(PRELIMINARY VERSION: PLEASE DO NOT QUOTE)

Guaranteed Renewable Insurance Under Demand Uncertainty

February 9, 2016

Abstract

Guaranteed renewability is a prominent feature in health and life insurance markets in a number of countries. It is generally thought to be a way for individuals to insure themselves against reclassification risk. We investigate how the presence of unpredictable flucutations in demand for life insurance over an individual's lifetime (1) affects the pricing and structure of such contracts and (2) can compromise the effectiveness of guaranteed renewability to achieve the goal of insuring against reclassifiction risk.

JEL Codes:

1 Introduction

Guaranteed renewability is a prominent feature in health and life insurance markets. The value of guaranteed renewable (GR) insurance is that it allows individuals to insure themselves against reclassification risk. This is thought to work as follows. Consider individuals who may purchase a ten year term life insurance contract that expires at the end of the ten year period. By the end of the contract period, some insureds may have discovered that their health status has changed. If this change is observable to insurers, then the price for a new insurance contract may be either lower or higher than what would be average for the population of individuals of their age. Although individuals cannot predict their future demand for life insurance, they recognize ex ante that their risk type will evolve over time and so prefer to avoid the prospect of premium risk associated with stochastic mortality prospects. In principle, GR insurance allows individuals in an earlier period of life to avoid this risk in a subsequent period through a guaranteed renewability clause or rider. GR contracts contain a promise to offer insurance at the expiry date of the first contract at an agreed upon price or at least one that is determined without being dependent on any changes in mortality risk. The premium for the insurance against reclassification risk is embedded in the first contract (earlier period) through an extra premium assessment – a phenomenon known as front loading. This allows insurers to offer insurance to those individuals who turn out to be higher risk types in the second period at a price below their actuarially fair rate, hence providing implicit insurance against reclassification (or premium) risk. As long as the amount of front loading is sufficient, the added profit from the first (period) contract compensates for losses from the second (period) contract.

The effectiveness of GR insurance, however, is compromised if individuals do not know what their future demands for life insurance will be. Such uncertainty is natural. The amount of coverage an individual desires at any point in time is affected by a number of factors, including marital status, income, number of children, earning options for other family members. expenditure requirements for the survivor family should death of the individual occur, the insureds pure (altruistic) preferences, etc.. All of these factors can change over time. We treat the impact of all of these characteristics as determining a particular demand type. Some of these characteristics are unobservable to the insurer and others, while observable, typically have idiosyncratic and unobservable implications on individuals' preferences for insurance. For example, the extra insurance demand resulting from an additional child in a family will be idiosyncratic and unobservable to the insurer. Given that these characteristics are generally either intrinsically or pragmatically noncontractibe, we treat them as unknown to the insurer even after the insured knows their realizations. Insureds, on the other hand, learn about their demand preferences and change their valuation of insurance accordingly. This represents a challenge to individuals when deciding how much guaranteed renewable insurance (GR) is appropriate to purchase at a given point in their lifetime, which in turn compromises the ability of guaranteed renewable insurance contracts to protect consumers against reclassification risk. Moreover, given the noncontractible nature of demand risk, the combination of variations in both morality risk and demand risk creates a type of adverse selection problem, as described below.

There is a substantial literature on guaranteed renewability of insurance. One important feature of interest regarding the performance of GR insurance is contract lapsation. If the renewal terms are not sufficiently attractive to people who discover they have become relatively low risk, then they will have an incentive to opt out of the first contract at or before the expiry date and not purchase a subsequent contract at the agreed upon price. Moreover, those with low insurance demand who turn out to be high risk will wish to renew more insurance than is efficient if the renewal price is below the actuarially fair rate for high risk types. This means the second contract will have a disproportionate share of demand from high risk types which creates a stress on the degree of front loading required to make GR insurance financially sustainable.

Reasons for lapsation offered in the literature include individuals learning, perhaps imperfectly, about their mortality risk resulting in lower risk types abandoning the contract or not renewing for a second period (e.g., see Richard, Richter, Steinorth (2015). Other reasons offered include unpredictable illiquidity shocks or decision making based on behavioural models (e.g., Nolte and Schneider (2015)) which imply “irrational” decision making (relative to the expected utility model).. Our model can be thought of as a more general model of demand uncertainty that includes liquidity shocks as a source of lapsation. Besides explaining the cause of lapsation some papers (e.g., Hendel and Lizzeri (2003)) also investigate welfare implications of this phenomenon either directly or indirectly.

Our paper contributes to this literature by providing an explicit welfare analysis of a general two-period model of decision making based on expected utility preferences which may evolve over time; that is, individuals may find their preference for insurance either rises or falls for the later (second) period under consideration. They know this is a possibility ex ante – i.e., at the time of purchasing their first contract – but the realization of this demand risk does not occur until the end of the first period. Their risk type also evolves over time in a similar manner; that is, they discover their second period risk type at the end of the first period. With this model we are able to generate welfare comparisons between the first-best allocation (social optimum), the allocation achieved in a market with only spot insurance (i.e., one period contracts) available, and when guaranteed renewable insurance contracts are also available. In this way we are able to shed light on the conditions under which GR insurance is of high value to consumers relative to an environment with only spot insurance contracts available. We also show how demand type risk compromises the ability of GR insurance to improve consumer welfare relative to the social optimum as well as relative to spot insurance. This is an important exercise as alternative policies, such as a regulation requiring community rating of insurance, will in some circumstances lead to higher social welfare than can GR insurance; e.g., see Polborn, Hoy, Sadandand (2006) and Hoy (2006).

Our paper also sheds light on the role of the balance between the amount of front loading of GR insurance contracts and the second period renewal price in the construction of the optimal contract. If the second period price is set at the actuarially fair price for low risk types in the case of risk type uncertainty only (i.e., in the absence of demand type risk), then high risk types can receive full insurance against reclassification risk. However, including demand type heterogeneity in the second period leads to the possibility that individuals who are both low demand but high risk will value the renewal terms so favourably that they will renew more of their first period purchases of GR insurance than is efficient for a low demand type.

The addition of demand uncertainty leads to a variety of deviations from optimal contract design when only risk type is uncertain. Perhaps surprisingly, it is possible that the optimal contract will involve a second period renewal price that is even lower than the actuarially fair price of low risk types. Of course, in this case one requires sufficiently high loading on the first period contract in order to ensure financial sustainability for the insurer. Also perhaps surprising is that even if there is no uncertainty or variation about individuals' second period mortality risk but only uncertainty about demand type, GR insurance can improve welfare relative to availability of only spot insurance. The intuition for this result is that front loading of GR insurance allows individuals to cross subsidize themselves in the second period should they turn out to be high demand types. High demand types, by definition, have higher marginal utility for insurance purchased in period two. Therefore, it is advantageous to plan for this possibility by having subsidized insurance available.

Our paper is closest to the papers of Polborn, Hoy, Sadanand (2006), PHS, and Fei, Fluet, and Schlesinger (2013), FFS. Both papers consider the role of contract design in providing second best efficient insurance in a dynamic framework (two periods) with demand (bequest) uncertainty. The important difference between the two papers is that PHS assume zero profit insurance pricing in both periods while FFS require only zero profit aggregating across the two periods. The model of PHS includes contracts for future insurance coverage (i.e., before risk or demand type is realized) but also allows for reselling of any amount deemed to be excessive.[1] This reselling occurs after individuals know

[1] We do not investigate the implications of reselling of insurance contracts through life settlement or

their risk type and so is subject to adverse selection. Neither of those papers explicitly considers the structure of GR insurance and both ignore any explicit link between the amounts of insurance demanded in the first and second periods. In FFS, allowing for cross subsidization over time periods generates an effect that improves ex ante welfare even when only demand (bequest) uncertainty persists but no risk type differences are present. As expected, we find that front loading of GR insurance results in a similar finding, albeit in a more restrictive contracting space. Illustrating the implications of demand uncertainty in this restrictive and realistic setting provides specific insights about the specific contract form of GR insurance that neither PHS nor FFS do.

The rest of the paper is organized as follows. The next section presents the basic model, including describing the first best (social) optimum, the allocation when (only) spot insurance markets are available, and the allocation when GR insurance is also available. We summarize the results in a series of propositions. Section 3 has a discussion of simulation results that provide further insight on the implications of demand type uncertainty on optimal GR insurance contracts. Section 4 provides conclusions.

2 Model

We assume that an individual who is the insurance buyer lives at most two periods.[2] Each such individual has a family associated with him. In case of the individual's death, we refer to his associated family as the *survivor family* while in any period that he lives we refer to his associated family as the *whole family*. No other members of the family may die. Preferences relate to those of the insurance buyer, albeit as he takes his family members well-being into account. For simplicity, we assume he is the only income earner in the family and receives income y_1 at the beginning of period 1. If he survives to period 2, he receives a further y_2 at the beginning of period 2. His risk and demand type evolves over time. Each individual has a probability of death of p, $0 < p < 1$, in the first period of life. If an individual survives the first period, then his probability of death depends on whether he is a high or low risk type. We describe risk type by index $i = L, H$ for low and high risk type, respectively, with associated probabilities p_L, p_H where $0 < p_L < p_H < 1$. Moreover, we assume all risk types have a higher mortality in period 2 than in period 1 (i.e.; $0 < p < p_L < p_H < 1$).[3]

viatical markets.

[2] For simplicity one can think of this person as the main or only earner (breadwinner) of a family composed of two adults and possibly some children. The main breadwinner is modeled as the decision maker although it is of course likely such decisions are made with input from his partner (i.e., the other adult in the family) but we don't model this explicitly.

[3] See Hendel and Lizzeri (2003).

The individuals (and associated families) are homogeneous in all respects in the first period and discover their risk type associated with second period mortality at the beginning of period 2. Insurers also observe individuals' risk type and so there is no asymmetric information in this regard. However, individuals also discover their demand type at the beginning of period 2 which insurers do not observe. In period 1 individuals perceive their prospects about both risk type and demand type development according to the actual population portions of q_i, $i = L, H$ for risk type and r_j, $j = l, h$ for demand type where $i = L, H$ represents low and high risk type while $j = l, h$ represents low and high demand type. Risk and demand type are not correlated (i.e., the probability of an individual being risk type i and demand type j is $q_i \cdot r_j$. These differing preferences (demand type) for life insurance in period 2 are reflected in the felicities for death state consumption in period 2 as described below.[4]

So, period 2 decisions depend on both the individual's risk and demand type, characterized by the pair ij, with $i \in \{L, H\}$ and $j \in \{l, h\}$. In cases where confusion may occur, we index the time period and the state (life or death) using superscripts. We refer to the death state by D and the life state by N (i.e., not death). Thus, consumption in period 2 for a person of type ij is represented by C_{ij}^{2D} in the death state and C_{ij}^{2N} in the life (i.e., non-death) state. We write their felicity for consumption in the life state for period t as $u_t(\cdot), t = 1, 2$. Their felicities in the period 2 death state, which depend on demand type j are modeled by the function $\theta_j v_2(\cdot)$ with $\theta_l < \theta_h$. The functions u_2 and v_2 satisfy the usual assumptions for risk averters (i.e., $u_2', v_2' > 0$ and $u_2'', v_2'' < 0$).

Individuals have homogeneous preferences in period 1 with their felicity in the life state being $u_1(\cdot)$ and that in the death state being $v_1(\cdot)$, the latter of which is meant to reflect the insurance purchaser's perspective on the survivor family's future utility (including prospects for period 2).[5] This can naturally be different from the felicity in the death state of period 2. Similar to the above notation for period 2, consumption in the death and life states of period 1 are C^{1N} and C^{1D}, respectively. Note that since individuals do not know their demand or risk type in period 1, there is no subscript pair ij attached to these consumptions.

Timing of information revelation and taking of decisions is as follows. At the beginning

[4]Note that one could instead introduce demand heterogeneity through different feliciities in the life state. This would have similar effects as in our model. Note that such an example may be a liquidity shock associated with the life state of the world.

[5]This is an indirect utility based on how the family's circumstances will evolve should the income earning insurance buyer die in the first period. The family may be expected to evolve in the sense that a surviving spouse has uncertain prospects of generating income in period 2 (as well as the remainder of period 1) and/or becoming attached to another main breadwinner. This simplistic "main breadwinner" sort of model could be transformed to one with two earners and two potential insurance buyers. However, that would lead to a much more complicated model and, we believe, not significantly improved insights.

Figure 1: Period 1 Decisons and Outcomes <see Appendix>

Figure 2: Period 2 Decisions and Outcomes <see Appendix>

of period 1 individuals decide on the amount of spot insurance to hold for period 1 (L^1), amount of guaranteed renewable insurance (L^{1G}), and an amount of savings, s. $L^1 + L^{1G}$ is the insurance coverage in period 1 and savings is also available to the survivor family should the insured die in period 1. L^{1G} is the amount of that coverage that could be renewed at a guaranteed (predetermined) rate in the second period should the insured survive to period 2. We let π^1 be the price of first period spot insurance. We assume risk neutral insurers in a competitive environment and having no administrative costs. Thus, since coverage from first period spot insurance expires at the end of period 1, competition leads to $\pi^1 = p$ (i.e., actuarially fair insurance).

Guaranteed renewable insurance allows an individual the option to renew at a price which earns the insurer expected losses. This implies that the unit price of this coverage, π^{1G} must exceed p, the expected unit cost of providing first period insurance cover. This is explained in greater detail later. At the beginning of period 2 the spot insurance from period 1 expires. Individuals learn about their risk type (i) and their demand type (j). Insurers know the risk type of insureds but not their demand type. Each insured then chooses how much guaranteed renewable insurance that was purchased in period 1 (L^{1G}) to renew (L^{2G}_{ij}) at the predetermined (guaranteed) price of π^{2G}. This amount will depend on both risk and demand type with (obviously) $L^{2G}_{ij} \leq L^{1G}$. An insured may also purchase spot insurance (L^2_{ij}) at his risk type specific price ($\pi^2_i = p_i$). Note that if $\pi^{2G} \geq p_L$, low risk types would not renew any of their guaranteed renewable insurance from period 1.[6] Expected utility from the perspective of the beginning of period 1 is

$$EU = pv_1(C^{1D}) + (1-p)u_1(C^{1N}) + (1-p)\left[\sum_i \sum_j q_i r_j [p_i \theta_j v_2(C^{2D}_{ij}) + (1-p_i)u_2(C^{2N}_{ij})]\right] \tag{1}$$

where:

$C^{1N} = y_1 - s - \pi^1 L^1 - \pi^{1G} L^{1G}$

$C^{1D} = y_1 + s + (1-\pi^1)L^1 + (1-\pi^{1G})L^{1G}$

[6] We also assume that low risk types do not renew any of L^{1G} if $\pi^{2G} = p_L$, the spot price for low risk types in period 2. This is of no consequence since competition means π^2_L is equal to the low risk type loss probability which means the lapsation-renewal decision has no consequence on insurer profits and hence on π^{1G} or π^{2G}.

$C_{ij}^{2N} = y_2 + s - \pi_i^2 L_{ij}^2 - \pi^{2G} L_{ij}^{2G}$
$C_{ij}^{2D} = y_2 + s + (1 - \pi_i^2) L_{ij}^2 + (1 - \pi^{2G}) L_{ij}^{2G}$
with constraints
$0 \leq L^1, \ 0 \leq L^{1G}, \ 0 \leq L_{ij}^{2G} \leq L^{1G}, \quad 0 \leq L_{ij}^2$

We now explain more explicitly the timing of events and decisions for the model. This is illustrated in Figures 1 and 2 (see Appendix). A literal approach to timing would recognize that in each period, which represents say 10 years of life, the main breadwinner earns income throughout the period and could die at any point in the period (i.e., both income generation and mortality are flow variables throughout the period). We simplify the problem by assuming that income is earned at the beginning of the period (before mortality is realized) and that decisions about savings (s) and life insurance purchases (L^1 for spot and L^{1G} for guaranteed renewable in the first period) are also made at the beginning of the period. If death occurs then it does so at the end of period 1 and felicity $v_1(\cdot)$ represents the future stream of expected utility (from the breadwinner's perspective) of the survivor family. In case of death in the first period, C^{1D} is not literally the consumption of the survivor family in period 1 but rather is the income received by the survivor family to use going forward in time (i.e., including period 2 and beyond). This income includes the savings decided upon by the individual as well as life insurance payments. If the main breadwinner lives through period 1, then consumption of the "whole family" for period 1 is C^{1N} (income earned in that period minus savings and the cost of any spot and guaranteed renewable insurance purchased).[7]

If the individual survives period 1, which he does with probability $(1 - p)$, then at the beginning of period 2 income y_2 is earned by the main breadwinner and saving from period 1 is also available for consumption. Again, the main breadwinner makes insurance purchasing decisions; i.e., how much spot insurance (L_{ij}^2) and how much of the first period guaranteed renewable insurance that he bought to renew (L_{ij}^{2G}). If he dies, which happens at the end of period 2 (with probability p_i), the survivor family receives C_{ij}^{2D} which includes second period insurance payments (and this generates felicity $\theta_j v_2(C_{ij}^{2D})$). If he lives, then the whole family receives C_{ij}^{2N} and this generates felicity $u_2(C_{ij}^{2D})$. These amounts $(C_{ij}^{2D}, C_{ij}^{2N})$ are meant to reflect not just consumption for period 2 but also consumption for an implicit third period and beyond. By not formally including a third period we admittedly omit modeling any further income generation or intertemporal income transfer possibilities (e.g., from period two income for consumption in period three, et cetera).

[7] One could, admittedly, quibble with this timing presumption since it requires the same amount of savings to be carried forward in both life and death states of the world, albeit by "different families". We believe, however, that the simplification is worthwhile and the model remains both rich and a reasonable reflection of the decison making environment.

The fact that $C_{ij}^{2D} > C_{ij}^{2N}$ even though the survivor family has fewer individuals can be accounted for by imagining that in a third period (and beyond) the main breadwinner earns income at the beginning of that period which accommodates for higher consumption for the whole family than would be available for the survivor family. This can readily be captured by the state contingent felicities $\theta_j v_2(\cdot)$ and $u_2(\cdot)$. Moreover, this would also be consistent with the usual rationale for life insurance demand (i.e., loss of income due to death which would be the loss of income that would have been generated by the breadwinner at the beginning of period 3 and beyond). It seems reasonable to leave these issues aside as explicitly including additional periods would unduly complicate the model.

We first describe the first best allocation which is the solution to the problem of maximizing ex ante utility (i.e., from the perspective of individuals in period 1) subject to a per capita expected resource constraint (i.e., expected consumption over the two periods must equal expected income over the two periods). Since the breadwinner dies in period 1 with probability p, and in this case does not earn income in period 2, then expected income is $\overline{y} = y_1 + (1-p)y_2$. Expected consumption ($EC$) is straightforward (see left side of constraint below). Therefore, the first best allocation is the solution to:

$$\max EU = pv_1(C^{1D}) + (1-p)u_1(C^{1N}) + (1-p)\left[\sum_i \sum_j q_i r_j [p_i \theta_j v_2(C_{ij}^{2D}) + (1-p_i)u_2(C_{ij}^{2N})]\right] \tag{2}$$

subject to:

$$pC^{1D} + (1-p)C^{1N} + (1-p)\left[\sum_i \sum_j q_i r_j [p_i C_{ij}^{2D} + (1-p_i)C_{ij}^{2N})\right] = \overline{y} \tag{3}$$

Using λ as the multiplier, the Lagrangian is $L = EU + \lambda[\overline{y} - EC]$, and the first-order conditions are:

$$\frac{\partial L}{\partial C^{1D}} = pv_1'(C^{1D}) - \lambda p = 0 \tag{4}$$

$$\frac{\partial L}{\partial C^{1N}} = pu_1'(C^{1N}) - \lambda(1-p) = 0 \tag{5}$$

$$\frac{\partial L}{\partial C_{ij}^{2D}} = (1-p)\left[q_i r_j p_i \theta_j v_2'(C_{ij}^{2D}) - \lambda q_i r_j p_i\right] = 0, \quad i = L, H; \quad j = l, h \tag{6}$$

$$\frac{\partial L}{\partial C_{ij}^{2N}} = (1-p)\left[q_i r_j (1-p_i) u_2'(C_{ij}^{2N}) - \lambda q_i r_j (1-p_i)\right] = 0, \quad i = L, H; \quad j = l, h \tag{7}$$

It is easy to show that the first-best allocation implies allocating expected per capita income to individuals in a way that equates marginal utility of consumption in each state. Specifically this implies:

$$v_1'(C^{1D}) = u_1'(C^{1N}) = \lambda \tag{8}$$

$$\theta_j v_2'(C_{ij}^{2D}) = u_2'(C_{ij}^{2N}) = \lambda, \quad i = L, H; \quad j = l, h \tag{9}$$

That is, the first-best allocation results in both life and death marginal utilities in the second period being equated across all risk and demand types and those, in turn, are also equated to life and death marginal utilities in period 1. This implies that, for a given demand type, consumption in the period 2 death state is the same for both risk types and likewise for the period 2 life state consumption. However, consumption in the death state is higher for the high demand type than for the low demand type. This is easily established as

$$\theta_l v_2'(C_{il}^{2D}) = \theta_h v_2'(C_{ih}^{2D}) \Rightarrow \frac{v_2'(C_{il}^{2D})}{v_2'(C_{ih}^{2D})} = \frac{\theta_h}{\theta_l} > 1 \Rightarrow C_{ih}^{2D} > C_{il}^{2D} \tag{10}$$

Note that the relationship between the period 2 death state consumption levels according to demand type is independent of risk type.

Now consider the equilibrium choices of individuals when only spot insurance is available in period 2. Determining each individual's optimal consumption requires first solving the second period optimization problem for each individual conditional on risk and demand type, which is known at that point in time, conditional on a given set of first period choices (i.e., for s and L^1). We then use the value functions from the second period optimization problem to determine optimal values for decision variables relating to the first period.

Second period choice problem is, given type ij:

$$\max_{\{L_{ij}^2\}} \left[p_i \theta_j v_2(C_{ij}^{2D}) + (1 - p_i) u_2(C_{ij}^{2N})\right] \tag{11}$$

where

$$C_{ij}^{2N} = y_2 + s - \pi_i^2 L_{ij}^2 \tag{12}$$

$$C_{ij}^{2D} = y_2 + s + (1 - \pi_i^2) L_{ij}^2 \tag{13}$$

which leads to the first order condition

$$p_i \theta_j v_2'(C_{ij}^{2D})(1 - \pi_i) - (1 - p_i) u_2'(C_{ij}^{2N}) \pi_i = 0 \tag{14}$$

Assuming spot market prices are actuarially fair (i.e., $\pi_i^2 = p_i$), we have

$$\theta_j v_2'(C_{ij}^{2D}) = u_2'(C_{ij}^{2N}) \tag{15}$$

i.e., ex post efficiency prevails.

Let $Z_{ij}(s)$ be the value function relating to the second period optimization problem. Since no GR insurance is available to purchase in period 1 for potential renewal in period 2, it follows that the only decision variable from period 1 that carries over to period 2 is s. $Z_{ij}(s)$ is strictly concave.

We go back to first period to complete the description of the optimal plan.

$$\max_{\{s,L^1\}} EU = pv_1(C^{1D}) + (1-p)u_1(C^{1N}) + (1-p)\left[\sum_i \sum_j q_i r_j Z_{ij}(s)\right] \tag{16}$$

where

$$C^{1N} = y_1 - s - \pi^1 L^1 \tag{17}$$

$$C^{1D} = y_1 + (1-\pi^1)L^1 \tag{18}$$

First order conditions are:

$$\frac{\partial EU}{\partial L^1} = pv_1'(C^{1D})(1-\pi^1) + (1-p)u_1'(C^{1N})(-\pi^1) = 0 \tag{19}$$

$$\frac{\partial EU}{\partial s} = (1-p)u_1'(C^{1N})(-1) + (1-p)\left[\sum_i \sum_j q_i r_j Z_{ij}'(s)\right] = 0 \tag{20}$$

Competition ensures first period insurance is actuarially fair, $\pi^1 = p$, and so we get

$$v_1'(C^{1D}) = u_1'(C^{1N}) \tag{21}$$

$$u_1'(C^{1N}) = (1-p)\left[\sum_i \sum_j q_i r_j Z_{ij}'(s)\right] \tag{22}$$

Noting that $Z_{ij} = p_i\theta_j v_2'(C_{ij}^{2D}) + (1-p_i)u_2'(C_{ij}^{2N})$, equation (20) demonstrates that the optimal savings amount equalizes the marginal utility of consumption in the first period life state to the expected marginal utility of consumption in the second period.

We now develop the model of primary interest; that is, the one that describes actual behaviour when guaranteed renewable insurance is available. Information assumptions are the same as in the preceding model. In this case, however, in the second period the individuals hold an amount of guaranteed renewable insurance (L^{1G}) that they purchased in the first period. They may renew any amount of this ($L_{ij}^{2G} \leq L^{1G}$) in period 2 at the predetermined price π^{2G}. Individuals may also purchase spot insurance in period 2 (L_{ij}^2) which, since insurers also observe risk type, is priced at the risk type specific actuarially fair price (π_i^2). Determining each individual's optimal consumption requires first solving the second period optimization problem for each individual conditional on risk and demand type, which is known at that point in time, based on a given set of first period choices (i.e., for s, L^1, and L^{1G}). We then use the value functions from the second period optimization problem, Z_{ij}, to determine optimal values for decision variables relating to the first period.

Second period choice problem is, given type ij:

$$\max_{\{L_{ij}^2, L_{ij}^{2G}\}} \left[p_i\theta_j v_2(C_{ij}^{2D}) - (1-p_i)u_2(C_{ij}^{2N})\right] \tag{23}$$

where

$$C_{ij}^{2N} = y_2 + s - \pi_i^2 L_{ij}^2 - \pi^{2G} L_{ij}^{2G} \tag{24}$$

$$C_{ij}^{2D} = y_2 + s + (1-\pi_i^2) L_{ij}^2 + (1-\pi^{2G}) L_{ij}^{2G} \tag{25}$$

$$L_{ij}^{2G} \leq L^{1G} \tag{26}$$

We denote the multipliers for each type pair's constraint by λ_{ij}. The first order condition with respect to the choice variables are:

$$L_{ij}^2 : p_i \theta_j v_2'(C_{ij}^{2D})(1-\pi_i^2) - (1-p_i) u_2'(C_{ij}^{2N}) \pi_i^2 = 0 \tag{27}$$

$$L_{ij}^{2G} : p_i \theta_j v_2'(C_{ij}^{2D})(1-\pi^{2G}) - (1-p_i) u_2'(C_{ij}^{2N}) \pi^{2G} - \lambda_{ij} = 0 \tag{28}$$

$$\lambda_{ij}(L^{1G} - L_{ij}^{2G}) = 0 \tag{29}$$

For the scenario with only spot insurance available, the resource constraints are trivial. That is, spot insurance is actuarially fair in each period which means $\pi^1 = p$, since individuals have the same first period mortality risk. In period 2 the spot market price is $\pi_i^2 = p_i$, $i = L, H$, since insureds observe risk type. There is an additional resource constraint for GR insurance since front period loading must be sufficient to cover any second period costs associated with high risk types renewing at a rate that is more favourable than their actuarially fair rate (i.e., for $\pi^{2G} < p_H$). The extent to which the first period contract must be front loaded (i.e., the difference $\pi^{1G} - p$) depends on the extent to which the renewal price falls below the actuarial cost of providing H types with insurance as well as the amount of L^{1G} that is purchased and available for renewal by high risk types of both low and demand type. Although insureds who turn out to be low demand types but are of high risk type may not renew all of L^{1G}, they have an incentive to renew more than would a low demand type who is also of low risk type since the price is more favourable to them. This means that low demand types who are high risk types typically end up with more second period insurance coverage than their low demand - low risk counterparts.[8] From the characterization of the social optimum, we know this cannot be efficient and so insureds would prefer contracts that are designed so this does not happen. However, once a person knows he is of high risk type, he cannot "resist" renewing more insurance than is necessarily efficient even though, from the ex ante perspective, everyone would like to prevent such an outcome. This "over renewal" by $Hl - types$ creates undesirable adverse selection costs which must be covered by a combination of increasing the front loading or the second period renewal price compared to what would be required if such inefficient

[8] For this to happen depends on both how different is the desired demand of these two types of individual as well as on how much GR insurance they hold L^{1G} entering the second period.

behaviour could be controlled. The following equation explains this additional resource constraint which ensures zero expected profits for insurers offering GR insurance.

$$\pi^{1G}L^{1G} = pL^{1G} + \sum_{ij} q_i r_j (p_i - \pi^{2G}) L_{ij}^{2G}$$

Note that the LHS of this equation represents the total revenue from sales of GR insurance in period 1. The first term of the RHS is the expected cost of insurance claims of GR insurance in period 1 while the second term is the sum of net expected costs of claims from all possible risk and demand types who pay π^{2G} to renew amount L_{ij}^{2G} of their holdings of GR insurance. In the case of $\pi^{2G} \geq \pi_L$, it follows that low risk types of either demand type do not renew any of L^{1G} since the spot price is more favourable (i.e., $L_{Lj}^{2G} = 0$). Using this result and performing a little algebra yields

$$\pi^{1G} = p + q_H(p_H - \pi^{2G}) \sum_j r_j \frac{L_{ij}^{2G}}{L^{1G}} \tag{30}$$

There are several important points regarding this constraint with some admittedly obvious. Firstly, the amount of front loading per contract, as measured by the difference $\pi^{1G} - p$, is increasing with the (average) fraction of GR insurance holdings from the first period that is in fact renewed in the second period by high risk types. It is also increasing in the amount of the effective cross subsidy $(p_H - \pi^{2G})$ to high risk types.[9] An increase in π^{1G} will affect the demand for GR insurance (L^{1G}) and so affect the amount of front loading that is required through the ratio $\frac{L_{ij}^{2G}}{L^{1G}}$. L^{1G} is also naturally a function of π^{2G} since GR insurance is more attractive the lower is its renewal price. This means that the way to control adverse selection problems arising from those who become low demand but high risk is not simply through increasing the renewal price as changing in both prices π^{1G} and π^{2G} affects the desirability of GR insurance in opposing ways.

It turns out that many possible configurations of variable combinations in the above equation can be part of an optimal GR contract. We demonstrate through simulations these possibilities. However, it is useful to see the first-order conditions and a characterization of an equilibrium from a welfare perspective for one such configuration. We do this below for the special case where the optimal contract is such $\pi^{2G} \geq p_L$ so that no low risk type person, whether of high or low demand type, renews any GR insurance.[10] In this case the optimal renewal price is

---

[9] Clearly, there will be no market if the renewal price equals or exceeds the actuarially fair cost of insurance of high risk types (i.e., if $\pi^{2G} \geq p_H$).

[10] Although the possibility of adverse selection due to low demand types who are also high risk types may compromise the efficiency of GR insurance, this can also be controlled by sufficiently high front loading that L^{1G} is low enough that even if it is entirely renewed by Hl types, this full renewal in itself does not create an efficiency problem and it may even be the case that $\pi^{2G} < p_L$ is part of an optimal GR insurance contracting scenario. The reason is that by setting π^{1G} sufficiently high blunts the incentive to purchase a large amount of GR insurance (L^{1G}) in the first place.

For type $i = L$, $L_{ij}^{2G} = 0 \Rightarrow \lambda_{ij} = 0$ and $\pi_i^2 = p_L$. Therefore we have:

$$\theta_j v_2'(C_{ij}^{2D}) = u_2'(C_{ij}^{2N}) : \quad \text{for types } Ll \text{ and } Lh \tag{31}$$

This follows since low risk types are offered the price p_L which is at least as low as the price of guaranteed renewable insurance carried over from period 1, and so they buy only spot insurance in period 2.

Assume that among those who discover they are high risk types, the high demand types renew 100% of their first period GR insurance (as will be shown later must be the case), in which case $\lambda_{Hh} = 0$. Those H-types who turn out to be low demand types may insure only some of their first period GR insurance and so $\lambda_{Hl} \geq 0$ with equality only if they renew all of it. Therefore, it follows that

$$\theta_j v_2'(C_{ij}^{2D}) \geq u_2'(C_{ij}^{2N}) : \quad \text{for types } Hj, j = h, l \tag{32}$$

We write value functions (indirect utilities) from this exercise as $Z_{ij}(L^{1G}, s; \pi^1, \pi_i^2, \pi^{2G})$. It follows that

$$\frac{\partial Z_{ij}}{\partial L^{1G}} = 0 \;\text{ for }\; Ll \;\text{ and }\; Lh, \;\; \lambda_{Hj} \text{ for } Hl \;\text{ and } Hh \tag{33}$$

We now turn our attention to the period 1 optimization problem to complete the description of the optimal plan.

$$\max_{\{s, L^1\}} EU = pv_1(C^{1D}) + (1-p)u_1(C^{1N}) + (1-p)\left[\sum_i \sum_j q_i r_j Z_{ij}(\cdot)\right] \tag{34}$$

where

$$C^{1N} = y_1 - s - \pi^1 L^1 - \pi^{1G} L^{1G} \tag{35}$$

$$C^{1D} = y_1 + (1-\pi^1)L^1 + (1-\pi^{1G})L^{1G} \tag{36}$$

First order conditions are:

$$\frac{\partial EU}{\partial L^1} = pv_1'(C^{1D})(1-\pi^1) - (1-p)u_1'(C^{1N})\pi^1 = 0 \tag{37}$$

$$\frac{\partial EU}{\partial L^{1G}} = pv_1'(C^{1D})(1-\pi^{1G}) - (1-p)u_1'(C^{1N})\pi^{1G} + (1-p)\left[\sum_j q_H r_j \lambda_{Hj}\right] = 0 \tag{38}$$

$$\frac{\partial EU}{\partial s} = -(1-p)u_1'(C^{1N}) + (1-p)\left[\sum_i \sum_j q_i r_j Z_{ij}'(s)\right] = 0 \tag{39}$$

Note that $Z_{ij}'(s) = [p_i \theta_j v_2'(\cdot) + (1-p_i)u_2'(\cdot)]$. First period insurance being actuarially fair means $\pi^1 = p$ and so, assuming $L^1 > 0$, which we refer to as case 1A, we get

$$v_1'(C^{1D}) = u_1'(C^{1N}) \tag{40}$$

$$v_1'(C^{1D}) = u_1'(C^{1N}) = \left[\sum_i \sum_j q_i r_j [p_i \theta_j v_2'(C_{ij}^{2D}) + (1 - p_i) u_2'(C_{ij}^{2N})]\right] \tag{41}$$

Noting that $Z_{ij} = p_i \theta_j v_2'(C_{ij}^{2D}) + (1-p_i) u_2'(C_{ij}^{2N})$, equation (39) demonstrates that the optimal savings amount equalizes the marginal utility of consumption in the first period life state to the expected marginal utility of consumption in the second period.

Note also that it is possible that an individual's purchase of L^{1G} may exceed first period insurance demand (i.e., $L^1 = 0$). In this case, we would have $v_1'(C^{1D}) \neq u_1'(C^{1N})$. In this case, we would not even have efficiency in first period state contingent consumptions. It also follows that second period allocations would also not be ex post efficient (i.e., for that period). Some of the possible configurations, including analysis of renewal (or lapsation) decisions by the various types as well as intertemporal pricing patterns for optimal GR insurance contracts are described in the following section using simulation results. First, we collect the important results from this section and place them into the following Propositions.

Proposition 1. *Characterization of Social optimum*

The socially optimal allocation requires satisfaction of the following conditions:

- Marginal utilities in all time/state contingent scenarios are equal across all risk and demand type combinations.
- Consumption in the life state or death state for both time periods should be independent of risk type.
- The period two consumption level for high demand types is higher than for low demand types (but independent of risk type, as noted above).

Proposition 2. *Characterization of Allocation Under Spot Insurance Only*

If the only markets for insurance in both periods is spot insurance, then it follows that:

- Ex post efficiency (in period 2) prevails; that is, for a given risk type, demand type combination, marginal utility of consumption in the death state is equal to marginal utility in the life state.
- Consumption in the life and death states in period 2 are **not** independent of risk type. Conditional on a given demand type, high risk types have lower consumption in both life and death states of the world than do low risk types. (This follows from the fact that high risk types face a higher price of insurance.)
- The period two consumption level for high demand types of a given risk type is higher than that for low demand types.

Proposition 3. *Characterization of Allocation with Guaranteed Renewable Insurance Available*

If there are markets for both spot and guaranteed renewable insurance, then it follows that

- Ex post efficiency (in period 2) will not generally prevails. In particular, marginal utility in the death state may exceed marginal utility in the life state for high demand types who are also low risk types.

- Consumption in the life and death states in period 2 are **not** necessarily independent of risk type. Conditional on a given demand type, high risk types may have lower consumption in both life and death states of the world than do low risk types. (This follows if second period spot purchases are nonzero due to the fact that high risk types face a higher spot price of insurance.)

- The period two consumption level for high demand types of a given risk type is at least as high as that for low demand types.

Upon comparing Propositions 1 and 2, it appears that there are at least as many tendencies towards inefficiency when GR insurance is available compared to the situation in which only spot markets are available. However, the presence of GR insurance allows for individuals who turn out to be high risk types to obtain some insurance coverage at a price below the actuarially fair rate. This ameliorates the inefficiency of reclassification risk (i.e., the pushing apart of consumption levels of any given demand type in both states of period 2 due to risk based pricing in period 2 spot markets). However, GR insurance may also lead to the phenomenon that low demand types who are also high risk types will renew so much of their GR insurance that they end up with greater consumption in the death state of period 2 than that of low demand but high risk types. This reflects a type of ex post inefficiency (see the second statement of Proposition 1). Nevertheless, it is always the case that the presence of GR insurance enhances ex ante efficiency relative to the scenario of only spot insurance availability since the incentive for designing the GR insurance is to make it as attractive as possible from an ex ante perspective. If it were not welfare improving to offer GR contracts, the optimal design of the contracts would be priced to exclude it from the market.

3 Simulations

We develop a set of simulations in order to demonstrate the types of properties of GR insurance when both risk and demand uncertainty persist and to investigate conditions under which availability of GR insurance does significantly better in terms of improving social welfare compared to the presence of spot insurance only. We adopt CRRA felicities which are varied according to time and state through use of a multiplicative constant. In particular, we have:

3.1 Period 1 Felicities

$u_1(C^{1N}) = \frac{1}{1-\beta}(C^{1N})^{1-\beta}$
$v_1(C^{1D}) = \alpha^D u_1(C^{1D})$

3.2 Period 2 Felicities

$u_2(C^{2N}_{ij}) = \frac{1}{1-\beta}(C^{2N}_{ij})^{1-\beta}$
$v_2(C^{2D}_{ij}) = \theta_j \frac{1}{1-\beta}(C^{2D}_{ij})^{1-\beta}$, $\theta_h > \theta_l > 1$

3.3 Common Assumptions

$p = 0.08$; $y_1 = y_2 = 100$
CRRA utility with $\beta = 2$
$\alpha^D = 8$

All of the cases described below contain the common assumptions. In each case, only the values of parameters that differ from the previous case are specified.

Case 1: Demand Difference Only

$\theta_l = 1.2$, $\theta_h = 20.0$; $p_L = p_H = 0.10$; $r_l = r_h = 0.5$ (i.e., 50% each of high and low demand types)

Case 2: Demand and Risk Differences

$p_L = 0.10$ and $p_H = 0.15$
$q_L = 0.80$, $q_H = 0.2$ (i.e., 80% low risk types, 20% high risk types).

Case 3: Risk Differences (Large) Only

$\theta_l = \theta_h = 20$; $p_l = 0.10$, $p_h = 0.50$;
$q_L = 0.90$; $q_H = 0.1$

Case 4: Demand and Risk Differences (Large)

$\theta_l = 1.2$, $\theta_h = 20.0$; $p_l = 0.10$, $p_h = 0.50$;
$r_l = r_h = 0.5$; $q_L = 0.90$, $q_H = 0.1$.

We compute the first-best optimal allocation for each case and report the results (period-state-contingent consumption levels) in Table 1. We also generate the compensating variation (CV) for each market outcome for each case.[11] The CV values reflect the extent to which efficiency is compromised relative to the social optimum for each scenario of No Insurance, Spot Markets Only, and GR Insurance available (along with spot insurance). These results are reported in Table 2. This allows us to compare how close to the social optimum each market scenario achieves. Note that the CV values represent loss of efficiency relative to the social optimum and so the lower is the CV value, the better is the market outcome. The case of No Insurance is useful as a benchmark by which we can determine how well each insurance market setting does in reducing the welfare loss due to mortality risk. Note that, given $y_1 = y_2 = 100$, the CV values describe the loss of welfare due to mortality risk in the various market scenarios as a percentage of a person's annual income. Tables 2 and 3 summarize those results and also include relevant information about the GR Insurance contracts (initial price for coverage and renewal price).

Table 1

Social Optimum (Part 1)

Case	C^{1N}	C^{1D}	C^{2N}_{ij}	C^{2D}_{ilow}	C^{2D}_{ihigh}
1. DD Only	86	243	86	94	385
2. DD/RD I	85	242	85	94	382
3. RD Only	76	216	76	341	341
4. DD/RD II	84	236	84	91	374

NOTE: For 3. RD Only and 4. DD/RD II, $p_H = 0.5$ (LARGE)

From Table 1 we see that it is optimal for individuals to augment their consumption in the period 1 death state by a factor of approximately 1.8 times their consumption in the life state. For the high demand types in the various scenarios it would be socially optimal for individuals to augment their consumption in the period 2 death state by a factor of approximately 3.5 relative to the life state. With spot insurance only available, we find individuals come reasonably close to first period optimal state contingent consumption

[11] This value is computed by subtracting the amount CV from the socially optimal level of each period-state-contingent conumption and solving implicitly by setting the resulting expected utility equal to the expected utility obtained under each market outcome.

by purchasing insurance of (roughly) amount 1.2 to 1.7 times their first period income depending on the relative (average) importance and cost of insurance that they "forecast" for period 2.[12] This is not surprising since individuals have homogeneous tastes, income, and mortality risk in period 1.

The allocation under spot insurance differs significantly from the social optimum when risk differences are large (i.e., Cases 3 and 4). In particular, consumption in the period 2 death and life states for individuals who are both high demand and high risk types is only about 50% the level of that for individuals who are high demand but low risk types. These pairs of consumption levels are the same (i.e., independent of risk type) in the social optimum. The divergence under spot insurance is due to the associated income effect created by the higher cost of second period life insurance for high risk types. These results highlight the problem of reclassification risk from a welfare perspective.

In Table 2, we repeat the consumption levels for the socially optimal allocation and report the amount of guaranteed renewable insurance that is purchased in period 1 (i.e., L^{1G}). In all reported cases, individuals purchase a substantial amount of this insurance (approximately 1.6 times their income) and meet all of their first period insurance needs through their GR insurance purchases. Purchasing GR insurance in period 1 not only serves their first period insurance needs but also offers some protection against reclassification risk for period 2 insurance. As a result, for the cases with $p_H = 0.5$ (i.e., p_H large), individuals essentially overinsure for period 1 (relative to the social optimum), ending up with consumption in the period 1 death state of 245 compared to the social optimum of 216 in the case of risk differences only and have period 1 death state consumption of 251 compared to the social optimum of 236 in the case with both demand differences and risk differences. In the case of risk differences only, the loss of efficiency is relatively low at 0.6% while in the case of risk and demand differences, it is 2.4%. In both cases there is a loss of efficiency in that individuals hold more than the socially optimal amount of first period insurance in order to provide protection against reclassification risk in period 2. In both cases the choice of L^{1G} is 168.

In the case of both risk and demand differences, the high risk but low demand types renew significantly more of their first period GR insurance (90 units) than is socially optimal: that is, Hl types end up with period 2 death state consumption of 172 compared to the socially optimal amount of 92 units. This overconsumption of insurance is due to the fact that high risk - low demand types value the GR insurance more highly than their demand type "warrants" due to the relatively attractive renewal price of 0.21 per unit of

[12] In case 3 with risk differences only, all individuals are high demand types and so in that scenario people shift more income from period 1 to period 2 due to a greater expectation of having high need for life insurance (i.e., having a high demand with probability 1).

insurance (i.e., compared to their actuarially fair price of 0.50). Note that low risk types of either demand type do not renew any of their first period GR insurance since the price is not attractive for them ($\pi^{2G} = 0.21$ compared to $\pi_L = 0.10$). In this sense, we have an adverse selection phenomenon with high risk types (of low demand type) purchasing more insurance than is efficient at a "subsidized" price and this has a spoiling effect on the market for GR insurance.

Table 2

Social Optimum and Efficiency Loss under GR Insurance

	C^{1N}	C^{1D}	C^{2N}_{ij}	C^{2D}_{ilow}	C^{2D}_{ihigh}	L^{1G}	L^{2G}_{Hl}	π^{1G}	π^{2G}	CV
1. DD Only	86	243	86	94	385	158	-	0.09	0.08	0.8
2. DD/RD I	85	242	85	94	382	157	53	0.097	0.08	1
3. RD Only	76	216	76	341	341	168	-	0.13	0.085	0.6
4. DD/RD II	84	236	84	91	374	168	90	0.10	0.21	2.4

NOTE: For 3. RD Only and 4. DD/RD II, $p_H = 0.50$ (LARGE)

$p = 0.8$; $p_L = 0.10$ in all cases

Table 3

Comparing Insurance Regime Efficiency (CV loss)

	No Insc	Spot Only	GR Insc
DD Only	27	1	0.8
DD/RD I	28	1.3	1
RD Only	38	4.2	0.6
DD/RD II	31	4.5	2.4

For 3. RD Only and 4. DD/RD II, $p_H = 0.50$ (LARGE)

In Table 3 we see that for all the cases, having no insurance available leads to a reduction in welfare equivalent to about two to three percent of income for each year/state. With demand differences only, availability of both Spot Insurance and GR Insurance reduce this loss substantially, with GR Insurance adding little performance value to the case of Spot Insurance Only. The same is true with case 2 (Demand/Risk Diff I) where the difference in risk is relatively moderate ($p_l = 0.10$ and $p_h = 0.15$). However, when there are only risk differences (case 3) and they are relatively high ($p_l = 0.10$ and $p_h = 0.50$, although there are only 10% of type H), *Spot Insurance Only* leads to a less substantial improvement in welfare than in the other cases while GR insurance still performs well - in fact, about an order of magnitude lower loss of welfare than *Spot Insurance Only.* The improvement of GR Insurance over Spot Insurance is relatively modest in the cases with both demand and risk type heterogeneity. These results are suggestive that the extent to which GR Insurance improves welfare more than Spot Insurance is not as large when demand differences are present. Of course, this may not more generally be the case as the relative performance of the two market scenarios will depend on all parameters involved.

It is interesting to dig a little deeper to understand just how GR insurance offers welfare improvements over spot insurance in the various cases of demand differences only, risk differences only, and instances with both types of heterogeneity. Consider the case for demand type differences only. There is, of course, no role of GR insurance to play in reducing reclassification risk (i.e., to smooth consumption across individuals who become different risk types). However, people who become higher demand types end up with a higher (average) marginal utility of consumption in period 2. Therefore, if period 2 insurance purchases are effectively subsidized through GR insurance purchased in period 1 which is then renewable in period 2 at a price lower than the actuarially fair price (for all homogeneous risk types), then consumption that delivers higher marginal utility can be enhanced. This is seen by comparing the outcomes in Case 1 under *Spot Insurance Only* and under *GR Insurance.* Under GR Insurance, L^{1G} is front loaded ($\pi^{1G} = 0.09$ while $p = 0.08$); that is, the price exceeds the actuarial cost of first period coverage. The renewal price of $\pi^{2G} = 0.08$ is less than the actuarial cost of second period insurance, which is $p_L = 0.10$. The result is that under GR insurance, individuals who are high demand types, and so have relatively high marginal utility of consumption, end up with period 2 consumption in the death state of $C^{2D}_{Lh} = 342$ while in the case of Spot Insurance Only, they end up with consumption of only $C^{2D}_{Lh} = 338$. This is a modest move in the direction of the socially optimal allocation of $C^{2D}_{Lh} = 385$. (Note: we use L to index the single risk type in this case.) GR insurance does not provide a perfect solution in that individuals who end up being low demand types have an incentive to renew too much insurance since the renewal price is below the actuarially fair price for them as well. This has a spoiling effect on the market for GR insurance as low demand types end up holding (renewing) too much of their first period GR insurance holding. In fact, under GR insurance these low demand types consume $C^{2D}_{Ll} = 121$ while the socially optimal level is $C^{2D}_{Ll} = 94$. This is a rather different type of adverse selection phenomenon that the customary one in that it occurs when the population of insureds are all of the same risk type. It is low demand types who are overinsuring rather than high risk types.

We described earlier how spot insurance naturally has no ability to provide protection against reclassification risk and so high risk types end up with significantly lower consumption in the (period 2) death state than do low risk types. These consumption levels are equal in the socially optimal allocation (i.e., full insurance against reclassification risk is socially and ex ante individually optimal). In case 3 (i.e., only risk differences which are "large"), GR insurance provides for substantial welfare gains compared to *Spot Insurance*

Only . The first-best allocation is not achieved in this case because to hold full protection against reclassification risk would require holding more GR insurance in period 1 than is optimal. Also, under GR insurance, the optimal renewal price is below the actuarial cost even for low risk types ($\pi^{2G} = 0.085$ while $\pi_L = 0.10$). Having such a low renewal price enhances the ability of GR to cross-subsidize and provide higher consumption to those with high marginal utility (i.e., individuals who are high risk type). But this also leads to overconsumption by people who are low risk types. (NOTE: In this case individuals are of the same demand type which we denote with the index h.) This is demonstrated by the result in Case 3 that the first-best level of consumption in the period 2 death state is independent of risk type with $C_{Lh}^{2D} = C_{Hh}^{2D} = 342$ while under GR insurance we have $C_{Lh}^{2D} = 349$. High risk types end up with $C_{Hh}^{2D} = 281$ which, although substantially less than the first-best level, is substantially more than under *Spot Insurance Only* ($C_{Hh}^{2D} = 189$). In this example, therefore, GR Insurance provides partial insurance against reclassification risk compared to having only spot insurance available. We also see that there is an income effect that leads low risk types to end up with more consumption in the death state than is first-best efficient (i.e., $C_{Lh}^{2D} = 349$ versus 342.) This income effect arises because both low and high risk types are active on the spot market in this example and this favours low risk types.

Finally, we see that in a scenario with heterogeneity of both demand and risk type (Case 4), GR Insurance may perform rather weakly, although still better than having only spot markets available. Introducing demand type differences (Case 4) into the scenario of only risk type differences (Case 3) leads to a worsening of efficiency from a loss of 0.6% in Case 3 to 2.4% in Case 4. The efficiency loss under *Spot Insurance Only* is approximately the same in these two cases. The reason GR insurance looses some of its advantage when demand type differences are present is that individuals who turn out to be both high risk but low demand type face a very favourable renewal price ($\pi^{2G} = 0.21$ compared to $\pi_H = 0.50$). These individuals end up renewing substantially too much of their GR insurance with the result that their second period death state consumption is $C_{Hl}^{2D} = 172$ while the first-best level is $C_{Hl}^{2D} = 112$. This is a sort of "normal" adverse selection (i.e., high risk types ending up being over-insured). Note, however, that it is the presence of low demand types who are also high risk types that creates this sort of adverse selection and not simply the presence of high risk types who are assessed the same (renewal) price as are low risk types.

4 Conclusions

We have developed a model with two periods in order to investigate the implications of unobservable (or at least noncontractible) demand type which evolves over time along with risk type. Individuals do not know in advance (of period 2) which risk or demand type they will become. The availability of GR insurance cannot provide full insurance against reclassification risk because individuals cannot determine the appropriate amount to purchase. Some GR insurance will typically be purchased and this may provide some protection against being classified as a high risk type. However, those who turn out to be high risk but low demand types will renew more of their GR insurance than is efficient and create an adverse selection problem at the renewal stage. This must be taken into account when insurers choose the degree of front loading to charge for the first period of the contract. Propositions based on the theoretical model demonstrate that the first-best allocation cannot be achieved as a result of the evolving demand uncertainty, realizations of which only insureds can observe. Our simulations demonstrate the effect of adverse selection in the second period renewal market with those who are low demand but high risk renewing more than is efficient and creating upward pressure on the price and effectiveness of GR insurance.

We demonstrate through use of simulations that, as explained in FFH, GR insurance can be effective in smoothing consumption across demand types and so can improve welfare even if there is no reclassification risk (i.e., individuals are of homogeneous risk type). The source of the welfare improvement is that first period purchases of GR with favourable (subsidized) renewal terms allows for shifting second period death-state consumption towards those with higher marginal utility of consumption (i.e., towards high demand types). The use of GR insurance, however, to smooth over risk or demand type is compromised if first period insurance needs are lower than second period insurance needs for high demand types. There are elements of adverse selection in either case of persistence of one or the other of demand or risk type. In the former case, it is the incentive for low demand types to renew too much GR insurance that is the source of adverse selection. Moreover, there is greater pressure or opportunity for adverse selection in a model with both types of heterogeneity. If the renewal price is significantly below the actuarially fair price for high risk types, which may be part of an optimal (second-best) GR insurance contract, then individuals who discover they are high risk but low demand may end up with an incentive to become substantially over-insured and so significantly spoiling the market.

It is important to recognize that the addition of GR insurance to spot insurance does enhance welfare. This should not be taken for granted. Given that the market setting is one of second-best due to the noncontractible nature of demand type heterogeneity, even optimal contract design of this additional market would not obviously lead to improved welfare. Other innovations in such a setting can compromise welfare as argued in PHS and FFH who show that introducing life settlement contracts or viatical contracts does not generally lead to a welfare improvement in environments such as the one we have designed. However, optimal GR insurance contracts do not create such entanglements with spot insurance since first-period pricing allows for constraining the amount of GR insurance available and hence limits - although does not remove - incentives that create adverse selection.

References

Fei, Wenan, Claude Fluet, Harris Schlesinger (2015), "Uncertain Bequest Needs and Long-Term Insurance Contracts," The Journal of Risk and Insurance, vol. 82, no. 1, pp. 125-148.

Hendel, I., and A. Lizzeri (2003), "The Role of Commitment in Dynamic Contracts: Evidence From Life Insurance, Quarterly Journal of Economics, vol. 118, no. 1, pp. 299-328.

Hoy, Michael (2006), "Risk Classification and Social Welfare," Geneva Papers on Risk and Insurance: Issues and Practice, vol. 31, no. 2;, pp. 321-336.

Nolte, Sven, Judith C. Schneider (2015), "Don't lapse into temptation: A Behavioral explanation for policy surrender," unpublished mimeo, pp. 41

Peter, Richard, Andreas Richter, Petra Steinorth (2015), "Yes, No, Perhaps? Premium Risk and Guaranteed Renewable Insurance Contracts with heterogeneous Incomplete Private Information," The Journal of Risk and Uncertainty, forthcoming, DOI: 10.1111/jori.12064.

Polborn, Mattias K., M. Hoy, A. Sadanand (2006), "Advantageous Effects of Regulatory Adverse Selection in the Life Insurance Market," The Economic Journal, pp. 327-354.

Figure 1

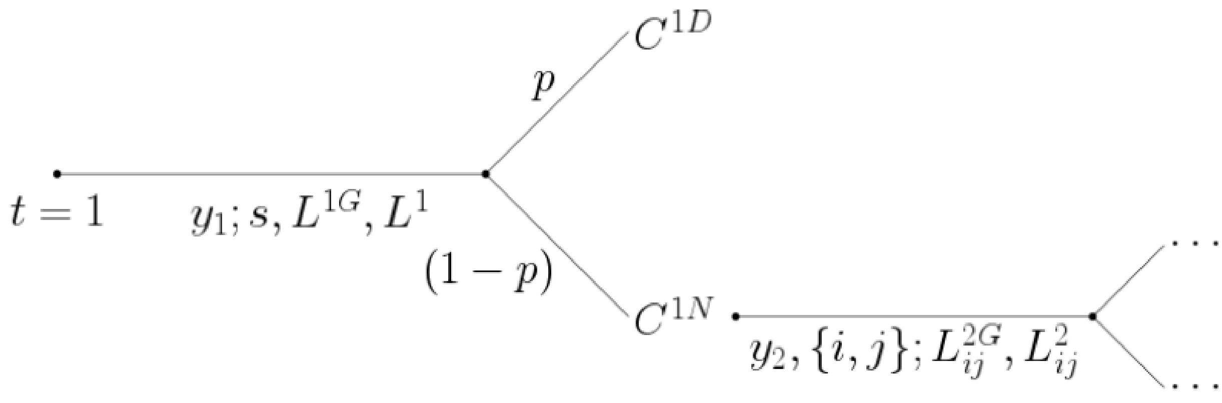

Figure 2