

PENSION RISK MANAGEMENT WITH FUNDING AND BUYOUT OPTIONS

ABSTRACT

There has been a surge of interest in recent years from defined benefit pension plan sponsors in de-risking their plans with strategies such as “longevity hedges” and “pension buyouts” (Lin et al., 2015). While buyouts are attractive in terms of value creation, they are capital intensive and expensive, particularly for firms with underfunded plans. The existing literature mainly focuses on the costs and benefits of pension buyouts. Little attention has been paid to how to capture the benefits of de-risking within a plan’s financial means, especially when buyout deficits are significant. To fill this gap, we propose two options, namely a *pension funding option* and *pension buyout option*, that provide financing for both underfunded and well funded plans to cover the buyout risk premium and the pension funding deficit, if a certain threshold is reached. To increase market liquidity, we create a transparent pension funding index, calculated from observed capital market indices and publicly available mortality tables, to determine option payoffs. A simulation based pricing framework is then introduced to determine the prices of the proposed pension options. Our numerical examples show that these options are effective and economically affordable. Moreover, our sensitivity analyses demonstrate the reliability of our pricing models.

Keywords: defined benefit pension plan, risk management, pricing, funding options, buyout options.

1. INTRODUCTION

A parade of bad news, from unprecedented market swings to sustained declines in interest rates, has caused double-digit losses of many defined benefit (DB) plan sponsors in several years of the last decade. Unanticipated improvements in mortality rates increase pension liabilities (Cox et al. (2006); Cox and Lin (2007); Lin and Cox (2008); Cox et al. (2010); Milidonis et al. (2011); Cox et al. (2013); Lin et al. (2013)). While the investment experience was favorable in 2014, due to a decline in interest rates, it was not enough to offset pension liability increases. In January 2015, the Milliman 100 Pension Funding Index (PFI) decreased to 79.6%, down from 83.5% in December 2014 (Milliman, 2015). The PFI is based on the 100 largest corporate DB pension plans in the United States. Pension funding deficits increase volatilities of corporate earnings, balance sheets, and cash flows and reduce share value (Bunkley, 2012). As a result, there has been a surge of interest from DB plan sponsors to de-risk their pensions with strategies such as longevity hedges and pension buyouts in recent years (Lin et al., 2015). A longevity hedge, such as a longevity swap, allows a pension plan to transfer its high-end longevity risk to a third party. In contrast, a pension buyout involves purchasing annuities from an insurance company. A buyout allows a firm to offload all pension liabilities from its balance sheet.

Compared with longevity hedges, pension buyouts are more effective in improving firm value. Longevity hedges only transfer extreme longevity risk and retain most of pension risk, so they prevent a firm from taking full advantage of pension de-risking. Pension buyouts, on the other hand, transfer the entire pension risk including investment risk, interest rate risk and longevity risk. Thus, pension buyouts provide more freedom for a firm, within its risk tolerance, to take on more risky projects with high positive net present values. Consistent with this observation, Lin et al. (2015) find that in the enterprise risk management framework, buyouts create more value than longevity hedges.¹

While buyouts are attractive in terms of value creation, they are capital intensive and relatively expensive. Mercer uses up-to-date pricing information to estimate the approximate costs of pension buyouts in four countries: U.S., U.K., Ireland and Canada. In December 2014, the price of a buyout

¹Enterprise risk management (ERM) assesses all enterprise risks and coordinates various risk management strategies in a holistic way (Lin et al., 2015). ERM is likely to create value because it integrates all risk factors and ensures individual decisions handling idiosyncratic risks compatible with a firm's overall risk appetite and global corporate agenda (Lam, 2001; Liebenberg and Hoyt, 2003; Nocco and Stulz, 2006).

annuity transaction across these four countries was 14% higher on average than the equivalent accounting liability (i.e. projected benefit obligation (PBO)) based on FASB Accounting Standard ASC 715 (Mercer LLC, 2014). This buyout cost estimate is based on the assumption that a plan consists of retirees only with a duration of nine years. In fact, the actual cost of a buyout could be higher. If predicted mortality rates are higher than assumed ones used to measure the balance sheet value of pension obligations and/or capital markets deteriorate in the future, annuity insurers will likely require a higher buyout price to cover pension risk.

In addition to high upfront annuity premiums, buyouts can be very expensive for firms with underfunded plans. To complete buyout transactions, these firms have to satisfy a minimum funded status by infusing cash to cover their funding deficits. In practice, pension shortfalls should be paid immediately or over an amortization period with a series of regular payments. Recent falling interest rates and subdued equity markets have driven up buy-out deficits of most pension plans. Thus, while many firms would like to de-risk their pensions through buy-outs, due to the high cost at present, it is not a good idea to use shareholder funds to finance buyout deals (LCP, 2012, page 6). This may explain why longevity swaps generally have had higher business volume than buyouts in recent years, even though buyouts can create greater value.

Lane Clark & Peacock LLP, a leading pension consulting firm, argues that we should explore well-designed structures that make buyouts easier for underfunded plans (LCP, 2012). Nevertheless, with only a few exceptions, cost effective strategies that make buyouts affordable have been largely unexplored. The existing literature focuses mainly on costs and benefits of pension buyouts (Cox et al., 2013; Lin et al., 2014, 2015). Little attention has been paid to how to capture the benefits of this de-risking option within a plan's financial means, especially when buyout deficits are significant. To fill this gap, we first investigate innovative ways to achieve pension buyouts for underfunded DB plans in a value-enhancing way.

Specifically, we propose two types of options that provide financing to underfunded plans if a pension funding index improves to a certain level. This certain level, called the trigger level, is set at a ratio less than the fully funded ratio 100%. To increase market liquidity, we create a transparent pension funding index to determine option payoffs. This pension funding index, to be explained in detail in a later section, is calculated from observed capital market indices and publicly available

mortality tables. Our first proposed option, the *pension funding option*, provides a payoff equal to the difference between a strike funding level and the pension funding index if the option is triggered. The second pension option, the *pension buyout option*, provides a payoff to cover the buyout risk premium and pension funding deficit if the pension funding index reaches a threshold. In reality, there is a misconception among pension sponsors that pension risk can be transferred only if a plan is well funded (Mathur and Kaplan, 2013). Indeed, our proposed options provide an innovative and promising venue for severely underfunded firms to execute buyouts before they reach full funding. The moral hazard and adverse selection problem is low in our setup because these options will pay out only if the index is triggered, irrespective of the actual pension performance of option buyers. While these options are index-based, basis risk may not be a serious concern because our proposed pension funding index can be flexibly designed to resemble the funding dynamics of a particular pension plan.

On the other hand, some firms with well funded DB plans are currently reluctant to adopt buyouts because they are comfortable with retaining market risk or they may feel the buyout annuity price is too high (Mathur and Kaplan, 2013). Regulation and legal issues related to buyout transactions further deter DB plan sponsors from shedding their pension obligations now. For example, Verizon retirees sued Verizon and challenged its buyout deal with Prudential that annuitized benefits of 41,000 retirees. While the claims were dismissed, Verizon retirees continued their fight in court. This lawsuit may stir a change in pension law (Buckmann, 2014). For example, the ERISA advisory council is considering new rules for buyouts. The possibility of changes to the law, therefore, may be another reason why fully funded pension firms hesitate to implement buyouts. If new laws on de-risking transactions remove ambiguities and disputes, we expect more pension sponsors to take actions to transfer their pension liabilities, especially when their DB pensions start to cause problems. The difficulty is that, at that time their funding status deteriorates, the cost of buyouts may be high due to large plan deficits. Therefore, we propose two more pension funding and buyout options that allow a currently well-funded firm to exercise the option and use the option payoff to fund their buyouts in the future, when the pension funding index falls below a given level. That is, the options provide funding for pension sponsors when they are highly motivated to remove their pension liabilities from their balance sheets.

A market for these options will develop if they are effective, economically affordable, and transparently priced. Thus, as the second objective of this paper, we show how to explicitly price these options. We first model the evolution of the pension funding index as the ratio of a flexibly designed pension asset index to a pension liability index. The pension asset index is calculated from several market indices based on predetermined investment weights, while the pension liability index is evaluated using a publicly available projected mortality table estimated with the Lee and Carter (1992) mortality model. The payoffs of pension funding options depend on the relation between a strike funding ratio and the pension funding index. If these pension options are only exercisable on valuation dates, they can be decomposed to a series of single period European “gap-type” options.² We use Monte Carlo simulation to obtain the payoffs and risk neutral prices for pension funding options. Unlike pension funding options, pension buyout options include an immediate buy-out feature when the funding threshold is triggered. We account for this pension take-over feature to properly price these options. Specifically, at the time the option is exercised, the payoff has a buyout risk premium, which can be evaluated using the approach introduced by Lin et al. (2015). The same pricing framework for funding options can be applied to buyout options with modified option payoffs.

This work contributes to the growing body of research on pension de-risking in three ways. First, this paper adds to the pension risk management literature by proposing new ways to reduce pension risk. Some plan sponsors view transferring DB risk to an insurer as too expensive. Our proposed options aim to fill the gap between the intentions and actions of pension sponsors with respect to pension risk transfer with buyouts. They provide a cost effective way to address pension risk, making buyouts feasible for underfunded pension firms and cheaper for well-funded firms that decide to defer their buyout decisions. Second, to increase market liquidity and reduce moral hazard and adverse selection problems, we introduce a transparent pension funding index based on market indices and publicly available mortality tables. It will lower transaction costs and help develop pension option markets. Third, we contribute to the existing studies on pension risk pricing. We show how to price these new pension de-risking securities while recognizing investment risk, longevity risk, and interest

²McDonald (2013, page 422) discusses gap options. There are pricing formulas for gap options in the Black-Scholes setting.

rate risk. Moreover, we propose how to incorporate buyout risk premiums for pricing pension buyout options.

2. SECURITIZATION OF PENSION RISK WITH OPTIONS

Pension buyouts, a pension de-risking strategy, have gained more and more attention from both scholars and practitioners. To facilitate development of this emerging market, we propose financial innovations that provide new venues to reduce pension risk exposures and make buyouts more affordable to pension sponsors. In a later section, we will show how to price these securities.

2.1. Pension Funding Index. Index-based securitization reduces moral hazard and adverse selection, increases liquidity, reduces transaction costs, and provides a standardized and transparent structure. Thus, our proposed pension options are based on a pension funding index calculated from various market indices and publicly available mortality tables. Specifically, the pension funding index at time t , PFI_t , is defined as the ratio of a pension asset index PAI_t to a pension liability index PLI_t :

$$PFI_t = \frac{PAI_t}{PLI_t}. \quad (1)$$

The pension liability index is based on a retired life cohort with $N(0)$ members of age x_0 at time 0. The number of lives $N(0)$ and the age x_0 may depend on a particular plan under consideration for a funding or buyout option.³ To ensure transparency, when calculating the pension liability index PLI_t at time t , we use a publicly available population mortality table based on the projected future mortality rates. Assume that each surviving member of the cohort receives an annual survival benefit of P , paid at the end of each year. For a given plan, we can use the average annual benefit for the members of the cohort of $N(0)$ at time $t = 0$. Then PLI_t is the present value at time t of all future benefit payments to $N(t)$ surviving retirees at time t :

$$PLI_t = N(t) \cdot Pa_{x_0+t} \quad t = 1, 2, \dots \quad (2)$$

At time $t = 0$, the number of retirees, $N(0)$, and the annual payment, P , are known. Thus at time 0, PLI_0 is known: $PLI_0 = N(0) \cdot Pa_{x_0}$. In (2), a_{x_0+t} is the immediate life annuity factor for age

³The group of retirees under consideration may have different ages at time $t = 0$. We can easily adjust the model if that is the case. The main thing is that the ages and number of retirees at each age are known at time $t = 0$.

$x = x_0 + t$ given the predicted mortality rates at time t :

$$a_x = a_{x_0+t} = \sum_{s=1}^{\infty} v_t^s {}_s\hat{p}_{x,t}, \quad (3)$$

where $v_t = 1/(1+r_{p,t})$ is the discount factor based on the pension valuation rate $r_{p,t}$. The conditional expected s -year survival rate for age x at time t , ${}_s\hat{p}_{x,t}$, is calculated as:

$${}_s\hat{p}_{x,t} = \mathbb{E} [{}_s\tilde{p}_{x,t} | \tilde{p}_{x,t}, \tilde{p}_{x+1,t+1}, \dots, \tilde{p}_{x+s-1,t+s-1}], \quad (4)$$

where $\tilde{p}_{x+s-1,t+s-1}$, $s = 1, 2, \dots$ is the probability that a plan member at age $x + s - 1$ at time $t + s - 1$ survives to age $x + s$ at the beginning of year $t + s$, based on a forecasted mortality table at time $t + s - 1$.

The pension asset index PAI_t is determined by the value of a market portfolio composed of I indices at time t . That is,

$$PAI_t = \sum_{i=1}^I A_{i,t-1}(1+r_{i,t}), \quad i = 1, 2, \dots, I; \quad t = 1, 2, \dots, \quad (5)$$

where $A_{i,t-1}$ is the amount invested in index i at time $t - 1$ and $r_{i,t}$ is the return of index i in period t . At time 0, PAI_0 is set at a predetermined value denoted PA_0 with $A_{i,0} = w_i PA_0$ where w_i is the weight of index i that represents a plan's typical weight in that asset category. That is, the initial fund amount PA_0 and the weights w_i can be tailored to a specific plan under consideration for a funding or buyout option. Given $N(t)$ survivors at time t , the following relation holds:

$$\begin{aligned} PA_t &= \sum_{i=1}^I A_{i,t} \\ &= PAI_t + k_t \cdot UL_t \cdot 1_{\{UL_t > 0\}} - N(t)P, \quad t = 1, 2, \dots, \end{aligned}$$

where UL_t is the funding deficit measured as

$$UL_t = PLI_t - PAI_t + N(t)P, \quad t = 1, 2, \dots. \quad (6)$$

Following Maurer et al. (2009) and Cox et al. (2013), the pension underfunding amortization factor k_t is equal to

$$k_t = \frac{1}{\sum_{i=0}^{m-1} (1 + r_{p,t})^{-i}}, \quad (7)$$

where $m > 1$ is the number of years of the amortization period. At the end of each period, the pension fund PA_t is rebalanced so that the weight invested in index i stays at $w_i, i = 1, 2, \dots, I$. That is,

$$A_{i,t} = w_i \cdot PA_t = w_i (PAI_t + k_t \cdot UL_t \cdot 1_{\{UL_t > 0\}} - N(t)P), \quad (8)$$

where $\sum_{i=1}^I w_i = 1$. With this setup, the evolution of the pension asset index PAI_t will closely resemble the dynamics of a typical pension plan's asset portfolio.

2.2. Pension Options. We propose several pension options for fully funded plans as well as underfunded plans, based on the pension funding index PFI_t .

2.2.1. Pension Options for Fully Funded Plans. Some well-funded pension sponsors choose not to execute pension buyouts (Mathur and Kaplan, 2013). However, a fully funded plan may want to obtain the right to a buyout in the future, if its funding deteriorates. Future asset values may be volatile and future liabilities may increase due to improving mortality of retirees. A plan can become underfunded and require cash injection in the future, in which case implementing a pension buyout may become very expensive. Thus, we propose a pension funding option that allows a plan to retain the option to execute a buyout, in the face of future underfunding. This option preserves flexibility to finance future buyouts at a low cost.

Consider a pension plan that is fully funded at time 0. Suppose its funding status is highly correlated with the pension funding index PFI_t . To manage its pension funding risk, the plan can purchase the following n -year funding option with a strike funding level K . The option is defined in terms of a notional amount, NA , the trigger funding index level, z , at which the option may be exercised, the strike level, $K > z$, and the option term n . It may be exercised at time $t = 1, 2, \dots, n$ but only if $PFI_t < z$. The payoff at time t is summarized as follows:

$$F_t^w = \frac{NA}{PLI_0} \times \begin{cases} PLI_t (K - PFI_t) & \text{if } PFI_t < z \\ 0 & \text{if } PFI_t \geq z \end{cases}, \quad \text{for } t = 1, 2, \dots, n. \quad (9)$$

PFI_t , as described in (1), is the pension funding index at time $t = 1, 2, \dots, n$. With this option,⁴ if the PFI_t falls below the trigger z , the plan may exercise the option and receive a payoff equal to $NA \frac{PLI_t}{PLI_0} (K - PFI_t)$. The term $\frac{PLI_t}{PLI_0}$ measures the remaining percentage of pension liabilities at time t as the number of surviving retirees in the plan declines overtime. The payoff allows the plan to make up its funding deficits and reduce costly external financing to complete the buyout transaction.

This funding option provides cash to the DB sponsor to cover a future funding deficit and satisfy a minimum funding requirement for a future buyout. However, the cash may not be sufficient to pay the full cost of a buyout, if the strike level K is less than or equal to the full funding ratio 100%. Additional cash may be required to complete the buyout. The buyout price, in general, is higher than the expected pension liabilities because buyout insurers require a risk premium. For example, recent UK buyout prices were about 14% above the value of pension liabilities covered.

We define a second option which recognizes the buyout provider's risk premium, as follows, with the same definitions of the trigger point z , strike level K , and term n . This buyout option provides a currently fully funded plan with the following payoff at time t :

$$B_t^w = \frac{NA}{PLI_0} \times \begin{cases} PLI_t (K - PFI_t + R_t) & \text{if } PFI_t < z \\ 0 & \text{if } PFI_t \geq z \end{cases}, \quad (10)$$

where R_t is the estimated buyout risk premium at time t . This buyout option has a higher payoff than the funding option in (9). Accordingly, the buyout option will have a higher price than the funding option.

2.2.2. Pension Options for Under-Funded Plans. In contrast to a fully funded pension plan, an underfunded plan may not have the cash for a buy-out immediately, even though the plan sponsor may want a buyout. To make the pension buyout feasible, such a plan can purchase a n -year funding option with a strike level K and a notional amount of NA , subject to a trigger $PFI_t > z$ where $z < K$. The

⁴In option terminology, this is an n -year Bermudian gap put option written on the pension funding index.

following expression describes the value of this option at time t :

$$F_t^u = \frac{NA}{PLI_0} \times \begin{cases} 0 & \text{if } PFI_t \geq K \\ PLI_t(K - PFI_t) & \text{if } z < PFI_t < K \\ 0 & \text{if } PFI_t \leq z \end{cases} \quad \text{for } t = 1, 2, \dots, n. \quad (11)$$

If the funding index PFI_t improves and exceeds the trigger z , but less than the strike K at time t , the option will be exercised, providing a payoff of $NA \frac{PLI_t}{PLI_0} (K - PFI_t)$. To avoid a negative option payoff, in (11), we set the payoff equal to 0 if $PFI_t \geq K$. As long as the funding status of the plan is highly correlated with the pension funding index PFI_t , the payoff from the option makes it easier for the plan to pay for a buyout.

An underfunded firm may want to enter a buyout transaction right after its funding ratio reaches z . As in the case of a fully funded plan, we introduce a buyout option that recognizes the provider's risk premium. For such an underfunded plan, the buyout option payoff B_t^u at time t is defined as follows:

$$B_t^u = \frac{NA}{PLI_0} \times \begin{cases} PLI_t \times R_t & \text{if } PFI_t \geq K \\ PLI_t(K - PFI_t + R_t) & \text{if } z < PFI_t < K \\ 0 & \text{if } PFI_t \leq z \end{cases} \quad (12)$$

The pension funding index PFI_t can be designed initially to track a plan's funding dynamics. Then, the payoff from this buyout option will very likely pay the entire buyout annuity premium for the underfunded plan.

2.3. Discussion. In the UK, pension plan deal volume in 2013 included £8.9 billion of longevity swaps and £7.45 billion of bulk annuity buyouts (Ward, 2014). The reinsurers and insurers providing these transfers might very well be interested in developing and including buyout options to expand the range of their provided risk management tools. The actuarial consulting firms currently advising plan sponsors could assist in obtaining appropriate pension buyout options. As we documented earlier, the interest from plan sponsors in managing pension risks is substantial. The demand for buyouts in the U.S. increases because "increasing PBGC premiums and increasing longevity" are making it harder for pension plan sponsors to maintain DB plans (Ebling, 2014). While many plans have

recovered from the 2008 crisis, the remaining under-funded plans may see buyout options as a means to eventually transfer unwanted risks. Importantly, these options allow plans that are not eligible for a traditional buyout to execute buyouts easier. In sum, it seems reasonable that buyout options will be attractive to a significant number of sponsors. Moreover, it appears feasible for the current providers of mortality swaps and buyouts to arrange buyout options to accompany their existing buyout and mortality swap products.

3. BASIC FRAMEWORK

3.1. Pension Asset Index. Suppose the pension asset index PAI_t is composed of three indexes at time t : the S&P 500 index ($i = 1$), the Merrill Lynch corporate bond index ($i = 2$), and the 3-month T-bill index ($i = 3$) with the weights of w_1 , w_2 and w_3 . Under the physical (P) risk measure, the process of the S&P 500 index at time t , $A_{1,t}$, is described by the following stochastic differential equation (SDE):

$$\frac{dA_{1,t}}{A_{1,t-}} = (\alpha_1 - \lambda_1 k_1) dt + \sigma_1 dW_{1t}^P + d\left(\sum_{j=1}^{N_{1t}} (V_{1j} - 1)\right), \quad (13)$$

where α_1 is the instantaneous expected return and σ_1 is the instantaneous volatility of the S&P 500 index. The standard Brownian motion W_{1t}^P has a mean 0 and variance t . The Poisson process N_{1t} has a mean number of arrivals λ_1 per unit of time. The independently and identically distributed jump size V_{1j} for $j = 1, 2, \dots$ is modeled as a lognormal random variable (Merton, 1976). Thus, its logarithm $Y_{1j} = \log V_{1j}$ is a standard normal random variable with mean m_1 and standard deviation s_1 . A Poisson event will cause an expected percentage change in the S&P 500 index equal to $k_1 \equiv E(V_{1j} - 1)$. Solving the SDE (13) provides an explicit expression for the S&P 500 index process:

$$A_{1,t} = A_{1,0} \exp[X_t] = A_{1,0} \exp\left[\left(\alpha_1 - \frac{1}{2}\sigma_1^2 - \lambda_1 k_1\right)t + \sigma_1 W_{1t}^P + \sum_{j=1}^{N_{1t}} Y_{1j}\right]. \quad (14)$$

Due to the market incompleteness introduced by the jump-diffusion process (14), we are no longer able to find a unique risk-neutral (martingale equivalent) measure. The transition from the physical world to the risk neutral world varies, depending on which type of distance minimization is selected. To preserve the same kind of jump-diffusion framework under measure change, we follow Gerber and

Shiu (1994) and adopt an Esscher transform of the logarithmic asset return as our martingale equivalent measure. It can be shown that such a risk neutral measure is the closest martingale equivalent measure to the physical measure in terms of a power utility function (Gerber and Shiu, 1994, p.175-177). Define the risk neutral Esscher measure $P^*(X_t, h)$ through its corresponding Radon-Nikodym derivative:

$$\left(\frac{dP^*}{dP} \right) \Big|_{\mathcal{F}_t} = \frac{e^{hX_t}}{\mathbb{E}[e^{hX_t}]}, \quad (15)$$

where h is a real constant such that $\mathbb{E}[e^{hX_t}]$ exists.

Under the risk neutral Esscher measure $P^*(X_t, h)$, the process of the S&P 500 index, $A_{1,t}$, still follows a Merton-type jump-diffusion process:

$$A_{1,t} = A_{1,0} \exp \left[\left(r - \frac{1}{2}\sigma_1^2 - \lambda_1^* k_1^* \right) t + \sigma_1 W_{1t}^{P^*} + \sum_{j=1}^{N_{1t}^*} Y_{1j}^* \right], \quad (16)$$

where r is the risk-free interest rate, $W_{1t}^{P^*} = W_{1t}^P - h\sigma_1 t$ is a standard Brownian motion under P^* , and the new Poisson and jump size parameters are

$$\lambda_1^* = \lambda_1 \exp \left(m_1 h + \frac{1}{2} s_1^2 h^2 \right), \quad m_1^* = m_1 + s_1^2 h, \quad (17)$$

with the standard deviation $s_1^* = s_1$ unchanged. The parameter h is determined such that $\{e^{-rt} A_{1,t}, t \geq 0\}$ is a martingale with respect to the Esscher risk measure P^* . More specifically, the parameter h satisfies

$$\alpha_1 - \lambda_1 k_1 + \sigma_1^2 h + \lambda_1^* k_1^* = r \quad (18)$$

(see, e.g. Bo et al. (2010)). Such explicit analytic relations among the parameters under the physical risk measure and the risk neutral Esscher measure provide significant convenience for the numerical illustrations in Section 5.

The dynamic processes of the Merrill Lynch corporate bond index ($i = 2$) and the 3-month T-bill index ($i = 3$) are modeled as geometric Brownian motions:⁵

$$\frac{dA_{i,t}}{A_{i,t}} = \alpha_i dt + \sigma_i dW_{it}^P, \quad i = 2, 3 \quad (19)$$

in the physical risk world and

$$\frac{dA_{i,t}}{A_{i,t}} = r dt + \sigma_i dW_{it}^Q, \quad i = 2, 3 \quad (20)$$

in the risk neutral (Q) world, where the constant α_2 (α_3) is the drift of the Merrill Lynch corporate bond index (the 3-month T-bill index) with an instantaneous volatility σ_2 (σ_3). In (19) and (20), W_{it}^P (W_{it}^Q) is a standard Brownian motion with mean 0 and variance t . The correlation between the standard Brownian motions W_{1t}^P (W_{1t}^{P*}) and W_{2t}^P (W_{2t}^Q) is denoted as ρ .⁶

3.2. Dynamic Pension Valuation Rate. The value of pension liabilities is closely related to the yield of long-term corporate bonds, which implies that considerable attention should be paid to the interest rate risk in pension risk management. Although long-term interest rates are far less volatile than short-term rates, the U.S. composite corporate bond rates⁷ published by the Internal Revenue Service (IRS) do show some smooth changes from time to time. As pointed out by Gökaynak et al. (2005), “long-term forward rates move significantly in response to many macroeconomic data releases and monetary policy announcements”. Consequently, it is of great importance to incorporate the dynamics of pension valuation rate into the pricing framework of pension options. While there exist a variety of short-term interest rate models in the literature (to name only a few, Cox et al. (1985); Hull and White (1990); Longstaff and Schwartz (1992)), few simple and effective models have been proposed to capture the movement of long-term interest rates. In this paper, we cautiously use the Cox-Ingersoll-Ross (CIR) model (Cox et al., 1985) to illustrate the dynamics of the pension valuation rate. That is, the pension valuation rate $r_{p,t}$ satisfies the following process:

$$dr_{p,t} = \nu (\theta - r_{p,t}) dt + \sigma_p \sqrt{r_{p,t}} dW_{p,t}^P, \quad (21)$$

⁵Our statistical test rejects the model with a jump for the processes of the Merrill Lynch corporate bond index and the 3-month T-bill index based on the monthly data from March 1988 to December 2010.

⁶We reject a significant correlation between W_{1t}^P and W_{3t}^P as well as between W_{2t}^P and W_{3t}^P in our sample period.

⁷<https://www.irs.gov/Retirement-Plans/Composite-Corporate-Bond-Rate-Table>.

TABLE 1. Maximum Likelihood Parameter Estimates of Pension Valuation Rates

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
ν	0.1821	θ	0.0569	σ_p	0.0035
ρ_{1p}	-0.2835	ρ_{2p}	0.1377	ρ_{3p}	0.1255

where ν is the mean-reversion rate, θ and σ_p are the long-term mean and instantaneous volatility of the pension valuation rate, and $W_{p,t}^P$ is a standard Brownian motion. The Brownian motion $W_{p,t}^P$ is further assumed to be correlated with W_{it}^P of pension asset i with a correlation coefficient ρ_{ip} ($i = 1, 2, 3$).

The mean-reversion characteristic of the CIR model could potentially enhance the stableness of the pension valuation rate process. In addition, by controlling the parameter σ_p , the fluctuation of the pension valuation rate can be relatively smooth and hopefully be consistent with the behavior of a long-term interest rate. Using the monthly IRS composite corporate bond rate data from January 2001 to December 2010, we obtain the maximum likelihood estimates as shown in Table 1.⁸

4. PRICING PENSION OPTIONS

Equipped with the aforementioned pension asset and liability index models, the funding and buyout options can then be evaluated using the traditional risk-neutral pricing techniques.

Given the option payoff function (9), the risk neutral price for a n -year funding option of a fully funded pension, in terms of the percentage of the nominal amount, can be expressed as

$$\begin{aligned}
PF_w &= \frac{1}{PLI_0} E^Q [e^{-r\tau_w} PLI_{\tau_w} (K - PFI_{\tau_w})^+] \\
&= \frac{1}{PLI_0} E^Q [e^{-r\tau_w} N^*(\tau_w) \cdot Pa_{x_0+\tau_w} (K - PFI_{\tau_w})^+] \tag{22}
\end{aligned}$$

where the stopping time $\tau_w = \inf \{t : PFI_t < z, t \in \{1, 2, \dots, n\}\}$ (∞ if the option is not triggered) denotes the first observed year-end that the pension funding index is below the trigger level z .⁹ The

⁸The estimation procedure has two steps. The maximum likelihood estimates are first obtained for ν , θ and σ_p . Then, the estimated values of $dW_{p,t}^P$ are further used to calibrate the correlation coefficients ρ_{ip} ($i = 1, 2, 3$) based on the data from January 2001 to December 2010.

⁹We assume the buyers immediately exercise the option when triggered. Underfunded pension firms are required to amortize their funding deficits over time (as long as the deficits are not covered), which will make it unattractive to hold a pension option if the option can be exercised.

survival evolution of the pension cohort $N^*(t)$ (and thus, PFI_t) in (22) is based on the transformed mortality rates to reflect the market expectation on mortality improvements.

If a direct buyout feature is added on top of the funding option, with exactly the same structure, the price of a n -year buyout option is given by

$$\begin{aligned} PB_w &= \frac{1}{PLI_0} \mathbb{E}^Q \left[e^{-r\tau_w} N^*(\tau_w) \cdot Pa_{x_0+\tau_w} \left((K - PFI_{\tau_w})^+ + R_{\tau_w} \right) \right] \\ &= PF_w + PR_w, \end{aligned} \quad (23)$$

where

$$PR_w = \frac{1}{PLI_0} \mathbb{E}^Q \left[e^{-r\tau_w} N^*(\tau_w) R_{\tau_w} \right]$$

is the option premium for the buyout feature. Note that R_t is the immediate buyout premium of a fully funded pension plan at time t . According to Lin et al. (2015), R_t can be decomposed into an investment risk premium and a longevity risk premium. More specifically, given that $PAI_t = PLI_t$ at time t ,

$$R_t = P_{invest,t} + P_{longevity,t}, \quad (24)$$

where the immediate buyout investment risk premium $P_{invest,t}$ is given by

$$P_{invest,t} = \frac{1}{PLI_t} \left(\sum_{j=1}^{\tau_N-t} e^{-rj} \cdot \mathbb{E}^Q \left[k_{t+j} \cdot UL_{t+j} \cdot 1_{\{UL_{t+j}>0\}} \right] - e^{-r(\tau_N+1-t)} \cdot \mathbb{E}^Q [PAI_{\tau_N+1}] \right), \quad (25)$$

for $\tau_N = \min \{[t] : N(t) = 0\}$, and the immediate buyout longevity risk premium $P_{longevity,t}$ is

$$P_{longevity,t} = \frac{a_{x_0+t}^*}{a_{x_0+t}} - 1. \quad (26)$$

In (26), $a_{x_0+t}^*$ is the life annuity value calculated based on the transformed mortality rates.

Similarly, for a n -year funding option of an underfunded plan with payoff F_t^u defined in (11), the option price can be obtained through

$$\begin{aligned} PF_u &= \frac{1}{PLI_0} \mathbb{E}^Q \left[e^{-r\tau_u} PLI_{\tau_u} (K - PFI_{\tau_u})^+ \right] \\ &= \frac{1}{PLI_0} \mathbb{E}^Q \left[e^{-r\tau_u} N^*(\tau_u) \cdot Pa_{x_0+\tau_u} (K - PFI_{\tau_u})^+ \right], \end{aligned} \quad (27)$$

where the stopping time $\tau_u = \inf \{t : PFI_t > z, t \in \{1, 2, \dots, n\}\}$ (∞ if the option is not triggered) is the first observed time that the pension funding index goes above the trigger level z . Finally, the price of a n -year buyout option with payoff B_t^u defined in (12), for an underfunded plan, is

$$\begin{aligned} PB_u &= \frac{1}{PLI_0} \mathbb{E}^Q [e^{-r\tau_u} N^*(\tau_u) \cdot Pa_{x_0+\tau_u} ((K - PFI_{\tau_u})^+ + R_{\tau_u})] \\ &= PF_u + PR_u, \end{aligned} \quad (28)$$

where

$$PR_u = \frac{1}{PLI_0} \mathbb{E}^Q [e^{-r\tau_u} N^*(\tau_u) R_{\tau_u}]$$

is again the premium for the buyout add-on.

5. NUMERICAL ILLUSTRATIONS

In this section, we use a numerical example to illustrate how to determine the values of pension asset and liability indexes and price pension funding and buyout options accordingly.

5.1. Pension Liability Index. The pension liability index PLI_t depends on the mortality experience of a population, for example, the U.S. male population. Suppose the mortality rates are modeled following Lee and Carter (1992). Lee and Carter (1992) provide a transparent and widely employed method to study mortality risk across time and age. This approach assumes a log-affine structure for the one-year death rate $q_{x,t}$ for age x ($x = 0, 1, 2, \dots$) in year t ($t = 1, 2, \dots, T$) and relies on the specification

$$\ln q_{x,t} = \kappa_x + b_x \gamma_t + \epsilon_{x,t}, \quad (29)$$

where κ_x is an age-specific parameter equal to the average population mortality level at age x ,

$$\kappa_x = \frac{\sum_{t=1}^T \ln q_{x,t}}{T}. \quad (30)$$

In (29), γ_t is a common risk factor that captures the overall decline in mortality of all ages. The age-specific parameter b_x accounts for differences among short-term death rate changes at different ages. The error term $\epsilon_{x,t}$ is normally distributed with a mean 0 and a standard deviation of σ_{ϵ_x} . Following

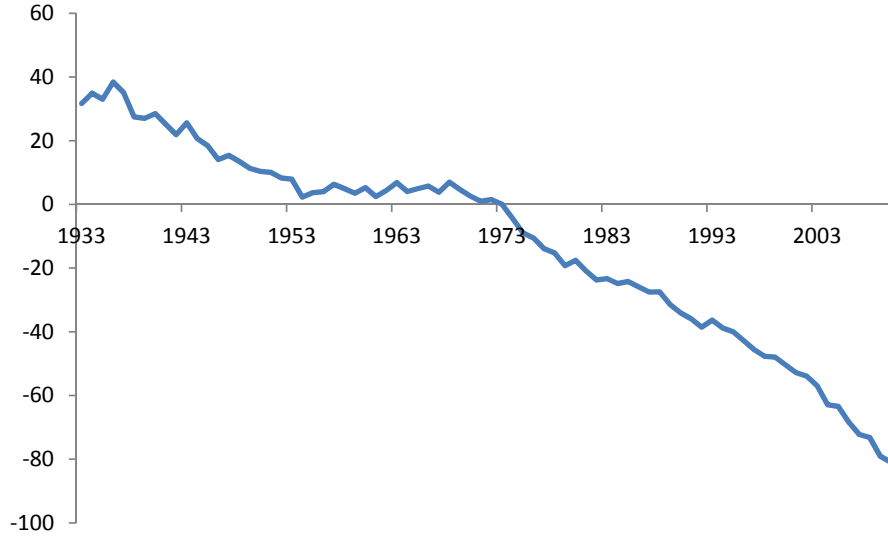


FIGURE 1. The vertical axis represents the estimated common risk factor γ_t and the horizontal axis is year t , $t = 1933, 1934, \dots, 2009, 2010$.

Lee and Carter (1992), we model the common risk factor γ_t as a random walk with drift,

$$\gamma_t = \gamma_{t-1} + g + e_t, \quad e_t \sim N(0, \sigma_\gamma) \quad (31)$$

where the error term e_t is i.i.d. standard Normal.

We assume the underlying population of the pension liability index PLI_t is the U.S. male population and the future mortality rates are simulated from the Lee and Carter (1992) model. We set year 2010 as our base year $t = 0$. Based on the U.S. male population mortality tables from 1933 to 2010 in the Human Mortality Database¹⁰ and the two-step estimation procedure described in Lee and Carter (1992), we calibrate the models (29) and (31) for predicting future mortality rates. Our estimated γ_t 's are depicted in Figure 1 with $g = -1.46$ and $\sigma_\gamma = 2.44$.¹¹ Overall, the common risk factor γ_t has a downward sloping trend, which suggests that mortality rates improve overtime.

To simplify our example, we assume the pension liability index PLI_t is determined by the mortality experience of a retired cohort at the age $x_0 = 65$ at time $t = 0$. While we use a simplified setup to illustrate the basic idea, our pension liability index has flexibilities in accommodating different cohort assumptions and configurations. To derive the value of PLI_t , we first simulate future mortality rates

¹⁰Available at www.mortality.org (data downloaded on June 21, 2013).

¹¹To conserve space, the parameter estimates of κ_x and b_x are not reported but available upon request.

TABLE 2. Maximum Likelihood Parameter Estimates of Three Asset Indexes

Parameter	Estimate	Parameter	Estimate
α_1	0.1046	α_2	0.0761
σ_1	0.1101	σ_2	0.0537
λ_1	3.9112	α_3	0.0418
m_1	-0.0242	σ_3	0.0069
s_1	0.0495	ρ	0.4248

based on model (29) as follows:

$$\tilde{q}_{x,t} = \exp(\kappa_x + b_x \tilde{\gamma}_t), \quad t = 1, 2, \dots,$$

where $\tilde{\gamma}_t$ is the simulated mortality common risk factor at time t . We use it to obtain the simulated one-year death rate $\tilde{q}_{x,t}$ for age x at time t given $x = x_0 + t = 65 + t$. Then the simulated one-year survival probability equals

$$\tilde{p}_{x,t} = 1 - \tilde{q}_{x,t}.$$

With the simulated $\tilde{p}_{x,t}$ and the simulated pension valuation rate $r_{p,t}$ from model (21), we can calculate $a_x = a_{65+t}$ in (3). Suppose the initial number of retirees at time 0 is $N(0) = 5,000$. If a retiree survives at the end of each year $t, t = 1, 2, \dots$, he or she will be entitled to receive a survival benefit of $P = 60,000$. Given this, the pension liability index PLI_t at time t equals

$$PLI_t = 60,000 \times N(t) \cdot a_{65+t} \quad t = 1, 2, \dots, \quad (32)$$

where $N(t)$ is the number of survivors at time t .

5.2. Pension Asset Index. To increase the number of observations for model calibration, we use the monthly data from January 1988 to December 2010 to estimate the parameters in (14) and (19) for the three indexes. The monthly historical data of the S&P 500 total return index, the Merrill Lynch corporate bond total return index and the 3-month T-bill total return index are obtained from the DataStream. Because the proposed pension options are only exercisable at the end of each year, we convert the monthly estimates to the annualized estimates. The annualized maximum likelihood estimates are reported in Table 2.

Conditional on no jumps, on average, the S&P 500 index has a higher expected annual return ($\alpha_1 = 0.1046$) than the Merrill Lynch corporate bond index ($\alpha_2 = 0.0761$) and the 3-month T-bill index ($\alpha_3 = 0.0418$) but it has a higher annual standard deviation $\sigma_1 = 0.1101$ than the other two indexes ($\sigma_2 = 0.0537$ and $\sigma_3 = 0.0069$). Moreover, the stock index and the corporate bond index is positively correlated with a correlation coefficient equal to $\rho = 0.4248$. If a jump occurs, the S&P 500 index has log jump size with a mean of $m_1 = -0.0242$ and a standard deviation of $s_1 = 0.0495$.

The DB firm can purchase a pension option based on a pension asset index PAI_t with the weights of w_1 , w_2 and w_3 invested in the above three indexes, similar to the asset allocation in its pension plan. With this setup, the pension asset index will nicely resemble the dynamics of the plan's assets. Specifically, suppose at time 0, the weights invested in the three asset indexes underlying PAI_0 are specified at $w_1 = 0.5$, $w_2 = 0.45$ and $w_3 = 0.05$. The pension asset index PAI_0 at time 0 can be expressed as a proportion of the pension liability index PLI_0 :

$$PAI_0 = \xi \cdot PLI_0,$$

where $\xi = PFI_0$ represents the funding ratio at time 0. A value of $\xi < 1$ implies that the plan is initially underfunded and a value of $\xi = 1$ suggests fully funded. After time 0, the pension asset index is rebalanced at the end of each year to keep this asset allocation.¹² Moreover, we use a 7-year amortization period to calculate the amortization factor k_t when there is a funding deficit.

5.3. Transformed Mortality Rates. To derive prices of pension funding and buyout options, we need to use transformed mortality rates that reflect the market expectation on future mortality improvements. We use the Wang transform (Wang, 1996, 2000, 2001, 2002; Lin and Cox, 2005) to distort the mortality rates. The random variable to be transformed is the remaining life time $T(x, 0)$ of a life age x at time 0. The transform applies to its cdf $F_{T(x,0)}(s) = \Pr(T(x, 0) \leq s)$ to yield the transformed cdc $F_{T(x,0)}^*(s)$ as follows:

$$F_{T(x,0)}^*(s) = \Phi[\Phi^{-1}(F_{T(x,0)}(s)) - \lambda], \quad (33)$$

¹²The pension asset index can be designed in a different way without rebalancing. The same pricing technique still can be applied to this situation.

where the cumulative probability

$$F_{T(x,0)}(s) = {}_s\widehat{q}_{x,0} = 1 - {}_s\widehat{p}_{x,0}$$

is the probability that a life age x at time 0 dies within s years. In (33), $\lambda > 0$ is the market price of risk that accounts for systematic longevity risk. $\Phi(\cdot)$ is the standard normal cdf. After the transform, we can obtain the risk-neutral s -year survival rate for age x , ${}_s\widehat{p}_{x,0}^*$, for pricing:

$${}_s\widehat{p}_{x,0}^* = 1 - {}_s\widehat{q}_{x,0}^* = 1 - \Phi[\Phi^{-1}({}_s\widehat{q}_{x,0}) - \lambda]. \quad (34)$$

Following Lin et al. (2015), we use the 25-year European Investment Bank (EIB) bond issued in November 2004 to derive the market price of risk λ for longevity risk. This bond was sold for £540 million (775 million euros). It covered the English and Welsh male population aged 65 years old and paid a fixed annuity, £50 million, multiplied by the percentage of the reference population still alive at each anniversary from the Office for National Statistics in U.K.. Following Lin et al. (2015), the market price of risk λ can be estimated by solving the following equation implicitly:

$$540,000,000 = 50,000,000a_{65:\overline{25}|}^*$$

where $a_{65:\overline{25}|}^*$ is the 25-year immediate life annuity factor based on the transformed survival rates in (34). Following the estimation procedure and data of Lin et al. (2015), our estimated market price of risk λ equals 0.0943. Assume the market price of longevity risk in U.S. is the same as that derived from the EIB bond. With this assumption, we use $\lambda = 0.0943$ to derive the risk premiums of pension funding and buyout options based on the U.S. male population mortality rates.

5.4. Pension Option Premium.

5.4.1. *Pricing Pension Options for Fully Funded Plans.* Suppose at time 0 all plan participants of Firm W reach the retirement age $x_0 = 65$. Each retiree will receive an annual survival benefit of $P = 60,000$ as long as he/she survives at the end of each year. This cohort with $N(0) = 5,000$ retirees at time 0 has the same mortality experience as the U.S. male population. So it follows the

same dynamics of the pension liability index PLI_t in (32). Further assume that Firm W invests a proportion $w_1 = 0.5$ of its pension assets in the S&P 500 index, $w_2 = 0.45$ in the Merrill Lynch corporate bond index, and $w_3 = 0.05$ in the 3-month T-bill index, the same as the composition of the pension asset index PAI_t described in Section 5.2. As a result, the pension funding index PFI_t tracks the performance of Firm W's pension plan.

At time 0, the pension plan of Firm W is fully funded with a funding ratio of 1. As the pension plan performs well and does not depress business operations, Firm W feels comfortable about retaining pension risk. While Firm W chooses not to de-risk its pensions now, it recognizes the risk that cash drains from required pension contributions caused by unprecedented market swings and unexpected mortality improvements could deplete its internal resources and cause financial difficulties. To hedge the risk that it cannot cover a serious funding deficit when it faces financing constraints in the future, Firm W purchases a life-time pension funding option at $t = 0$. This option allows Firm W to exercise the option when the pension funding index PFI_t falls below the trigger level z throughout the life of the plan. When this funding option is triggered at time t , given the strike level K , Firm W will receive a payoff determined by (9).

Suppose the pension valuation rate at time 0 is $r_{p,0} = 0.057$ and the long-term risk-free rate is 1% lower, which equals $r = 0.047$. Following (22), we calculate the funding option premiums PF_w at different trigger levels z and strike funding ratios K . The results are shown in Table 3. When the trigger level is $z = 0.8$ and the strike level is $K = 1.00$, Firm W will exercise this option if the funding index PFI_t deteriorates to 80%. When this happens, the option will make a payment sufficient to cover the entire funding shortfall because the funding status of the plan follows PFI_t . To obtain this option, Firm W has to pay a premium, stated as a percentage of the initial pension liability PLI_0 , equal to $PF_w = 8.69\%$ at time 0.

Firm W can purchase other funding options with different trigger and strike levels. As the trigger and strike levels decrease, the funding option premium goes down. This is because it is less likely to trigger the option and if the option is triggered, the option payoff is lower. For example, when the trigger level z decreases from 0.8 to 0.7 and the strike funding ratio K decreases from 1.00 to 0.90, as shown in Table 3, the funding option premium PF_w decreases from 8.69% to 4.83%.

If Firm W struggles to fund pensions for its retirees after a serious funding deficit in the future, it may want to de-risk its pensions with a buyout. In this case, however, the pension buyout cost will be prohibitively high. In addition to making up the funding deficit with cash infusions, Firm W has to pay a buyout risk premium. To hedge against this risk, Firm W can purchase a pension buyout option. The last column of Table 3 shows the price PR_w for a buy-out add-on at a given trigger. For example, when the trigger is $z = 0.8$, the buy-out add-on costs $PR_w = 3.54\%$. It means that Firm W needs to pay a premium of $PB_w = 12.23\%$ ($= PF_w + PR_w = 8.69\% + 3.54\%$) to purchase this buyout option. This buyout option allows Firm W to transfer its entire pension liability to the option seller when the pension funding index PFI_t falls to $z = 0.8$. Similar to the pattern of funding options, the prices of buyout options decrease with trigger levels. When the trigger level z decreases from 0.8 to 0.7, the buyout option premium PB_w goes down significantly from 12.23% ($= PF_w + PR_w = 9.29\% + 3.23\%$) to 8.73% ($= PF_w + PR_w = 6.83\% + 1.90\%$).

Are buyout options expensive for well-funded pension firms? To answer this question, we compare the costs of buyout options and an immediate pension buyout at time 0. Following equations (24), (25) and (26), we obtain the buyout price equal to $R_0 = 10.43\%$ if Firm W transfers its pension obligations to a buyout risk taker at time 0. When the trigger is $z = 0.8$, the buyout option premium is $PB_w = 12.23\%$. While the buyout option requires a higher cash outflow than the buyout at time 0, it provides extra protections by covering the entire funding deficit, more than 20% of pension liabilities, when the option is triggered. If the trigger is lowered to $z = 0.7$, the total cost of the buyout option is only $PB_w = 8.73\%$, lower than the price of an immediate buyout $R_0 = 10.43\%$ at time 0. If Firm W wants to transfer pension risk only when it needs the protection the most, it can purchase a buyout option with a low trigger level (e.g. $z = 0.7$).

5.4.2. Pricing Pension Options for Under-Funded Plans. In contrast to firms with fully funded plans like Firm W, companies with poorly funded plans may want to remove pension liabilities from their balance sheets since mandatory pension contributions impose a huge financial burden. With pension buyouts, they have to wait until their plans are fully funded and they have financial resources to pay buyout risk premiums. Buyout options make pension buyouts easier and closer. Suppose the pension plan of Firm U has the same member and asset compositions as those of Firm W except that it is

TABLE 3. Life-Time Funding and Buyout Option Premiums for Fully Funded Plans

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	0.80	1.00	8.69%	3.54%
		0.95	6.93%	–
		0.90	5.17%	–
	0.70	1.00	6.83%	1.90%
		0.90	4.83%	–
		0.80	2.83%	–

underfunded by 20% (with a funding ratio of $PFI_0 = 80\%$) at time 0. It purchases a life-time funding option that provides funding when the pension funding index PFI_t improves to the trigger level z . The prices of life-time funding options with different trigger and strike levels are shown in Table 4. If Firm U purchases a life-time funding option with a trigger $z = 0.85$ and a strike level $K = 1.00$, it needs to pay a funding option premium of $PF_u = 5.43\%$. This funding option will cover the entire funding deficit when the pension funding index PFI_t improves to 0.85. If Firm U wants to execute a buyout when the option is triggered, it will need to pay an extra buyout add-on premium of $PR_u = 5.88\%$ to achieve this goal. In this case, the total premium of pension buyout option is $PB_u = 11.31\%$ ($= PF_u + PR_u = 5.43\% + 5.88\%$). Note that it is significantly cheaper compared to an immediate buyout, which will cost 30.43% ($= 20\%$ funding deficit + 10.43% buyout premium). If Firm U chooses a buyout option with a higher trigger, the cost of pension buyout option will be lower. For example, when $z = 0.9$, the buyout option premium decreases to $PB_u = 6.70\%$ ($= PF_u + PR_u = 2.25\% + 4.45\%$).

We also find that the funding and buyout option premiums decrease with the initial funding ratio PFI_0 . For example, when $z = 0.90$ and $K = 1.00$, the funding (buyout) option premium with $PFI_0 = 0.75$ is 1.78% (5.20%), lower than that with $PFI_0 = 0.8$, 2.25% (6.70%). This is because it is more difficult for a deeply underfunded plan to improve and trigger a given funding target z . Accordingly, for each combination of z and K , the funding and buyout options are cheaper for more underfunded plans.

TABLE 4. Life-Time Funding and Buyout Option Premiums for Under-Funded Plans

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_u	Buyout Add-on PR_u
80%	0.85	1.00	5.43%	5.88%
		0.95	2.90%	–
		0.90	0.88%	–
	0.90	1.00	2.25%	4.45%
		0.95	0.69%	–
75%	0.85	1.00	4.37%	4.56%
		0.95	2.35%	–
		0.90	0.72%	–
	0.90	1.00	1.78%	3.42%
		0.95	0.54%	–

6. SENSITIVITY ANALYSIS

We now assess the impact of the parameter values. In doing these analyses, we explore how the implications of parameter choice affect pension funding and buyout option prices. In general, the analyses below have illustrated that our formulation leads to intuitive results in terms of how the pension option premiums depend on the maturity, unexpected mortality improvements, risk-free rates, pension asset allocations, etc. These help demonstrate the reliability of our pricing models. Our sensitivity analyses also show that our proposed pension options are reasonable and affordable to DB plans.

6.1. Impact of Option Maturity. To explore the impact of maturity on premiums, we analyze two more terms of pension options, 5-year and 10-year funding and buyout options. Tables 5 and 6 present the results for 5-year funding and buyout options for fully funded and under-funded plans, respectively, and Tables 7 and 8 are for 10-year options. Given all other parameters the same as those in the base scenario, our results show the longer the tenor of a pension option, the higher the price of the option. For example, the premium of the 5-year funding option with the trigger $z = 0.80$ and the strike level $K = 1.00$ for fully funded plans is 5.21%, which is lower than 7.70% based on the 10-year option and 8.69% based on the life-time option. As the maturity of a pension option increases, the underlying pension funding index PFI_t will have more time—and thus a greater chance—to trigger an option payment.

TABLE 5. 5-Year Funding and Buyout Option Premiums for Fully Funded Plans

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	0.80	1.00	5.21%	2.25%
		0.95	4.15%	–
		0.90	3.08%	–
	0.70	1.00	2.18%	0.67%
		0.90	1.53%	–
		0.80	0.89%	–

TABLE 6. 5-Year Funding and Buyout Option Premiums for Under-Funded Plans

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_u	Buyout Add-on PR_u
80%	0.85	1.00	4.43%	4.90%
		0.95	2.37%	–
		0.90	0.72%	–
	0.90	1.00	1.62%	3.30%
		0.95	0.50%	–
		1.00	3.07%	3.30%
75%	0.85	0.95	1.66%	–
		0.90	0.51%	–
		1.00	1.06%	2.10%
	0.90	0.95	0.33%	–
		1.00	–	–
		–	–	–

TABLE 7. 10-Year Funding and Buyout Option Premiums for Fully Funded Plans

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	0.80	1.00	7.70%	3.21%
		0.95	6.13%	–
		0.90	4.57%	–
	0.70	1.00	4.89%	1.44%
		0.90	3.45%	–
		0.80	2.01%	–

6.2. Impact of Risk-Free Rate. In addition to the maturity, we also compare the prices of pension options subject to different risk-free rate parameters. As the risk-free rate r decreases, a pension plan is more likely to be underfunded, which increases the probability that a funding option will be triggered for a currently well funded plan. Consistent with this, we find that the price of a funding option for a fully funded plans, PF_w , is negatively associated with r . For example, in the base case

TABLE 8. 10-Year Funding and Buyout Option Premiums for Under-Funded Plans

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_u	Buyout Add-on PR_u
80%	0.85	1.00	5.19%	5.67%
		0.95	2.78%	–
		0.90	0.84%	–
	0.90	1.00	2.09%	4.18%
		0.95	0.64%	–
75%	0.85	1.00	4.04%	4.26%
		0.95	2.18%	–
		0.90	0.67%	–
	0.90	1.00	1.57%	3.08%
		0.95	0.48%	–

with $r = 4.7\%$, PF_w for a fully funded plan with $z = 0.80$ and $K = 1.00$ is 8.69%. When the risk-free rate decreases by 0.5% to $r = 4.2\%$, the premium PF_w increases to 10.09% shown in Table 9. As r increases, the funding option price for a well funded plan decreases. If r increases to 5.2%, given $z = 0.80$ and $K = 1.00$, Table 10 indicates that the premium PF_w decreases to 7.30%.

However, we find a positive relation between the risk-free rate r and the funding option premium PF_u for a poorly funded plan presented in Tables 11 and 12. For example, as the risk-free rate increases from $r = 4.2\%$ to $r = 5.2\%$, given $PFI_0 = 75\%$, $z = 0.85$ and $K = 1.00$, PF_u increases from 4.10% to 4.62%. This can be explained by the fact that a higher risk-free rate r reduces pension liabilities and makes it easier for an under-funded plan to improve its funding status. This implies that triggering the funding option is more likely, leading to a higher PF_u .

Moreover, the buyout add-on premiums PR_w and PR_u decrease significantly when r increases, for both fully funded and poorly funded plans. This result is intuitively reasonable: the higher the risk-free rate r , the lower the investment risk premium required for a buyout, resulting in a lower immediate buyout price and also a smaller buyout option premium. For example, for a fully funded plan, given $r = 4.2\%$, $z = 0.80$ and $K = 1.00$, Table 9 shows that the buyout add-on premium is $PR_w = 5.91\%$. It decreases to $PR_w = 1.74\%$ when the risk-free rate increases to $r = 5.2\%$ presented in Table 10. Tables 11 and 12 report the same pattern for under-funded plans.

TABLE 9. Life-Time Funding and Buyout Option Premiums for Fully Funded Plans with Risk-Free Rate $r = 4.2\%$

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	0.80	1.00	10.09%	5.91%
		0.95	8.05%	–
		0.90	6.02%	–
	0.70	1.00	8.56%	3.40%
		0.90	6.06%	–
		0.80	3.56%	–

TABLE 10. Life-Time Funding and Buyout Option Premiums for Fully Funded Plans with Risk-Free Rate $r = 5.2\%$

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	0.80	1.00	7.30%	1.74%
		0.95	5.81%	–
		0.90	4.32%	–
	0.70	1.00	5.30%	0.88%
		0.90	3.74%	–
		0.80	2.19%	–

TABLE 11. Life-Time Funding and Buyout Option Premiums for Under-Funded Plans with Risk-Free Rate $r = 4.2\%$

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_u	Buyout Add-on PR_u
80%	0.85	1.00	5.17%	8.04%
		0.95	2.78%	–
		0.90	0.85%	–
	0.90	1.00	2.12%	5.97%
		0.95	0.65%	–
		1.00	4.10%	6.10%
75%	0.85	0.95	2.22%	–
		0.90	0.69%	–
	0.90	1.00	1.61%	4.42%
		0.95	0.50%	–

6.3. Impact of Asset Allocations. To explore the impact of asset allocations on the funding and buyout option risk premiums, we reduce the weight invested in the S&P 500 index from the base case $w_1 = 0.50$ to $w_1 = 0.25$ but increase the weight invested in the Merrill Lynch corporate bond

TABLE 12. Life-Time Funding and Buyout Option Premiums for Under-Funded Plans with Risk-Free Rate $r = 5.2\%$

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_u	Buyout Add-on PR_u
80%	0.85	1.00	5.61%	3.51%
		0.95	2.99%	–
		0.90	0.90%	–
	0.90	1.00	2.38%	2.75%
		0.95	0.72%	–
75%	0.85	1.00	4.62%	2.81%
		0.95	2.48%	–
		0.90	0.76%	–
	0.90	1.00	1.90%	2.17%
		0.95	0.58%	–

index from the base scenario $w_2 = 0.45$ to $w_2 = 0.70$. That is, the pension plan has a less risky investment portfolio. Given all other parameters are the same as those in the base case, Tables 13 and 14 provide the estimated funding option premiums and buyout add-on premiums for fully funded plans and under-funded plans.

As expected, the buyout add-on premiums with a lower weight in the S&P 500 index w_1 are lower than those in the base case. A lower proportion of the pension funds invested in the S&P 500 index reduces the volatility of the pension funding index PFI_t , which makes it less likely to trigger a pension option and provide a payoff to cover the buyout risk premium. As a result, the buyout add-on premiums decrease with the weight w_1 in the S&P 500 index. For example, as shown in Table 13, given $z = 0.80$ and $K = 1.00$, when w_1 is 0.25, half of the base case 0.50, the buyout add-on premium PR_w decreases from 3.54% to 2.42%, a 31.6% drop.

The funding option premiums for fully funded plans are determined by two factors: the probability of triggering a funding option and the payoff given the funding option is trigger. When a funding option is issued to a fully funded plan, with a lower weight $w_1 = 0.25$ in the S&P 500 index, it is less likely to trigger this funding option because the volatility of the pension fund is lower. Moreover, a lower volatility suggests a lower payoff if the funding index PFI_t falls below the trigger level z . Both effects explain lower funding option premiums for fully funded plans, which are shown in Table 13. For example, with $z = 0.70$ and $K = 0.90$, the funding option premium PF_w is only 2.48%, much lower than 4.83% in the base case.

TABLE 13. Life-Time Funding and Buyout Option Premiums for Fully Funded Plans with Investment Weights $w_1 = 0.25$, $w_2 = 0.70$ and $w_3 = 0.05$

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	0.80	1.00	5.86%	2.42%
		0.95	4.59%	–
		0.90	3.33%	–
	0.70	1.00	3.57%	0.94%
		0.90	2.48%	–
		0.80	1.40%	–

In contrast, the probability of triggering a funding option and the payoff given the funding option is trigger have opposing effects on funding option premiums for under-funded plans. On the one hand, a lower w_1 decreases the triggering probability. On the other hand, a lower w_1 increases the funding option payoff if the option is triggered. Which effect dominates depends on which effect imposes a greater effect. For example, Table 14 shows that when $PFI_0 = 80\%$, $z = 0.85$ and $K = 0.95$, with $w_1 = 0.25$, the funding option premium is $PF_u = 3.22\%$, higher than the base case $PF_u = 2.90\%$. This suggests that an increase in the funding option payoff dominates a decrease in the triggering probability. However, when $PFI_0 = 75\%$, $z = 0.85$ and $K = 1.00$, with $w_1 = 0.25$, the funding option premium is only $PF_u = 4.02\%$, lower than the base case $PF_u = 4.37\%$. In this case, a decrease in the triggering probability that reduces the funding option premium imposes a greater effect.

6.4. Impact of Longevity Risk. Finally, we analyze the impact of the longevity risk parameter λ on the prices of pension options. The longevity risk parameter λ will affect the option prices in two folds. First, the transformed morality rates are used to simulate the survival path of the pension cohort and thus, affect the value of a pension funding index and the premium of a funding option. On the other side, the longevity risk parameter λ is crucial in determining the longevity risk premium of an immediate buyout, which will then affect the buyout option price.

If the longevity risk parameter λ increases, the pension cohort is expected to live longer. All else equal, it will potentially deteriorate the funding status of a pension plan and make the funding options of a fully funded plan easier to be triggered. Changing λ by 50% from the base scenario $\lambda = 0.0943$,

TABLE 14. Life-Time Funding and Buyout Option Premiums for Under-Funded Plans with Investment Weights $w_1 = 0.25$, $w_2 = 0.70$ and $w_3 = 0.05$

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_u	Buyout Add-on PR_u
80%	0.85	1.00	5.61%	5.07%
		0.95	3.22%	–
		0.90	1.08%	–
	0.90	1.00	2.12%	3.28%
		0.95	0.71%	–
75%	0.85	1.00	4.02%	3.50%
		0.95	2.30%	–
		0.90	0.77%	–
	0.90	1.00	1.50%	2.25%
		0.95	0.50%	–

we see from Tables 15 and 16 that, as λ increases from 0.0472 to 0.1415, the funding option prices increase moderately for all tested scenarios. For example, the price of the funding option PF_w with $z = 0.80$ and $K = 0.95$ goes up from 6.54% to 7.22%. In contrast, a higher λ makes the funding option of a under-funded plan less likely to be triggered. Tables 17 and 18 show that the funding option prices of an under-funded plan PF_u decrease when λ increases from 0.0472 to 0.1415.

Compared to the moderate price change in funding options, the price movement of buyout add-ons is more significant. For fully funded plans, the buyout add-on premiums increase notably when the longevity risk parameter λ increases, as shown in Tables 15 and 16. For example, when λ increases from 0.0472 to 0.1415, for fully funded plans with $z = 0.80$ and $K = 1.00$, the buyout add-on premium PR_w increases from 2.80% to 4.29%, a 53% rise. This is consistent with the intuition that a larger λ will result in a higher longevity risk premium, and hence, a higher buyout add-on premium.

Tables 17 and 18 show the results for under-funded plans. In contrast to fully funded plans, a higher λ reduces the triggering probability of pension options for under-funded plans, resulting in lower funding option prices PF_u . However, a higher λ increases buyout option premia PR_u for poorly funded plans. For instance, in Table 17, the price of the buyout add-on for a 75% funded plan with $z = 0.85$ is $PR_u = 3.90\%$ when $\lambda = 0.0472$. Table 18 shows that the buyout add-on premium increases to 5.17% when λ goes up to 0.1415, even if the chance of triggering the option is lower in this case. This result shows the opposing effects of λ on the buyout add-on and the funding option for under-funded plans.

TABLE 15. Life-Time Funding and Buyout Option Premiums for Fully Funded Plans with Longevity Risk Parameter $\lambda = 0.0472$

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	0.80	1.00	8.20%	2.80%
		0.95	6.54%	–
		0.90	4.88%	–
	0.70	1.00	6.36%	1.46%
		0.90	4.49%	–
		0.80	2.63%	–

TABLE 16. Life-Time Funding and Buyout Option Premiums for Fully Funded Plans with Longevity Risk Parameter $\lambda = 0.1415$

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	0.80	1.00	9.05%	4.29%
		0.95	7.22%	–
		0.90	5.39%	–
	0.70	1.00	7.30%	2.38%
		0.90	5.17%	–
		0.80	3.03%	–

TABLE 17. Life-Time Funding and Buyout Option Premiums for Under-Funded Plans with Longevity Risk Parameter $\lambda = 0.0472$

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_u	Buyout Add-on PR_u
80%	0.85	1.00	5.45%	4.99%
		0.95	2.91%	–
		0.90	0.88%	–
	0.90	1.00	2.31%	3.84%
		0.95	0.71%	–
		1.00	4.44%	3.90%
75%	0.85	0.95	2.39%	–
		0.90	0.73%	–
	0.90	1.00	1.82%	2.94%
		0.95	0.56%	–

7. CONCLUSION

Operating DB pensions has become a more and more difficult business for a firm. Unexpected mandatory contributions to DB plans reduce resources available for business investments and adversely affect a firm's business performance. As a result, the phrase "pension buyout" has become

TABLE 18. Life-Time Funding and Buyout Option Premiums for Under-Funded Plans with Longevity Risk Parameter $\lambda = 0.1415$

Initial Funding Ratio PFI_0	Trigger z	Strike K	Funding Option Premium PF_u	Buyout Add-on PR_u
80%	0.85	1.00	5.38%	6.66%
		0.95	2.88%	–
		0.90	0.88%	–
	0.90	1.00	2.22%	5.05%
		0.95	0.68%	–
75%	0.85	1.00	4.30%	5.17%
		0.95	2.32%	–
		0.90	0.72%	–
	0.90	1.00	1.73%	3.88%
		0.95	0.53%	–

part of the pension plan lexicon. Despite this, removing pension liabilities from balance sheets can be expensive for DB firms, especially for those with poorly funded plans at the time when they decide to transfer pension risk with buyouts. This may explain why the buyout market has not fully taken off. To help develop this market, in this paper, we propose different funding and buyout options for currently well-funded and under-funded plans. To increase market liquidity and lower costs, we design a pension funding index with which to determine the trigger, strike level and payoff of a pension option. This pension funding index is based on publicly available mortality information and market indexes. A pension firm can select a pension option based on a pension funding index that could resemble its pension liability and asset dynamics. These pension options provide an effective way to reduce the impact of pension risk. They are likely to attract pension sponsors due to a low cost. With their expertise in managing pension risk, buyout annuity insurers and reinsurers could be the providers of these options.

We also study how to price pension options. We first model the evolutions of pension asset index and pension liability index, from which the dynamics of pension funding index are obtained. Then we calculate investment risk premium that compensates an insurer for its bearing the risk of uncertain pension asset returns and longevity risk premium that accounts for the risk that pension participants live longer than expected. We illustrate that pension options are a cost-effective way for underfunded plans to execute buyouts sooner. To show the robustness of our results, we explore how a value

change in a parameter affects pension option price. Our numerical examples show the reliability of our pricing models with reasonable results.

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