06(-0.08)	85	-2.12(-2.34)	-0.02(-0.03)
16	-7.15(-7.91)	-0.07(-0.06)	51	-5.20(-5.65)	-0.06(-0.07)	86	-2.04(-2.25)	-0.02(-0.03)
17	-7.04(-7.81)	-0.07(-0.07)	52	-5.10(-5.55)	-0.06(-0.07)	87	-1.96(-2.16)	-0.02(-0.02)
18	-6.96(-7.74)	-0.07(-0.07)	53	-5.00(-5.45)	-0.06(-0.07)	88	-1.88(-2.07)	-0.02(-0.02)
19	-6.92(-7.68)	-0.06(-0.06)	54	-4.91(-5.35)	-0.07(-0.07)	89	-1.80(-1.99)	-0.02(-0.02)
20	-6.91(-7.65)	-0.06(-0.06)	55	-4.81(-5.24)	-0.07(-0.07)	90	-1.73(-1.9)	-0.02(-0.02)
21	-6.92(-7.63)	-0.05(-0.06)	56	-4.72(-5.14)	-0.07(-0.07)	91	-1.65(-1.82)	-0.01(-0.02)
22	-6.94(-7.62)	-0.04(-0.05)	57	-4.62(-5.05)	-0.07(-0.07)	92	-1.58(-1.74)	-0.01(-0.01)
23	-6.97(-7.62)	-0.04(-0.05)	58	-4.53(-4.95)	-0.06(-0.07)	93	-1.51(-1.66)	-0.01(-0.01)
24	-7.01(-7.62)	-0.03(-0.05)	59	-4.44(-4.85)	-0.06(-0.07)	94	-1.44(-1.58)	-0.01(-0.01)
25	-7.04(-7.63)	-0.03(-0.04)	60	-4.35(-4.75)	-0.06(-0.06)	95	-1.37(-1.51)	-0.01(-0.01)
26	-7.07(-7.63)	-0.02(-0.04)	61	-4.26(-4.65)	-0.05(-0.06)	96	-1.30(-1.43)	0.00(-0.01)
27	-7.08(-7.62)	-0.02(-0.04)	62	-4.17(-4.55)	-0.05(-0.06)	97	-1.24(-1.36)	0.00(0.00)
28	-7.08(-7.6)	-0.01(-0.04)	63	-4.07(-4.45)	-0.05(-0.06)	98	-1.18(-1.29)	0.00(0.00)
29	-7.07(-7.58)	-0.01(-0.04)	64	-3.98(-4.35)	-0.05(-0.06)	99	-1.12(-1.22)	0.00(0.00)
30	-7.04(-7.55)	-0.01(-0.04)	65	-3.89(-4.25)	-0.05(-0.06)	100	-1.06(-1.16)	0.00(0.00)
31	-6.99(-7.5)	-0.01(-0.04)	66	-3.80(-4.15)	-0.05(-0.06)	101	-1.00(-1.10)	0.00(0.00)
32	-6.94(-7.44)	-0.02(-0.04)	67	-3.70(-4.05)	-0.05(-0.06)	102	-0.95(-1.03)	0.01(0.01)
33	-6.87(-7.38)	-0.02(-0.04)	68	-3.61(-3.96)	-0.05(-0.05)	103	-0.90(-0.98)	0.01(0.01)
34	-6.79(-7.31)	-0.02(-0.04)	69	-3.52(-3.86)	-0.04(-0.05)	104	-0.85(-0.92)	0.01(0.01)

As explained in Section 3.5, we assume an investor considers three asset classes in his investment portfolio, *i.e.*, equities, bonds, and risk-free assets.

China's stock markets were established in early 1990's. Since the establishment of the Shanghai Stock Exchange (SHSE) in 1990 and the Shenzhen Stock Exchange (SZSE) in 1991, the stock markets have expanded rapidly. We use the SSE (Shanghai Stock Exchange) Composite Index as a proxy for the equity class in our model.

In the last two decades, the Chinese bond market has grown from virtually nonexistent to the third largest in the world [9]. The major types of bonds available in the Chinese market can be grouped into four broad categories, i.e., the government bonds, the central bank notes, the financial bonds, and the non-financial corporate bonds. Among the four bond categories, the financial bonds issued by policy banks, commercial banks and other financial institutions are the most actively traded bonds in China. We select the S&P China Bond Index to represent the bond class in our model. The S&P China Bond Index is designed to track the performance of the Yuan-denominated government and corporate bonds in China. It covers a diverse collection of bonds from the Chinese sovereign bonds (23%), the policy bank bonds (32%), and the central state-owned enterprise bonds (45%). In addition, we use the 3-Month Treasury Securities in China to represent the risk-free asset.

We estimate the expected log rate of return and standard deviation of the SSE Composite Index based on the daily data from January 3, 2000 to March 5, 2015 provided by Yahoo Finance. To account for the China's bond market explosion in recent years, we estimate the parameters of the S&P China Bond Index and 3-Month Treasury Securities for China based on the daily data from January 1, 2007 to Nov 3, 2014 [10]. The correlations among these three financial assets are estimated using the data from 2007 to 2014. We present those annualized estimates in Table 2. The p-values of the Pearson correlation test are reported in the paraphyses. Table 2 also shows the assumed correlations between the returns of the three assets and the labor income for the base case.

As expected, the stock index provides the highest expected rate of return but is the riskiest among the three financial assets. The stock index is positively correlated with the bond index

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[9] Source: Bank for International Settlements, as of September 2013.

[10] The data of the S&P China Bond Index and the 3-Month Yield of Treasury Securities for China were retrieved from http://us.spindices.com/indices/fixed-income/sp-china-corporate-bond-index and http://research.stlouisfed.org, respectively, on March 17, 2015. Since the Federal Reserve Bank of St. Louis only provides monthly data for the China's 3-Month Treasury Securities, the daily data are obtained through linear interpolation of the monthly data.

TABLE 2. Assessment of Risky Assets and Labor Income

Expected Returns and Standard Deviations				
	SSE	S&P Bond China	3-M Yield	Labor Income
Annual expected return	8.55%	6.36%	3.92%	5.50%
Annual standard deviation	29.49%	4.92%	0.07%	15.00%
Correlations				
	SSE	S&P Bond China	3-M Yield	Labor Income
SSE	1			
S&P Bond China	5.64% ($p = 0.0128$)	1		
3-M Yield	-5.34% ($p = 0.0185$)	–	1	
Labor Income	0.5	0.5	0.5	1

but negatively correlated with the 3-month Treasuries for China [11]. The correlation coefficient between the bond index and the 3-month Treasuries for China is not statistically significant. We further perform a Kruskal-Wallis test and reject the null hypothesis that these two series are from the same distribution at p-value 0.001. Therefore, we assume the bond index and the 3-month Treasuries for China are independent. Furthermore, we assume the individual's real labor income grows at an expected rate of 5.5% with a volatility of 15%.[12]

4.3. **Preliminary Results.** We have obtained some preliminary results. Our numerical results show that the optimal life insurance demand for a Chinese individual is much higher than the actual premium per capita collected, indicating that Chinese households on average are seriously underinsured.[13]

5. CONCLUSION

In summary, this paper contributes to the life insurance and risk management literature in two ways. First, we propose a dynamic optimization model that explains the household underinsurance problem in China. Second, our results provide a useful reference to the Chinese households on their optimal life-cycle insurance demand and asset allocation. As the first paper that models the optimal

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[11] The Pearson test shows that both the correlation coefficients are statistically significant at 2.5% level.

[12] Based on the statistics reported by the National Bureau of Statistics (NBS), the annual growth rate of the disposable income of China's urban residents ranged 8.4%-9.7% in the last few years. Assume the average inflation rate is 3.5%. Accordingly, we set the expected growth rate of the labor income of a Chinese individual at 5.5%.

[13] In 2010, life insurance premiums paid were only $158 per capita in China. Source: http://www.forbes.com/sites/jackperkowski/2011/12/06/life-insurance-in-china/.

life insurance demand in China, it offers strong theoretical support to the continuous growth of the China's life insurance market.

REFERENCES

Brown, J. R. and J. M. Poterba (2000). Joint life annuities and annuity demand by married couples. *Journal of Risk and Insurance 67*(4), 527–554.

Cairns, A. J. G., D. Blake, and K. Dowd (2006). Pricing death: frameworks for the valuation and securitization of mortality risk. *ASTIN Bulletin 36*(1), 79–120.

Campbell, R. A. (1980). Demand for life insurance: An application of the economics of uncertainty. *Journal of Finance 35*(5), 1155–1172.

Chan, C. S.-C. (2009). Creating a market in the presence of cultural resistance: the case of life insurance in china. *Theory and Society 38*(3), 271–305.

Chen, P., R. G. Ibbotson, M. A. Milevsky, and K. X. Zhu (2006). Human capital, asset allocation, and life insurance. *Financial Analysts Journal 62*, 97–109.

Duarte, I., D. Pinheiro, A. A. Pinto, and S. R. Pliska (2011). An overview of optimal life insurance purchase, consumption and investment problems. *Springer Proceedings in Mathematics 1*, 271–286.

Durkheim, E. (1965). The elementary forms of the religious life.

Economides, N. (1982). Demand for life insurance: An application of the economics of uncertainty: A comment. *Journal of Finance 37*(5), 1305–1309.

Fang, H. (2013). *Insurance Markets in China*, Chapter The Oxford Companion to the Economics of China. Oxford University Press.

Gollier, C. (1994). Insurance and precautionary capital accumulation in a continuous-time model. *Journal of Risk and Insurance 61*(1), 78–95.

Huang, H., M. A. Milevsky, and J. Wang (2008). Portfolio choice and life insurance: The crra case. *Journal of Risk and Insurance 75*(4), 847–872.

Hwang, T. and S. Gao (2003). The determinants of the demand for life insurance in an emerging economy - the case of china. *Managerial Finance 29*, 82–96.

Hwang, T. and B. Greenford (2005). A cross-section analysis of the determinants of life insurance consumption in mainland china, hong kong, and taiwan. *Risk Management and Insurance Review 8*, 103–125.

Jeantet, T. (2012). *Outlook for health and life insurance in China for use by foreign insurers*. Ph. D. thesis, Enass, Institut du Cnam.

Lee, R. D. and L. Carter (1992). Modelling and forecasting the time series of US mortality. *Journal of the American Statistical Association 87*(419), 659–671.

Leverty, T., Y. Lin, and H. Zhou (2009). Wto and the chinese insurance industry. *The Geneva Papers on Risk and Insurance ł Issues and Practice 34*, 440–465.

Li, N. and R. Lee (2005). Coherent mortality forecasts for a group of population: An extension of the lee-carter method. *Demography 42*(3), 575–594.

Lin, Y. and M. F. Grace (2007). Household life cycle protection: Life insurance holdings, financial vulnerability and portfolio implications. *Journal of Risk and Insurance 74*, 141–173.

Mao, H., K. M. Ostaszewski, , and Y. Wang (2008). Risk analysis of mortality improvement: The case of chinese annuity markets. *The Geneva Papers on Risk and Insurance ł Issues and Practice 33*, 234–249.

Maurer, R., O. S. Mitchell, R. Rogalla, and V. Kartashov (2013). Lifecycle portfolio choice with systematic longevity risk and variable investmentłlinked deferred annuities. *Journal of Risk and Insurance 80*, 649–676.

Purcal, S. and J. Piggott (2008). Explaining low annuity demand: An optimal portfolio application to japan. *Journal of Risk and Insurance 75*(2), 493–516.

Shi, X., H.-J. Wang, and C. Xing (2015). The role of life insurance in an emerging economy: Human capital protection, assets allocation and social interaction. *Journal of Banking & Finance 50*, 19–33.

Zietz, E. N. (2003). An examination of thedemand for life insurance. *Risk Management and Insurance Review 6*(2), 159–191.