Capital, Risk, Insurance Prices and Regulation

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Abstract

We develop a unified equilibrium model of competitive insurance markets that incorporates the demand and supply of insurance as well as insurers' asset and liability risks. Insurers' assets may be exposed to both idiosyncratic and aggregate shocks. We obtain new insights into the relationship between insurance premia and insurers' internal capital that potentially reconcile the conflicting predictions of previous theories that investigate the relation using partial equilibrium frameworks. Equilibrium effects lead to a non-monotonic U-shaped relation between insurance price and internal capital. We study the effects of aggregate risk on the Pareto optimal asset and liquidity management by insurers as well as risk-sharing between insurers and insurees. When aggregate risk is low, both insurees and insurers hold no liquidity reserves, insurees are fully insured, and insurers bear all the aggregate risk. When aggregate risk takes intermediate values, both insurees and insurers still hold no liquidity reserves, but insurees partially share aggregate risk with insurers. When aggregate risk is high, however, it is optimal to hold nonzero liquidity reserves, and insurees partially share aggregate risk with insurers. The efficient asset and liquidity management policies as well as the aggregate risk allocation can be implemented through a regulatory intervention policy that combines a minimum liquidity requirement when aggregate risk is high, *ex post* taxation contingent on the aggregate state, comprehensive insurance policies that combine insurance with investment, and reinsurance.

1 Introduction

Financial institutions such as insurers and banks are usually required to hold sufficient equity capital on the liability side of their balance sheets and liquid reserves on the asset side as a buffer against the risk of insolvency, especially when their loss portfolios are imperfectly diversified and/or returns on their assets shrink dramatically. The financial crisis of 2007-2008 was precipitated by the presence of insufficient liquidity buffers and excessive debt levels in the financial system that made banks vulnerable to large aggregate negative shocks. In the context of insurers, the imperfect incorporation of the externality created by aggregate risk on their investment decisions when markets are incomplete may lead them to hold insufficient liquidity buffers to meet insurance liabilities. The resulting increase in insurer insolvency risk has an impact on the amount of insurance they can supply to insurees and, therefore, the degree of risk-sharing in the insurance market. Indeed, empirical evidence shows that, in response to Risk Based Capital (RBC) requirements, under-capitalized insurers not only increase their capital holdings to meet minimum capital requirements, but also take more risks to reach higher returns (Cummins and Sommer, 1996; Shim, 2010; Sager, 2002). Insurers' propensity to "reach for yield" contributes to their overall insolvency risk.[1] Aggregate risk may, therefore, lead to misallocation of capital and suboptimal risk sharing among insurees and insurers when markets are incomplete. To the best of our knowledge, however, the above arguments have yet to be theoretically formalized in an equilibrium framework that endogenizes the demand and supply of insurance as well as insurers' asset and liability risks. Such a framework could potentially shed light on the optimal regulation of insurance firms taking into account both the asset and liability sides of insurers' balance sheets.

We contribute to the literature by developing an equilibrium model of competitive insurance markets where insurers' assets may be exposed to idiosyncratic and aggregate shocks. In the unregulated economy, we show that the equilibrium insurance price varies non-monotonically in a U-shaped manner with the level of internal capital held by insurers. In other words, the insurance price decreases with a positive shock to internal capital when the internal capital is below a threshold, but increases when the internal capital is above the threshold. We thereby reconcile conflicting predictions in previous literature on the relation between insurance premia and internal capital

[1] Cox(1967) describes bank's tendency to invest in high risk loans with higher returns. Becker and Ivashina (2013)) support insurers' reaching for yield behavior by examining insurers' bond investment decisions

that are obtained in partial equilibrium frameworks that focus exclusively on either demand-side or supply-side forces. We also obtain the additional testable implications that an increase in insurers' asset risk, which raises the default probability, raises insurance premia and reduces coverage. We then proceed to derive insights into the solvency regulation of insurers by studying the benchmark "first best" economy in which there is perfect risk-sharing among insurers and insurees (so that they are only exposed to aggregate risk) and the effects of aggregate risk are fully internalized. We analyze the effects of aggregate risk on the Pareto optimal allocation of insurer capital to liquidity reserves and risky assets as well as risk sharing among insurees and insurers. We show that, when aggregate risk is below a threshold, it is Pareto optimal for insurers and insurees to hold zero liquidity reserves, insurees are fully insured, and insurers bear all aggregate risk. When aggregate risk takes intermediate values, both insurees and insurers still hold no liquidity reserves, but insurees partially share aggregate risk with insurers. When the aggregate risk is high, however, both insurees and insurers hold nonzero liquidity reserves, and insurees partially share aggregate risk with insurers. We demonstrate that the efficient allocation can be implemented through regulatory intervention that comprises of comprehensive insurance policies that combine insurance and investment, reinsurance, a minimum liquidity requirement when aggregate risk is high, as well as ex post budget-neutral taxation and subsidies contingent on the realized aggregate state.

Our model features two types of agents: a continuum of ex ante identical, risk averse insurees each facing a risk of incurring a loss in their endowment of capital, and a continuum of ex ante identical risk neutral insurers each endowed with a certain amount of internal "equity" capital. There is a storage technology/safe asset that provides a constant risk free return and a continuum of risky assets that generate higher expected returns than the risk free asset. Although both insurees and insurers can directly invest in the safe asset, only insurers have access to the risky assets. In addition to their risk-sharing function, insurance firms, therefore, also serve as intermediaries to channel individual capital into productive risky assets. Insuree losses are independently and identically distributed, but insurers' assets are exposed to aggregate risk. Specifically, a certain proportion of insurers is exposed to a common asset shock, while the remaining insurers' asset risks are idiosyncratic. A priori, it is unkown whether a particular insurer is exposed to the common or idiosyncratic shock. The proportion of insurers who are exposed to the common shock is, therefore, the natural measure of the aggregate risk in the economy. Insurees invest a portion of their capital in the risk-free asset and use the remaining capital to purchase insurance. Insurers invest their internal capital and the external capital raised from selling insurance claims in a portfolio of risk-free and risky assets.

We first derive the market equilibrium of the unregulated economy. In the unregulated economy, asset markets are incomplete because there are no traded securities contingent on the asset realizations of individual insurers or the aggregate state. Insurees make their insurance purchase decisions rationally anticipating insurers' investment strategy and default risk given their observations of insurers' internal capital, the size of the insurance pool, and the menu of traded insurance contracts that comprise of the insurance price (the premium per unit of insurance) and the face value of coverage. *Ceteris paribus*, an increase in insurers' internal capital or a decrease in asset risk increases the demand for insurance due to the lower likelihood of insurer insolvency. An increase in the risk of insuree losses leads to a decrease in insurance demand because it increases the proportion of insurees who suffer losses and, therefore, decreases the amount that each insuree recovers if he incurs a loss, but the insurance company is insolvent. Insurers, in turn, take the menu of traded insurance contracts as given and choose how many units of each contract to sell. There is free entry in that any contract that is expected to make positive expected profits for insurers is supplied. Competition among insurers then ensures that, in equilibrium, each insurer earns zero expected economic profits that incorporate the opportunity costs of internal capital that is used to make loss payments when insurers are insolvent. An increase in the insurance price, therefore, lowers the amount of insurance that each insurer sells in equilibrium leading to a downward sloping "zero economic profit" or "competitive" supply curve for insurance. An increase in the internal capital or an increase in asset risk, *ceteris paribus*, increases the opportunity costs of providing insurance, thereby increasing the amount of insurance that provides zero economic profits to insurers. An increase in the loss proportion increases the cost of claims, thereby pushing up the competitive supply level.

In competitive equilibrium, the insurance price is determined by market clearing—the demand for insurance must equal the supply—and zero economic profits for insurers. The insurance demand curve and the "zero economic profit" or "competitive" supply curve are both downward sloping with the demand curve being steeper due to the risk aversion of insurees. Consequently, any factor that increases the insurance demand curve, *ceteris paribus*, decreases the equilibrium price, while a factor that increases the competitive supply curve has a positive effect. We analytically characterize the competitive equilibrium of the economy and explore its comparative statics.

We demonstrate that there is a U-shaped relation between the insurance price and insurers' internal capital. Specifically, the insurance price decreases with a positive shock to internal capital when the internal capital is below a threshold, but increases when the internal capital is above the threshold. The intuition for the non-monotonic U-shaped relation hinges on the influence of both demand-side and supply-side factors. An increase in insurers' internal capital increases the competitive supply of insurance coverage because of the increased opportunity costs of internal capital. Because insurers are risk-neutral, however, the change in the competitive supply of insurance coverage is linear in the internal capital. On the demand side, an increase in insurers' internal capital increases insurers' insolvency buffer, thereby increasing the demand for insurance coverage. An increase in internal capital also increases the funds available for investment that further has a positive impact on the demand for insurance. The demand, however, is concave in the internal capital due to insurees' risk aversion. Because the competitive insurance supply varies linearly with capital, while the insurance demand is concave, there exists a threshold level of capital at which the demand effect equals the supply effect. Consequently, the demand effect dominates the supply effect so that the equilibrium premium rate goes down when the internal capital level is lower than the threshold. When the capital is above the threshold, the supply effect dominates so that the premium rate increases.

As suggested by the above discussion, equilibrium effects that integrate both demand side and supply side forces play a central role in driving the U-shaped relation between the insurance price and insurer capital. Our results, therefore, reconcile and further refine the opposing predictions for the relation in the literature that stem from a focus on only demand or only supply effects in partial equilibrium frameworks. Specifically, the "capacity constraints" theory, which focuses on the supply of insurance, predicts a negative relationship between insurance price and capital by assuming that insurers are free of insolvency risk (Gron, 1994; Winter, 1994). In contrast, the "risky debt" theory incorporates the default risk of insurers, but predicts a positive relationship between insurance price and capital (Doherty and Garven, 1986; Cummins, 1988, Cummins and Danzon, 1997). Empirical evidence on the relationship is also mixed. We make the simple, but fundamental point that the insurance price reflects the effects of capital on both the demand for insurance and the supply of insurance in equilibrium. We show that the relative dominance of demand-side and supply-side forces depends on the level of internal capital, thereby generating a U-shaped relation between price and internal capital.

Next, we show that an increase in insurers' asset risk, which increases their insolvency probability, increases the insurance price and reduces the insurance coverage in equilibrium. The intuition for the results again hinges on a subtle interplay between the effects of an increase in asset risk on insurance supply and demand. A positive shock to insurers' asset risk, *ceteris paribus*, has the *direct* effect of increasing the competitive supply of insurance coverage, that is, the level of insurance supply at which insurers earn zero economic profits. Consequently, the amount of funds available to pay loss claims in distress increases, thereby having the *indirect* effect of increasing the demand for insurance. On the other hand, an increase in the asset risk increases the insurers' insolvency probability that has a negative effect on the demand for insurance. We show that, under reasonable conditions, the direct effect outweighs the indirect effect. Consequently, an increase in asset risk reduces insurance demand, but increases the zero-economic-profit supply level, thereby increasing the insurance price and decreasing the coverage level in equilibrium. Our results imply that the response to the increased asset risk of insurance firms is the shift of insuree's capital accumulation from indirect investment in risky assets to direct storage in safe assets.

We then proceed to analyze the implications of our framework for the solvency regulation of insurers by analyzing the benchmark "first best" economy in which aggregate risk is fully internalized and there is perfect risk-sharing among insurees and insurers. We derive the Pareto optimal allocation of insurer capital between the safe asset (liquidity reserves) and risky assets as well as the sharing of risk between insurers and insurees. When the aggregate risk is low, there is sufficient aggregate capital in the economy to provide full insurance to insurees so that insurers bear all the aggregate risk. Further, because the expected return from risky assets exceeds the risk-free return, it is optimal to allocate all capital to risky assets so that neither insurers nor insurees have holdings in the risk-free asset. When aggregate risk takes intermediate values, insurees cannot be provided with full insurance because of the limited liability of insurers in the bad aggregate state. Consequently, insurers and insurees share aggregate risk, but it is still optimal to exploit the higher expected surplus generated by the risky assets so that all the capital in the economy is invested in the risky assets. When aggregate risk is very high, however, risk-averse insurees would bear excessively high losses in the bad aggregate state if all capital were invested in risky assets. Consequently, both insurees and insurers hold positive liquidity reserves, and share aggregate risk.

We demonstrate that a regulator/social planner can implement the first-best allocation policies through a combination of comprehensive insurance policies sold by insurers that combine insurance with investment, reinsurance, a minimum liquidity requirement, and ex post budget-neutral taxation that is contingent on the aggregate state. The comprehensive insurance policies provide direct access to the risky assets for insurees. Reinsurance achieves risk-sharing among insurers, while ex post taxation transfers funds from solvent to insolvent insurers. The minimum liquidity requrement, which is only imposed when aggregate risk exceeds a threshold, forces insurers to maintain the first best level of liquidity reserves.

The plan for the paper is as follows. We further discuss related literature in Section 2. We present the model in Section 3 and derive the equilibrium of the unregulated economy in Section 3.1. We analyze the impact of aggregate risk and regulation in Section 4. Section 5 concludes. We relegate all proofs to the Appendix.

2 Related Literature

Two streams of the literature investigate the relation between insurer capital and insurance premia. The first branch proposes the “capacity constraint" theory, which assumes that insurers are free from insolvency risk. The prediction of an inverse relation between insurance price and capitalization crucially hinges on the assumption that insurers are limited by regulations or by infinitely risk averse policyholders so that they can only sell an amount of insurance that is consistent with zero insolvency risk (e.g.,Gron, 1994; Winter, 1994). Winter(1994) explains the variation in insurance premia over the “insurance cycle" using a dynamic model. Empirical tests using industry-level data prior to 1980 support the predicted inverse relation between insurance capital and price, but data from the 1980s do not support the prediction. Gron (1994) finds support for the result using data on short-tail lines of business. Cagle and Harrington (1995) predict that the insurance price increases by less than the amount needed to shift the cost of the shock to capital given inelastic industry demand with respect to price and capital.

Another significant stream of literature—the “risky corporate debt” theory—incorporates the possibility of insurer insolvency and predicts a positive relation between insurance price and capitalization (e.g., Doherty and Garven, 1986; Cummins, 1988). The studies in this strand of the literature emphasize that, because insurers are not free of insolvency risk in reality, the pricing of insurance should incorporate the possibility of insurers' financial distress. Higher capitalization levels reduce the chance of insurer default, thereby leading to a higher price of insurance associated with a higher amount of capital. Cummins and Danzon (1997) show evidence that the insurance price declines in response to the loss shocks in the mid-1980s that depleted insurer's capital using data from 1976 to 1987. While the "capacity constraint" theory concentrates on the supply of insurance, "the pricing of risky debt" theory focuses on capital's influence on the quality of insurance firms and, therefore, the demand for insurance. The empirical studies support the mixed results for different periods and business lines.

We complement the above streams of the literature by integrating demand-side and supply-side forces in an equilibrium framework. We show that there is a U-shaped relation between price and internal capital. In contrast with the literature on "risk debt pricing", which assumes an exogenous process for the asset value, we endogenize the asset value which depends on the total invested capital including both internal capital and capital raised through the selling of insurance policies. Insurers' assets and total liabilities are, therefore, simultaneously determined in equilibrium in our analysis.

Our paper is also related to the studies that examine the relation between capital holdings and risk taking of insurance companies. Cummins and Sommer (1996) empirically show that insurers hold more capital and choose higher portfolio risks to achieve their desired overall insolvency risk using data from 1979 to 1990. It is argued that insurers response to the adoption of RBC requirements in both property-liability and life insurance industry by increasing capital holdings to avoid regulation costs, and by investing in riskier assets to obtain high yields (e.g.,Baranoff and Sager, 2002; Shim, 2010). Insurers are hypothesized to choose risk levels and capitalization to achieve target solvency levels in response to buyers' demand for safety. Filipovic, Kremslehner and Muermann (2015) show that limited liability creates an incentive for insurers to engage in risk-shifting, thereby transferring wealth from policy holders, and that solvency capital requirements that restrict investment and premium policies can improve efficiency. Our paper fits into this literature by studying the response of the market price to shocks to insurers' internal capital as well as aggregate and idiosyncratic shocks to insurers' ssets in an equilibrium framework. Our results shed light on the solvency regulation of insurance firms by incorporating the liability and asset sides of insurers' balance sheets. We show that efficient allocations can be implemented through comprehensive insurance policies sold by insurers that combine insurance with investment, reinsurance, a minimum liquidity requirement, and ex post budget-neutral taxation contingent on the aggregate state. The tradeoff between holding sufficient capital to meet insurance liabilities and diverting capital to the most productive assets implies that a liquidity requirement should be imposed only when aggregate risk is sufficiently high.

Our paper also contributes to the literature on capital allocation and insurance pricing. Zanjani (2002) argues that price differences across markets are driven by different capital requirements to maintain solvency assuming that capital is costly to hold. Our paper endogenizes the cost of capital in terms of the opportunity cost of holding internal capital, which is used to pay for loss claims when insurers default. We highight insurees' and insurers' responses to internal capital shocks. Consequently, insurance prices reflect insurees' demand for safety and insurers' abilities to provide insurance with imperfect protection.

3 The Model

We consider a single-period economy with two dates 0 and 1. There is a single consumption/capital good. There are two types of agents: a continuum of measure 1 of risk-averse *insurees* or *policy holders* and a continuum of measure 1 of risk-neutral *insurance firms*. Each insuree is endowed with 1 unit of capital at date 0 and has a logarithmic utility function. Each insurance firm is endowed with K units of "internal" capital. There is a storage technology/safe asset that is in sufficiently large supply that it provides a constant return of R_f per unit of capital invested.

At date 1, an insuree i can incur a loss $l \leq 1$ so that a portion of each insuree's endowment is at risk. Losses are independently and identically distributed across insurees. Each insuree's loss probability is p. At date 0, each insuree invests a portion of her capital in the safe asset and the remainder in buying an insurance contract, (κ, C), where κ is the premium per unit of loss and C is the face value of insurance coverage. Similar to Rothschild and Stiglitz (1976), we consider an insurance market in which insurance contracts, $\Phi \equiv \{(\kappa, C); \kappa > 0, C > 0\}$ that combine the "price" of insurance and the "quantity" of insurance are traded. Each insuree chooses a single contract from the set of traded contracts. Insurers have internal capital K and raise external capital by selling insurance contracts. Insurers and insurees take the set of insurance contracts Φas given in making their supply and demand decisions, respectively. The set Φ is such that any contract that is demanded and expected to be profitable for an insurance company is supplied.

Each insurance firm j has access to a risky technology that generates a return of R_H per unit of invested capital with probability $1-q$ when it "succeeds" but $R_L < R_H$ with probability q when it "fails." Insurance firms first raise capital in insurance markets and then invest it. Further, insurance firms *cannot commit* to their investment policy when they raise capital. A proportion $1-\tau$ of insurance firms are exposed to idiosyncratic technology shocks, that is, the technology shocks are independently and identically distributed for this group of insurance firms. The remaining proportion τ of insurers are, however, exposed to a *common* shock, that is, the technology shock described above is the *same* for these insurers. Although insurers know that a proportion τ of them is exposed to a common shock, an individual insurer does not know whether it is exposed to an idiosyncratic or common shock a priori. τ is a measure of the *aggregate risk* in the economy.

We assume that

$$(1-q)R_H + qR_L \geq R_f. \tag{1}$$

The above condition ensures that the expected return on the risky project is at least as great as the risk-free rate. Although policy holders can directly invest in the safe asset, only insurance firms have access to the production technology. Consequently, in addition to the provision of insurance to policy holders, insurance firms also play important roles as financial intermediaries who channel the capital supplied by policy holders to productive assets. In addition to the fact that insurees do not have direct access to asset markets in the unregulated economy, asset markets are incomplete because there are no traded securities contingent on the asset realizations of individual insurers or the realization of the aggregate shock.

Let C_j be the *total* face value of insurance contracts sold by insurer j and K_j be the external capital it raises. The insurer can invest its total capital, $K + K_j$ in a portfolio comprising of the risk-free storage technology and the risky project. In an autarkic economy with no regulation, it follows from condition (1), and the fact that insurance firms cannot commit to their investment policy when they raise capital by selling insurance contracts, that it is optimal for risk-neutral insurance firms to invest their entire capital in the risky technology. By our earlier discussion, the total liability of the insurer j is pC_j because a proportion p of its pool of insurees incur losses.

Insurers default if their total liability cannot be covered by the total investment returns when the risky technology fails, that is when

$$pC_j > (K + K_j)R_L. \tag{2}$$

In the event of default, the total available capital of an insurer is split up among insurees in proportion to their respective indemnities. The internal capital plays the role of a buffer that increases an insurer's capacity to meet its liabilities and, thereby, the amount of insurance it can sell. The cost of holding internal capital in our model is an opportunity cost, which refers to the returns from the invested internal capital that are depleted to pay out liabilities when insurers default.

Each individual insuree observes the total capital, $K + K_j$, held by each insurer j in marking her insurance purchase decision. In making the decision on the level of insurance coverage to purchase, insurees rationally anticipate the possibility of default, and the amount they will be paid for a loss when insurers' asset returns are insufficient to pay out the aggregate loss claims as shown by (2).

3.1 The Equilibrium of the Unregulated Economy

We now derive the equilibrium of the unregulated economy by analyzing the demand and supply of insurance by insurees and insurers, respectively. In equilibrium, the demand for insurance equals the supply.

3.1.1 Insurance Demand

Each insuree chooses its portfolio, which comprises of his investment in the safe asset (self-insurance) and his choice of insurance contract, to maximize his expected utility. Without insurance, each insuree i's expected utility is given by the autarkic utility level,

$$\text{Autarkic Utility } = p\ln(R_f - l) + (1-p)\ln(R_f). \tag{3}$$

Insurees take the set, Φ, of traded insurance contracts as given in making their purchase decisions. Each insuree observes the total capital held by each insurer and, therefore, rationally anticipates the possibility that she may not be fully indemnified in the scenario where the insuree incurs losses, but the insurer is insolvent. Insurees also rationally incorporate insurers' investment portfolio choices in making their insurance demand decisions. As previously stated, insurers invest all their capital in the risky technology, thereby causing insurers to be likely to default in the "bad" state where the technology fails. The likelihood that insurees' loss claims may not be fully indemnified is then affected by the risk in the investment portfolio of insurance firms and the total liabilities insured by them. In general, the loss payment obtained by each insuree is determined by three factors: the proportion of insurees in the insurer's pool who incur losses, the total amount of capital held by the insurer, and the return of the insurer's investment project.

Given that insurees and insurers are ex ante identical, we focus on *symmetric equilibria* where insurees make identical portfolio choices and insurers have ex ante identical pools of insurees. Without loss of generality, therefore, we focus on a representative insurer and a representative insuree. Suppose that the representative insuree chooses the contract (κ, C_d). If C_s is the *total face value* of the insurance contracts sold by the insurer, its total capital is $K + \kappa C_s$. The insurer's available capital if its project fails is, therefore, $(K + \kappa C_s)R_L$. Consequently, the payment received by each insuree who incurs a loss when the insurer's project fails is $\min(C_d, \frac{(K+\kappa C_s)R_L}{p})$. It is clear from our subsequent results that it is suboptimal for the insurer to sell so much coverage that it is unable to meet losses in the "good" state where its project succeeds. In the following, therefore, we assume this result to avoid unnecessarily complicating the exposition.

If the representative insuree chooses a contract from the subset of available contracts, $\{(\kappa, C_d)\,; C_d > 0\}$, for which the premium per unit of coverage is κ, she chooses the contract that maximizes her expected utility, that is, the insuree's choice of the face value of coverage solves

$$\begin{aligned}\max_{C_d} & \overbrace{p(1-q)\ln\left[(1-\kappa C_d)R_f - l + C_d\right]}^{\text{insuree incurs loss in insurer's ``good'' state}} + \\ & \overbrace{pq\ln\left[(1-\kappa C_d)R_f - l + \min(C_d, \frac{(K+\kappa C_s)R_L}{p})\right]}^{\text{insuree incurs loss in insurer's ``bad'' state}} \\ & + \overbrace{(1-p)\ln\left[(1-\kappa C_d)R_f\right]}^{\text{insuree does not incur loss}} \qquad (4)\end{aligned}$$

such that

$$\kappa C_d \leq 1 \tag{5}$$

As is clear from the above, the representative insuree makes her insurance purchase decision based on her probability of a loss and the probability that the insurer's assets fail. Because she observes the insurer's total capital when she makes her decision, the insuree's decision rationally incorporates the proportion of the population of insurees that will incur losses.

The properties of the logarithmic utility function guarantee that it is suboptimal for insurees to invest all their capital in risky insurance so that the budget constraint, (5) is not binding. The necessary and sufficient first order condition for the insuree's optimal choice of insurance coverage, C_d^*, is

$$\begin{aligned}&\left\{\frac{p(1-q)(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{pq\kappa R_f}{(1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s)R_L}{p}} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f}\right\} \cdot \mathbf{1}_{\{pC_d^* \geq (K+\kappa C_s)R_L\}} \\ &+ \left\{\frac{p(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f}\right\} \cdot \mathbf{1}_{\{pC_d^* < (K+\kappa C_s)R_L\}} = 0\end{aligned} \tag{6}$$

The solution to the above equation can be expressed as a function, $C_d^*(K, C_s, \kappa)$, where we suppress the dependence of the optimal demand on the liability and asset risk parameters, p and q, and the safe asset return, R_f, to simplify the notation.

The following lemma characterize the insuree's optimal demand for insurance coverage for a given insurance premium rate, κ. The optimal demand depends on whether or not the representative insurer defaults in the bad state where its assets fail.

Lemma 1 • *If the representative insurer defaults in the "bad" state where its assets fail, the optimal insurance demand C_d^* is given by*

$$C_d^* = C_d^*(K, C_s, \kappa), \tag{7}$$

where $C_d^*(K, C_s, \kappa)$ satisfies equation

$$\frac{p(1-q)(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{pq\kappa R_f}{(1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s)R_L}{p}} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f} = 0 \tag{8}$$

- *If the representative insurer does not default in the "bad" state where its assets fail, the optimal insurance demand* C_d^**is given by*

$$C_d^* = C_d^*(\kappa), \tag{9}$$

where $C_d^*(\kappa)$ *satisfies*

$$\frac{p(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f} = 0 \tag{10}$$

By (8) and (10), we note that the insurer's internal capital, K, total supply, C_s, and asset risk parameter, q, influence the optimal demand for insurance coverage only when insurees foresee insurer insolvency in the "bad" state, where its assets fail. For generality, we allow for the case that the market insurance premium rate might lead to over insurance, i.e., $C_d > l$.

The following lemma shows how the optimal demand for insurance coverage varies with the fundamental parameters of the model that will be useful when we derive the equilibrium of the economy.

Lemma 2 (Variation of Insurance Demand) *The optimal demand for insurance,* C_d^**, (i) decreases with the premium rate,* κ*; (ii) decreases with the return,* R_f*, on the safe asset; (iii) increases with insurers' internal capital,* K*; (iv) increases with the total face value of policies sold by the insurer,* C_s*; (v) increases with the insurer's asset return in the low state,* R_L*; and (vi) decreases with the insurer's expected probability of failure;* q*.*

The optimal demand for insurance claims reflects the tradeoff between self-insurance through investments in the safe asset and the purchase of insurance coverage with potential default risk for the insurer and, therefore, imperfect insurance for the insuree. Capital allocated in safe assets plays an alternative role in buffering the losses that cannot be indemnified by insurers when their assets fail. The insurance demand decreases with the insurance premium rate, that is, the demand curve is downward-sloping, since the utility function of insurees satisfies the properties highlighted by Hoy and Robson (1981) for insurance to be a normal good. An increase in the risk-free return raises the autarkic utility level, thereby diminishing the demand for insurance coverage.

In addition to functioning as a risk warehouse, which absorbs and diversifies each insuree's idiosyncratic loss, insurance firms also serve as financial intermediaries who channel external capital supplied by policyholders to productive assets. In our model, the overall insolvency risk faced by insurance firms are simultaneously determined by the asset and liability sides of insurer's balance sheets. An increase in the aggregate loss proportion of the insuree pool; a decrease in the internal capital held by insurers; a decrease in the amount of external capital raised by the insurer from selling insurance; and a decrease in the asset return in the low state all lower the insurance coverage of an insuree when the insurer is insolvent so that the optimal insurance demand declines.

3.1.2 Insurance Supply

Each insurer chooses which contracts from the set, Φ, to supply and the number of units of each contract to maximize its total net expected payoffs from providing insurance for insurees and investing the capital it raises. As discussed earlier, in the absence of regulatory intervention, it is optimal for each insurer to invest its entire capital in the risky project due to its risk neutrality and the asset return condition (1). Recall that an insurer cannot commit to its investment policy when it raises external capital by selling insurance contracts. An insurer chooses to supply insurance if and only if its expected net profits are at least as great as its autarkic expected payoff, that is, its expected payoff from not selling insurance and investing its internal capital. An insurer's autarkic expected payoff is

$$\text{Autarkic Expected Payoff} = K\left(\left(1-q\right)R_H + qR_L\right). \tag{11}$$

Each insurer makes its supply decision knowing the proportion, p, of its pool of insurees who will incur losses. In the bad state where its technology fails, if its available capital is lower than the total loss payments to insurees, then the capital is divided equally among the insurees. If the insurer chooses to sell contracts for which the premium per unit of coverage is κ, the optimal supply of insurance coverage solves

$$\max_{C_s} \left\{\left(1-q\right)\left(\left(K+\kappa C_s\right)R_H - pC_s\right)\right\} + \left\{q\left(\left(K+\kappa C_s\right)R_L - pC_s\right)\right\} \cdot \mathbf{1}_{\{pC_s \le (K+\kappa C_s)R_L\}} \tag{12}$$

such that

$$\{(1-q)\left(\left(K+\kappa C_s\right)R_H - pC_s\right)\} + \{q\left(\left(K+\kappa C_s\right)R_L - pC_s\right)\} \cdot \mathbf{1}_{\{pC_s \leq (K+\kappa C_s)R_L\}}$$
$$\geq K\left((1-q)R_H + qR_L\right) \qquad (P.C) \tag{13}$$

The participation constraint, (13), ensures that the insurer chooses to sell a nonzero amount of coverage if and only if its expected net profit exceeds its expected payoff in autarky,, that is, its expected *economic profit* (profit in excess of the autarkic level) is nonnegative. From (12) and (13), it is clear that it is optimal for the insurer to supply no coverage if the premium rate, $\kappa < \frac{p}{R_H}$ and infinite coverage if $\kappa > \frac{p}{R_L}$. In equilibrium, therefore, we must have $\kappa \in [\frac{p}{R_H}, \frac{p}{R_L}]$. It also follows from the linearity of the objective function, and the fact that any insurance contract that makes nonnegative expected economic profit for an insurer is supplied, that the participation constraint, (13), must bind in equilibrium, that is, insurers make zero expected *economic profits.* Consequently, the *zero economic profit supply* of insurance coverage for any premium rate $\kappa \in [\frac{p}{R_H}, \frac{p}{R_L}]$, which we hereafter refer to as the *competitive insurance supply* for expositional convenience, is

$$C_s^*(K,\kappa) = \frac{qKR_L}{(1-q)(\kappa R_H - p)} \tag{14}$$

Lemma 3 (Competitive Insurance Supply) *For $\kappa \in (\frac{p}{R_H}, \frac{p}{R_L})$, the competitive insurance supply level, $C_s^*(K,\kappa)$, (i) decreases with the insurance premium rate, κ; (ii) increases with insurers' internal capital, K; (iii) increases with insurers' expected default probability, q; (iv) increases with the asset return, R_L, in the bad state; and (v) increases with the loss probability of insurees, p.*

An increase in the premium rate increases the expected return from supplying insurance and, therefore, decreases the coverage level at which each insurer's participation constraint, (13), is binding. For given $\kappa \in (\frac{p}{R_H}, \frac{p}{R_L})$, an increase in the insurer's internal capital, asset risk, or the aggregate risk of the pool of insurees lowers the expected returns from providing insurance and, therefore, increases the competitive insurance supply level.

3.1.3 Insurance Market Equilibria

We now derive the insurance market equilibrium that is characterized by the insurance price (per unit of coverage) κ^*. The equilibrium satisfies the following conditions.

1. The face value of coverage supplied by each insurer is $C_s^*(K, \kappa^*)$ given by (14) and insurers make zero expected economic profits.

2. The coverage purchased by each insuree is $C_d^*(K, C_s^*(K, \kappa^*), \kappa^*)$ given by (7) and (9).

3. The equilibrium price κ^* clears the market, that is, $C_d^*(K, C_s^*(K, \kappa^*), \kappa^*) = C_s^*(K, \kappa^*) = C^*$.

The following proposition characterizes the equilibria of the insurance market. We begin with some necessary definitions. Define the expected return from the insurer's risky technology,

$$ER = (1 - q)R_H + qR_L. \tag{15}$$

Define the excess demand function

$$F(K, \kappa) = C_d^*(K, C_s^*(\kappa), \kappa) - C_s^*(K, \kappa), \tag{16}$$

where $C_d^*(K, C_s^*(\kappa), \kappa)$ is the demand function described by Lemma (1) and $C_s^*(K, \kappa)$ is given by ((14)).

Proposition 1 (Insurance Market Equilibria)

- *Suppose $K \leq \overline{K}_1$, where $\overline{K}_1$is given by*

$$F(K = \overline{K}_1, \kappa = \frac{p}{ER}) = 0 \tag{17}$$

In equilibrium, insurers default in the "bad" state when their assets fail. The equilibrium price, κ^*, satisfies:

$$F(K, \kappa^*) = 0. \tag{18}$$

- *Suppose $K > \overline{K}_1$. In equilibrium, insurers do not default in the "bad" state where their assets fail. The equilibrium insurance price is $\kappa^* = \frac{p}{ER}$ and the equilibrium coverage level, C^*, is* *given by*

$$C^* = \frac{p}{\kappa^*} - \frac{(1-p)(R_f - l)}{1-\kappa^* R_f} > l.$$

The above proposition shows that there are two possible equilibria that are determined by the internal capital of insurers. When the internal capital is lower than the threshold level $\overline{K}_1$, the representative insurer defaults in the "bad" state that is rationally foreseen by all agents. When the internal capital is higher than the threshold $\overline{K}_1$, the insurer faces no insolvency risk and this is rationally anticipated by all agents. The equilibrium insurance price is determined by the aggregate loss proportion of insurees adjusted by the expected return from the risky technology, $\frac{p}{ER}$, at which the insurer's participation constraint (13) is binding.

We focus on the more interesting first scenario in which insurers with insufficient internal capital default after their technologies fail. Figure 1 shows the equilibrium. The equilibrium price κ^* and coverage level C^*, satisfy the system of equations (37),(8) and (14). In addition, the condition (38) ensures that the insurer, indeed, defaults in the bad state. The equilibrium premium rate must be greater than $\frac{p}{R_H}$. To ensure that the insurer defaults in its "bad" state, the equilibrium insurance rate must be less than $\frac{p}{ER}$.[2] The demand curve for insurance coverage and the competitive supply curve are both downward sloping, but the demand curve is steeper than the supply curve. Thus the existence condition for a solution κ^* to the system of equations is

$$F(\kappa)|_{\kappa \to \frac{p}{ER}} = \lim_{\kappa \to \frac{p}{ER}} C_d^*(\kappa, C_s^*(\kappa)) - \lim_{\kappa \to \frac{p}{ER}} C_s^*(\kappa) > 0. \tag{19}$$

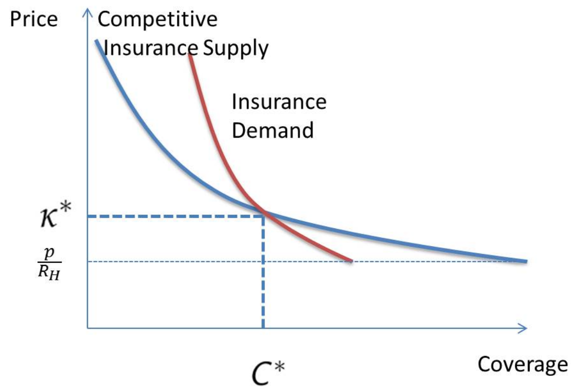
As shown in the proof of Proposition 1 in Appendix, condition (19) are satisfied when internal capital, K, is less than $\overline{K}_1$. Any solution, κ^*, to the system of equations also satisfy the following constraint:

$$\frac{qKR}{(1-q)(\kappa^* R_H - p)} < \frac{1}{\kappa^*}$$

so that κ^* is indeed the equilibrium premium rate. Since an individual insurer can deviate and supply contracts with lower premia that still ensure nonnegative economic profits, the equilibrium

[2] In general, the equilibrium insurance price can not exceed $\frac{p}{ER}$. The intuition is that,if the insurer's internal capital is level greater than $\overline{K}_1$, price higher than $\frac{p}{ER}$ will make the insurer positive economic profit; if insurer's internal capital is less than $\overline{K}_1$, price higher than $\frac{p}{ER}$ will make the insurer still solvent in the "bad" state where its asset fails.

Figure 1: Insurance Market Equilibrium



premium must be the smallest κ^* at which $\frac{\partial F(\kappa)}{\partial \kappa}|_{\kappa^*} > 0$.

We next identify the effects of shocks in the economy on the equilibrium insurance price, coverage level and social welfare.

3.2 The Effects of Capital and Risk

3.2.1 Internal Capital

Internal capital influences the equilibrium insurance price through the demand for and supply of insurance. By (14), an increase in internal capital increases the competitive insurance supply level. There are both direct and indirect effects of an increase in internal capital on the demand for insurance. An increase in internal capital has the direct effect of increasing the demand for insurance because of the higher available capital to meet insurance claims. The demand for insurance coverage is further enlarged by the insurees' anticipation of the increase in the competitive supply of insurance with the increase in internal capital. Consequently, the overall effect of internal capital on the demand for insurance is also positive. The net effects of an increase in internal capital on the equilibrium premium rate depend on the relative dominance of demand-side and supply-side effects.

The equilibrium price κ^* satisfies $\frac{\partial F(K,\kappa)}{\partial \kappa}|_{\kappa=\kappa^*} > 0$. The marginal effects of internal capital on the insurance price can be understood through its effects on the excess demand function.

$$\frac{\partial F(K,\kappa^*)}{\partial K} = \underbrace{\frac{\partial C_d^*(K,C_s^*,\kappa^*)}{\partial K}}_{\text{direct effect on demand}} + \underbrace{\frac{\partial C_d(K,C_s^*,\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*(K,\kappa^*)}{\partial K}}_{\text{indirect effect on demand}} - \underbrace{\frac{\partial C_s^*(K,\kappa^*)}{\partial K}}_{\text{direct effect on competitive supply}}.$$

The following proposition describes the effects of internal capital on the equilibrium insurance price.

Proposition 2 **(The Effects of Internal Capital)**

- *Suppose* $\frac{\partial F(K,\kappa^*)}{\partial K}|_{K\to 0} > 0$*. There exist a threshold* $\widetilde{K}$ *such that the equilibrium premium rate* κ^* *decreases with internal capital when* $K < \widetilde{K}$*, and increases when* $K > \widetilde{K}$*.*
- *Suppose* $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} < 0$*, the equilibrium premium rate increases with the amount of internal capital.*

Let $\widetilde{\kappa}$ *be the equilibrium premium rate corresponding to the internal capital level* $\widetilde{K}$ *.* $\widetilde{K}$ *and* $\widetilde{\kappa}$ *are jointly determined by the following two equations:*

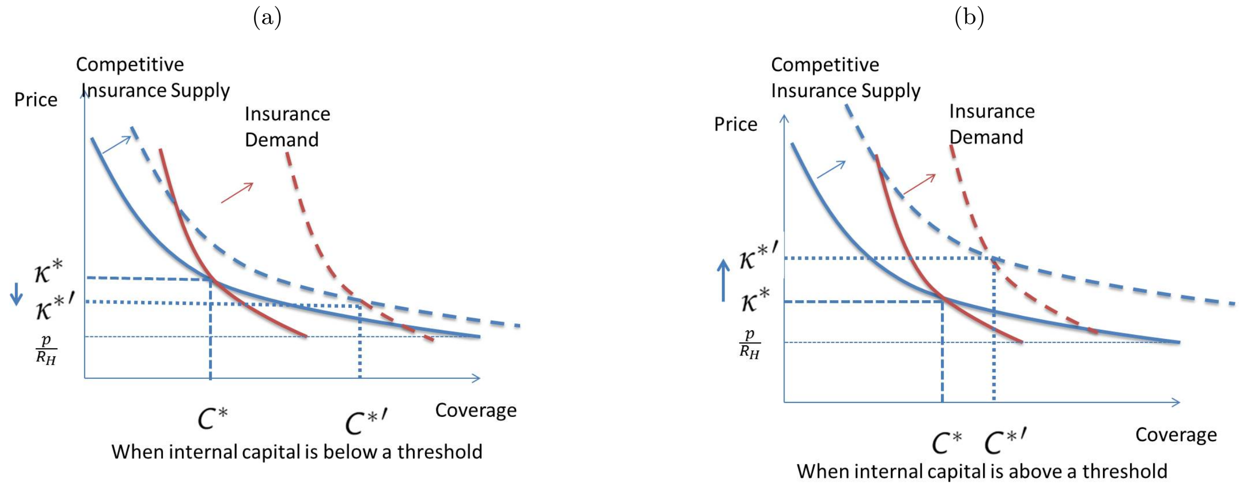
$$\frac{\partial F(K,\kappa^*)}{\partial K}|_{K=\widetilde{K},\kappa^*=\widetilde{\kappa};} = 0 \tag{20}$$

$$C_d^*\left(\widetilde{K}, C_s^*(\widetilde{\kappa},\widetilde{K}),\widetilde{\kappa},\right) = C_s^*(\widetilde{K},\widetilde{\kappa}). \tag{21}$$

The above proposition shows that the insurance premium decreases with insurers' internal capital when the internal capital level is relatively low, while it increases with insurers' internal capital when its level is relatively high. This result reconciles the conflicting predictions on the relation between insurance price and capital in previous literature. The "capacity constraint" theory relies on the assumption that insurance firms are free of insolvency. Winter (1990) argues that insurance firms can only write the volume of business consistent with zero insolvency due to regulation. The total capital amount determines the capacity of the insurance market. A significant negative shock to insurer capital shrinks the supply of insurance in imperfect capital markets. It follows that the insurance price increases and insurance coverage declines while the demand for insurance is not affected in the absence of insurer insolvency. The "pricing of risky debt" theory incorporates the insolvency risk of insurance firms. Cummins and Sommer (1996) theoretically show both a postive and negative relation between price and a retroactive loss shock based on an optimal endogenous capitalization structure of insurance firms.

As mentioned earlier, an increase in internal capital increases the insurance demand and supply so the net impact depends on which of the two effects is dominant. By (14), the competitive supply

Figure 2: Effects of Internal Capital



of insurance is linear in the internal capital level. Because insurees are risk-averse, the demand for insurance is concave in the insurer's internal capital. Consequently, the excess demand function, $F(K, \kappa^*)$, is concave in the internal capital, that is,

If $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} > 0$, then there exists, in general, a threshold level of internal capital, $\widetilde{K}$, at which the marginal effect of internal capital on the excess demand is zero. It follows from the concavity of the excess demand that the marginal effect of internal capital on the excess demand is positive for $K < \widetilde{K}$ and negative for $K > \widetilde{K}$. In other words, the risk aversion of insurees causes the "demand effect" of an increase in internal capital on the insurance price to dominate the "supply effect" for $K < \widetilde{K}$ and vice versa for $K > \widetilde{K}$. Hence, the equilibrium insurance premium varies in a U-shaped manner with the level of internal capital. If $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} \leq 0$, then the marginal effect of internal capital on the excess demand is always non-positive so that the equilibrium insurance premium increases with internal capital.

3.2.2 The Effects of Asset Risk

We now address the impacts of the representative insurer's asset risk on the equilibrium premium rate and insurance coverage. The presence of asset induced insolvency complicates the decisions on both the demand and supply sides. The impact of asset risk on insurance supply indirectly influences insurance demand by affecting the total capital available to the insurer to meet liabilities in insolvency. Specifically, it follows from (14) that an increase in asset risk increases the competitive insurance supply level. Because insurees rationally foresee the likelihood that their losses will not be fully indemnified by insurers, the direct effect of an increase in asset risk on insurance demandis negative. The increase in the competitive supply level with asset risk, however, increases the amount each insuree is able to recover if it incurs a loss, but the insurer is insolvent. The indirect impact of an increase in asset risk on insurancey demand is, therefore, positive. The net impact of asset risk on the equilibrium premium rateis determined via its effect on the excess demand function,

$$\frac{\partial F(\kappa^*, q)}{\partial q} = \underbrace{\frac{\partial C_d(\kappa^*, q)}{\partial q}}_{\text{direct effect on demand}<0} + \underbrace{\frac{\partial C_d(\kappa^*, q)}{\partial C_s^*}\frac{\partial C_s^*}{\partial q}}_{\text{indirect effect on supply}>0} - \underbrace{\frac{\partial C_s^*(\kappa^*, q)}{\partial q}}_{\text{direct effect on zero-economic-profit supply}>0}, \tag{22}$$

where we explicitly indicate the dependence of the demand and supply functions on the asset risk parameter, q. The following proposition characterizes the effect of asset risk on the equilibrium insurance price and coverage.

Proposition 3 (The Effects of Asset Risk) *Suppose $\frac{R_L}{p} < R_f$. The equilibrium insurance price increases with the asset risk, q, while the coverage level declines. If $\frac{R_L}{p} \geq R_f$, then the effect of asset risk on the insurance price is ambiguous.*

The intuition for the condition $\frac{R_L}{p} < R_f$ is as follows. $\frac{R_L}{p}$ captures the marginal contribution of an increase in the supply of insurance claims to the marginal utility of each insuree in the default state is, while R_f measures the marginal contribution of an increase in insurance demand to marginal utility of each insuree in the default state. Consequently, the condition $\frac{R_L}{p} < R_f$ implies that the marginal contribution of insurance supply to the marginal utility is less than that of insurance demand. In other words, one unit increase in insurance supply will induce less than one unit increase in insurance demand. It follows that the indirect effect of an increase in asset risk on insurance demand through the increase in the competitive insurance supply level is less than the direct effect on the competitive supply level. Consequently, the excess demand decreases with asset risk so that the equilibrium price increases.

4 Regulation

There are three sources of inefficiencies in the unregulated economy as analyzed in the previous section that stem from the fact that markets are incomplete. First, each insurer makes its insurance supply decisions and investment decisions incorporating its *individual* asset return distribution without fully internalizing the potential correlation of asset returns across insurers arising from the fact that a proportion τ of insurers is exposed to a common shock. Without considering aggregate risk, insurers may hold insufficient liquidity reserves and over-invest their capital in risky assets. Second, insurees' idiosyncratic losses may not be fully insured by insurers when insurers' internal capital is relatively low. Insurees bear insurers' default risk driven by the asset side of their balance sheets when there are no effective risk sharing mechanisms among insurers to share their asset risk because there are no traded Arrow-Debreu securities contingent on insurers' individual asset realizations or the realization of the aggregate shock. Third, insurees do not have direct access to the risky assets with insurance firms also serving as intermediaries that channels the insurees' capital into more productive risky assets. Insurers, however, cannot effectively share the investment risk with insurees through the insurance policies that only protect insurees' losses without combining investment returns to insurees.

The equilibrium price and insurance coverage level in the unregulated economy, therefore, do not internalize the externalities created by aggregate risk of insurers' assets and the lack of instruments that achieve full risk-sharing. Consequently, we potentially have a misallocation of insuree capital to the purchase of insurance and misallocation of insurer capital to safe and risky assets. Regulatory intervention could improve allocative efficiency by internalizing the externalities created by aggregate risk, imposing necessary liquidity reserve requirements to influence insurers' investment decisions, and also providing *risk sharing mechanisms* through *ex post* taxation transfers among insurers.

4.1 Benchmark First Best Scenario

We begin by studying a hypothetical benchmark scenario that full internalizes the inefficiencies in the unregulated economy due to aggregate risk and imperfect risk sharing mechanisms among insurees and insurers. In this benchmark economy, there is perfect sharing of the idiosyncratic risk of insuree losses among insurees and idiosyncratic risks of asset returns among insurers. Consequently, insurers and insurees are only exposed to the aggregate shock. Without loss of generality, we can assume that there is a single representative risk averse insuree with 1 unit of the capital good and a single representative risk neutral insurer with K units of the capital good. Both the insuree and the insurer have access to risky assets that may be subject to aggregate shocks.

We examine efficient (Pareto optimal) allocations in the benchmark economy. Pareto optimal allocations must only be contingent on the aggregate state of the economy. With probability q, the economy is in the "bad" aggregate state where a proportion τ of risky investments earn a low return R_L. In the bad aggregate state, the return per unit of capital invested is M^L, where $M^L = (1-q)(1-\tau)R_H + q(1-\tau)R_L + \tau R_L$. With probability $1-q$, the economy is in the "good" aggregate state where a proportion τ of the risky investments earn a high rate of return R_H. In the good aggregate state, the return per unit capital invested is M^H, where $M^H = (1-q)(1-\tau)R_H + q(1-\tau)R_L + \tau R_H$. The insurer provides insurance to cover the insuree's loss, but also shares the aggregate risk associated with investments in the risky assets.

Let $C^H$and C^L be the representative insurer's combined returns from investing capital in the risky technology and selling insurance in the good and bad aggregate states, respectively. The representative insurer invests a proportion α of its capital in the safe asset and the remaining proportion $1-\alpha$in the risky asset. The insuree invests a proportion β of its capital in safe assets and the rest in purchasing risky insurance. Let $D^H$and D^L be the net payoffs received by the representative insuree in the good and bad state, respectively, which includes individual losses and returns from risky assets and/or insurance. We focus on the Pareto optimal allocation in which the representative insurer receives its autarkic payoff. Consequently, the planning problem is

$$\max_{\beta,\alpha,D^L,D^H} \underbrace{q\ln(\beta R_f + D^L)}_{\text{bad aggregate state}} + \underbrace{(1-q)\ln(\beta R_f + D^H)}_{\text{good aggregate state}} \tag{23}$$

subject to

$$\alpha K R_f + [(1-q)C^H + qC^L] = K[(1-q)R_H + qR_L] \tag{24}$$

$$D^L + C^L = [(1-\beta) + (1-\alpha)K]M^L - pl \tag{25}$$

$$D^H + C^H = [(1-\beta) + (1-\alpha)K]M^H - pl \tag{26}$$

$$\alpha K R_f + C^L \geq 0 \tag{27}$$

$$\alpha K R_f + C^H \geq 0 \tag{28}$$

Equations (25) and (26) capture the fact that the total payoffs to the representative insuree and insurer in the two aggregate states must equal the aggregate payoff from the investments net of the loss incurred by the insuree. Because there is perfect sharing of insuree loss risks, the total loss incurred by the representative insuree is pl. Equations (27) and (28) are limited liability constraints for the representative insurer in the two aggregate states.

The following proposition shows the optimal asset allocation between risky and safe assets, and the optimal risk allocation among the representative insuree and insurer.

Proposition 4 (Benchmark Asset allocation and Risk Sharing among Insurees and Insurers)

1. *Suppose $K \geq \frac{ER-R_L}{R_L}$. Regardless of the value of τ, $\beta^* = 0, \quad \alpha^* = 0$, that is, both the insuree and insurer invest nothing in the safe asset. The representative insuree is fully insured against losses and investment returns, that is, the returns to the insuree per unit of capital invested in the good and bad aggregate states are equal.*

$$D^H = D^L = D^* = ER - pl \tag{29}$$

2. *Suppose*

   (a) *(i.) either $q < 0.5$, and $K < \min\left(\frac{ER-R_L}{R_L}, \frac{(ER-pl)(ER-R_f)}{ER \cdot R_f}\frac{1-q}{1-2q}\right)$, or (ii.) $q > 0.5$ and $K < \frac{ER-R_L}{R_L}$*

   (b) *$(ER + R_f - R_H - R_L)R_f + pl(ER - R_f) < 0$*

   - *When $\tau \leq \tau_1$, where $\tau_1 = \frac{K \cdot ER}{(1+K)(ER-R_L)}$, $\beta^* = 0, \quad \alpha^* = 0$, that is, both the insuree and insurer invest nothing in the safe asset. The representative insuree is fully insured against losses and investment returns, that is, the returns to the insuree per unit of capital invested in the good and bad aggregate states are equal, same as (29)*

- *When* $\tau_1 < \tau < \tau_2$, *where*

$$\tau_2 = \frac{(1-q)(1+K)(R_H+R_L-2ER)ER + q\cdot ER\cdot K(ER-R_L)}{2(1-q)(1+K)(R_H-ER)(ER-R_L)}$$
$$+ \frac{\sqrt{\begin{array}{c}\Big((1-q)(1+K)(R_H+R_L-2ER)ER + q\cdot ER\cdot K(ER-R_L)\Big)^2 \\ -4\Big((1-q)(1+K)(R_H-ER)(ER-R_L)\Big)\Big(\big(qK \\ -(1-q)(1+K)\big)\cdot ER + pl(1-q)\Big)(ER-R_f)\end{array}}}{2(1-q)(1+K)(R_H-ER)(ER-R_L)},$$

$\beta^* = 0$, *and* $\alpha^* = 0$, *that is, the insuree and insure continue to invest nothing in the safe asset. Insurees are imperfectly insured; the returns per unit of capital invested in the good and bad aggregate states are, respectively*

$$D^H = (1+K)M^H - pl - \frac{ER}{1-q}K, \qquad D^L = (1+K)M^L - pl$$

The insurer's limited liability constraint 27 binds, and its returns per unit of capital of invested in the in good and bad aggregate states are, respectively

$$C^H = \frac{ER}{1-q}K \qquad C^L = 0$$

- *when* $\tau > \tau_2$, *there is nonzero investment in the safe asset with*

$$\beta^* + \alpha^* K = \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER-R_f) - \frac{qER\cdot K\cdot(R_f-M^L)}{1-q}}{(M^H-R_f)(R_f-M^L)}$$

The insuree is imperfectly insured, and its returns per unit of capital invested in the good and bad aggregate states are, respectively

$$D^H = \big(1+K-(\beta^*+\alpha^*K)\big)M^H - pl - \frac{ER}{1-q}K + \alpha^* K R_f$$
$$D^L = \big(1+K-(\beta^*+\alpha^*K)\big)M^L - pl + \alpha^* K R_f$$

The insurer's limited liability constraint 27 binds, and its returns per unit of capital of *investing in the good and bad aggregate states are, respectively*

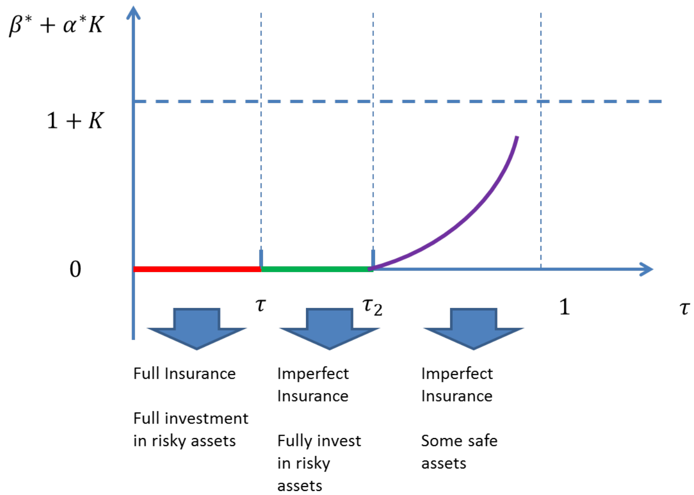
$$C^H = \frac{ER}{1-q}K - \alpha^* K \cdot R_f \qquad\qquad C^L = -\alpha^* K \cdot R_f$$

The above proposition shows the effects of aggregate risk on the optimal asset allocation and risk sharing among insurees and insurers. When the aggregate risk τ is relatively low, insuree's idosyncratic losses and returns from investment in risky assets can be fully insured by insurers. Thus it is optimal to invest all social capital in risky assets to produce the highest expected returns from investments. When the aggregate risk measure τ takes intermdiate values, there may not be enough capital in the bad aggregate state to cover insuree losses. The representative insurer, therefore, defaults and its limited liability constraint in that state is binding. It is, however, still optimal for all capital to be invested in risky assets.[3]When the aggregate risk τ is above a high threshold, however, the marginal increase in the expected return from investments in the risky assets is insufficient to compensate for the disutility to the representative insuree arising from the imperfect insurance payoffs due to aggregate shocks. It is, therefore, optimal to hold a certain amount in safe assets, that is, to maintain a nonzero liquidity buffer. Figure 3 summarizes the relationship between aggregate risk measure τ and the optimal investment in safe assets. It reflects the tradeoffs between total allocative returns from investments and the risk sharing among insurees and insurers. When the aggregate risk is low, the total allocative capital reaches the maximum level, and insurees are fully insured, and insurers take all aggregate risk. When the aggregate risk is in the intermediate level, the total allocative capital also reaches the highest level, and insurees are imperfectly insured, and insurees and insurers share the aggregate asset risk. When the aggregate risk is high, the marginal decrease in the total allocative capital due to some investment in safe assets trades off the wedge between insurance claims received by insurees in good and bad aggregate states.

We next analyze how the benchmark level of investment portfolios and risk sharing can be implemented through regulatory intervention.

[3]When asset default probability is sufficiently high and insurer's internal capital is relatively low, the marginal increase in total expected allocative capital returns from risky assets may be insufficient to compensate the disutility arising from the imperfect insurance payoffs due to aggregate shocks. It, thus may be optimal to hold some safe assets as in the third case.

Figure 3: Aggregate Risk and Liquidity Reserves Buffer



4.2 Regulatory Intervention

As discussed earlier, the inefficient investment allocation and imperfect risk-sharing in the unregulated economy relative to the first-best benchmark arises from three factors: the imperfect sharing of idiosyncratic loss risk among insurees, imperfect asset risk sharing among insurees and insurers, and the incomplete internalization of the effects of aggregate risk on insurers' investment portfolios and the provision of insurance. The above three factors provide regulators the room to reduce the market inefficiency using comprehensive tools.

Taxation and Idiosyncratic Risk

In the unregulated economy, there is no effective idosyncratic risk sharing mechanism among insurers. It follows that insurees bear the default risk driven by the idosyncratic component of an insurer's asset risk when the insurer's internal capital is sufficiently low. In the regulated economy, the regulators can serve as a "reinsurer" by taxing the insurers whose risky assets succeed and reinsuring the insurers whose risky assets fail. This *ex post* taxation contingent on the aggregate state is very similar to "insurance guarantee funds" run by state regulators. This mechanism can fully insure insurer's idiosyncratic asset risk, but not the aggregate risk. Taxation and reinsurance, therefore, depend on the aggregate state of the economy. Let T_S^H and T_F^H be the taxation transfers from sucessful and failed insurers, respectively in the good aggregate state. Also, let T_S^H and T_F^H be the taxation transfers from successful and failed insurers, respectively in the bad aggregate state.

A positive transfer means receiving a subsidy, while a negative transfer means a tax payout. Thus the tax balance condition in both good and bad aggregate state are:

$$\big((1-q)(1-\tau)+\tau\big)T_S^H + q(1-\tau)T_F^H = 0$$

$$(1-q)(1-\tau)T_S^L + \big(q(1-\tau)+\tau\big)T_F^L = 0$$

Comprehensive Insurance and Optimal Risk Sharing

In the unregulated economy, insurers provide insurance to cover individual insuree losses, and also serve as financial intermediaries to channel insuree capital to more productive assets. Because asset markets are incomplete, there is imperfect sharing of aggregate asset risk among insurees and insurers. In the regulated economy, we can implement the first best allocation if insurers sell *comprehensive insurance* policies that combine loss proection and investment returns. Let d_l^H/d_{nl}^H be the returns per unit of capital invested in comprehensive insurance policies in the good aggregate state where insurees incur idiosyncratic loss/no loss, and d_l^L/d_{nl}^L be the returns per unit of capital invested in comprehensive insurance policies in the bad aggregate state where insurees incur idiosyncratic loss/no loss.

Liquidity Requirement and Aggregate Risk

Proposition 4and Figure 3 show the optimality of investing a nonzero amount of the total capital in the safe asset when systemci risk is above the threshold, τ_2. The regulator can enforce this asset allocation by imposing a minimum liquidity requirement when aggregate risk is high enough. It is worth emphasizing here that what matters for the allocation of capital is the *total amount*, $\beta^* + \alpha^* K$, in the safe asset. The regulator can also implement this outcome through *ex ante* taxation. Specifically, the regulator can tax insuree capital at the rate β^*, insurer internal capital at the rate α^*, and invest the proceeds in the safe asset. The regulator can then use the proceeds from this investment to make transfers to insurers and insurees and, thereby, implement the efficient allocation.

The following proposition describes how the above comprehensive tools can be used to achieve the first best benchmark level of investment allocation and aggregate risk sharing among insurees and insurers.

Proposition 5 (Regulatory Intervention) *Suppose*

1. *(i.) either* $q < 0.5$, *and* $K < \min\left(\frac{ER-R_L}{R_L}, \frac{(ER-pl)(ER-R_f)}{ER \cdot R_f}\frac{1-q}{1-2q}\right)$, *or (ii.)* $q > 0.5$ *and* $K < \frac{ER-R_L}{R_L}$

2. $(ER + R_f - R_H - R_L)R_f + pl(ER - R_f) < 0$

- *When* $\tau \leq \tau_1$, *the regulator imposes no liquidity requirement. Insurees and insurers invest everything in risky assets so that*

$$\beta^* = 0, \qquad \alpha^* = 0$$

The optimal returns per unit of capital invested in the comprehensive insurance policy in the good and bad aggregate states are the same, that is

$$d_{nl}^H = d_{nl}^L = ER - pl \qquad d_l^H = d_l^L = ER + (1-p)l$$

Insurees are fully insured against idiosyncratic losses and asset risk. Insurers bear idiosyncratic and aggregate asset risk through the following taxation scheme

$$\begin{cases} T_S^L &= (1+K)(M^L - R_H) \\ T_F^L &= (1+K)(M^L - R_L) \\ T_S^H &= (1+K)(M^H - R_H) \\ T_F^H &= (1+K)(M^H - R_L) \end{cases} \tag{30}$$

- *When* $\tau_1 < \tau \leq \tau_2$, *the regulator imposes no liquidity requirement. Insurees and insurers invest everything in risky assets so that*

$$\beta^* = 0, \qquad \alpha^* = 0$$

The optimal returns per unit of capital invested in the comprehensive insurance policy in the *good and bad aggregate states are unequal and are given by*

$$d^H_{nl} = (1+K)M^H - \frac{ER}{1-q}K - pl \qquad d^L_{nl} = (1+K)M^L - pl \tag{31}$$

$$d^H_l = (1+K)M^H - \frac{ER}{1-q}K + (1-p)l \qquad d^L_l = (1+K)M^L + (1-p)l \tag{32}$$

Insurers bear the idiosyncratic loss risk of insurees as well as idiosyncratic asset risk through the taxation scheme as 30. Aggregate risk is, however, shared among insurees and insurers through the comprehensive insurance policy 31.

- *When $\tau > \tau_2$, the regulator imposes a liquidity requirement on insurers and insurees that is given by*

$$\alpha^* \in \left(\max\left\{ \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{K(M^H - R_f)(R_f - M^L)} - \frac{1}{K},\ 0 \right\},\ 1 \right)$$

$$\beta^* = \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} - \alpha^* K,$$

Alternately, the regulator can levy ex ante taxes at the rate $\alpha^*$for insurers and $\beta^*$for insurees and invest the proceeds in the safe asset.

- *The optimal return per unit of capital invested in the comprehensive insurance policy in the good and bad aggregate states are unequal and given by*

$$\begin{aligned}
d^H_{nl} &= \frac{(1+K)M^H + (\beta + \alpha K)(R_f - M^H) - \frac{ER}{1-q}K - \beta^* R_f - pl}{1-\beta^*} \\
d^L_{nl} &= \frac{(1+K)M^L + (\beta^* + \alpha^* K)(R_f - M^L) - \beta^* R_f - pl}{1-\beta^*} && (33) \\
d^H_l &= \frac{(1+K)M^H + (\beta + \alpha K)(R_f - M^H) - \frac{ER}{1-q}K - \beta^* R_f + (1-p)l}{1-\beta^*} \\
d^L_l &= \frac{(1+K)M^L + (\beta^* + \alpha^* K)(R_f - M^L) - \beta^* R_f + (1-p)l}{1-\beta^*} && (34)
\end{aligned}$$

Insuree' idiosyncratic losses and idiosyncratic asset risk are fully taken by insurers through *the taxation scheme as follows:*

$$\begin{cases} T_S^L &= (1+K-(\beta^*+\alpha^* K))(M^L - R_H) \\ T_F^L &= (1+K-(\beta^*+\alpha^* K))(M^L - R_L) \\ T_S^H &= \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_H) \\ T_F^H &= \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_L) \end{cases} \tag{35}$$

aggregate risk is shared by insurees and insurers through the comprehensive insurance policy as 34.

The above proposition implies that the comprehensive tools can be used by regulators to reduce the inefficiencies of unregulated economy. *Ex post* taxation contingent on the aggregate state, plays the role of "insurance guarantee funds", which induces insurers to fully absorb insurees' idiosyncratic loss risk when their internal capital is relatively low. *Comprehensive insurance policies* combining insurance with investment, together with *ex post* taxation, enhance aggregate risk sharing between insurees and insurers. The *liquidity requirement* adjusts inefficiencies arising from insurer's misallocation of their assets and the optimal aggregate risk sharing among insurees and insurers. Thus, when aggregate risk is high enough, the optimal investment allocation reflects the tradeoff between the growth of total assets and insurees' aversion to aggregate risk.

5 Conclusions

We develop an equilibrium model of competitive insurance markets where insurers' assets may expose to both idiosyncratic shocks and aggregate shocks. We reconcile the conflicting predictions in previous literature and provide new insights into the relationship between insurance premia and internal capital that stem from the influence of both demand and supply side forces. The insurance price varies non-monotonically in a U-shaped manner with the level of internal capital held by insurers. We also obtain additional testable implications for the effects of insurers' asset risks on premia and the level of insurance coverage.

We then derive insights into the solvency regulation of insurers by deriving the Pareto optimal allocation of insurer capital to liquidity reserves and risky assets as well as risk sharing among insurees and insurers. We show that, when aggregate risk is below a threshold, it is Pareto optimal for insurers and insurees to hold zero liquidity reserves, insurees are fully insured, and insurers bear all aggregate risk. When aggregate risk takes intermediate values, both insurees and insurers still hold no liquidity reserves, but insurees partially share aggregate risk with insurers. When the aggregate risk is high, however, both insurees and insurers hold nonzero liquidity reserves, and insurees partially share aggregate risk with insurers. We demonstrate that the efficient allocation can be implemented through regulatory intervention that comprises of comprehensive insurance policies that combine insurance and investment, reinsurance, a minimum liquidity requirement when aggregate risk is high, and ex post budget-neutral taxation and subsidies contingent on the realized aggregate state.

In future research, it would be interesting to develop a dynamic structural model of insurance markets. The analysis of such a model that is suitably calibrated to data could generate quantitative insights into the optimal regulation of insurance markets.

Appendix

Proof of Lemma 1

Proof. We first consider the case where the representative insurer defaults in the "bad "state where its assets fail; that is, $C_d p \leq (K + \kappa C_s)R_L$. It follows that $\min\left(C_d, \frac{(K+\kappa C_s)R_L}{p}\right) = \frac{(K+\kappa C_s)R_L}{p}$. The necessary and sufficient first order condition for insuree's optimal choice of coverage, C_d^*, is simplified as equation (8). We next consider the other case where the representative insurer does not default in the "bad "where its assets fail; that is, $C_d p > (K + \kappa C_s)R_L$. It then follows that $\min\left(C_d, \frac{(K+\kappa C_s)R_L}{p}\right) = C_d$. The optimal choice of insurance coverage, therefore, has to satisfy equation (10). ■

Proof of Lemma 2

Proof. We consider the case where the representative insurer defaults in the "bad"state where its assets fail; that is, $C_d p \leq (K + \kappa C_s)R_L$. From the first section in Lemma 1, we first define the implicit function for the optimal demand for insurance coverage $\mathcal{G}(C_d^*, \kappa, p, q, R_f, R_L, K, C_s)$ as

$$\mathcal{G}(C_d^*, \kappa, p, q, R_f, R_L, K, C_s) = \frac{p(1-q)(1-\kappa R_f)}{W_1} - \frac{pq\kappa R_f}{W_2} - \frac{(1-p)\kappa R_f}{W_3} \tag{36}$$

where $W_1 = (1-\kappa C_d^*)R_f - l + C_d^*$, $\quad W_2 = (1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s^*)R_L}{p}$, $\quad W_3 = (1-\kappa C_d^*)R_f$. We then show how the optimal demand for insurance coverage varies with the fundamental parameters of the model. It is easy to derive the signs for the following two equation:

$$\frac{\partial \mathcal{G}(C_d)}{\partial C_d} = -\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} - \frac{pq\kappa^2 R_f^2}{W_2^2} - \frac{(1-p)\kappa^2 R_f^2}{W_3^2} < 0$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial \kappa} = -\frac{p(1-q)R_f(R_f - l)}{W_1^2} - \frac{pqR_f(R_f - l + \frac{KR_L}{p})}{W_2^2} - \frac{(1-p)R_f^2}{W_3^2} < 0$$

It then follows that $\frac{\partial C_d^*}{\partial \kappa} < 0$. The optimal demand for insurance C_d^*, therefore, decreases with the insurance price. Similarly, it is easy to show the following:

$$\begin{aligned}\frac{\partial \mathcal{G}(C_d)}{\partial R_f} =& -\frac{p(1-q)\kappa}{W_1} - \frac{p(1-q)(1-\kappa R_f)(1-\kappa C_d)}{W_1^2} - \frac{pq\kappa(l - \frac{(K+\kappa C_s)R_L}{p})}{W_2^2} \\ & - \frac{(1-p)\kappa R_f(R_f - C_d)}{W_3^2} < 0\end{aligned}$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial K} = \frac{pq\kappa R_f \frac{R_L}{p}}{W_2^2} > 0 \qquad \frac{\partial \mathcal{G}(C_d)}{\partial C_s} = \frac{pq\kappa^2 R_f \frac{R_L}{p}}{W_2^2} > 0$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial R_L} = \frac{q\kappa R_f(K + \kappa C_s)}{W_2^2} > 0 \qquad \frac{\partial \mathcal{G}(C_d)}{\partial q} = -\frac{p(1-\kappa R_f)}{W_1} - \frac{p\kappa R_f}{W_2} < 0$$

Consequently, the optimal demand for insurance coverage, C_d, decreases with the return, R_f, on the safe asset and the default probability of insurer's risk assets, q, while increases with insurer's internal capital, K, the total face value of insurance contracts sold by the insurer, C_s, and the insurer's asset return in the low state, R_L. ■

Proof of Lemma 3

Proof. We show the effects of fundamental parameters on the competitive insurance supply by checking the signs for the following equations based on the competitive insurance supply in the case where insurer's asset may fail in "bad" state, C_s^*, given by equation (14). It is obvious that

$$\frac{\partial C_s^*}{\partial \kappa} = -\frac{qKR_LR_H}{(1-q)(\kappa R_H - p)^2} < 0 \qquad \frac{\partial C_s^*}{\partial K} = \frac{qR_L}{(1-q)(\kappa R_H - p)} > 0$$
$$\frac{\partial C_s^*}{\partial q} = \frac{KR_L}{(1-q)^2(\kappa R_H - p)} > 0 \qquad \frac{\partial C_s^*}{\partial R_L} = \frac{qK}{(1-q)(\kappa R_H - p)} > 0$$
$$\frac{\partial C_s^*}{\partial p} = \frac{qKR_L}{(1-q)(\kappa R_H - p)^2} > 0$$

It follows that the competitive insurance supply level, C_s^*, decreases with the insurance price, κ, while increases with insurers' internal capital, K, the default probability of insurer's risky assets, q, the risky asset return, R_L, in the bad state, and the loss probability of insurees, p. ■

Proof of Proposition 1

Proof. The insurance market equilibria depends on the internal capital level, K, held by insurance companies. We first conjecture that the representative insuree rationally foresees that the representative insurer will default in the "bad" state if the insurer's internal capital level is below $\overline{K}_1$ (where $\overline{K}_1$ satisfies equation (17)), whereas the representative insuree will anticipate that the representative insurer will still be solvent in the "bad" state if its internal capital level is above $\overline{K}_1$. We then derive the equilibrium insurance contracts, which consists of insurance price κ^* and the face value of insurance coverage C^* for each case, and later validate that the equilibrium where insurers defaults in "bad" state cannot exist given any level of the internal capital level above the threshold level, $\overline{K}_1$.

1. Suppose $K \le \overline{K}_1$, we conjecture that the insurer is expected to default in the "bad" state. It follows that the optimal demand for insurance coverage, C_d^*, has to satisfy equation (8), and the competitive insurance supply level, C_s^*, has to satisfies (14). The equilibrium insurance price, therefore, have to satisfy the following equation:

$$F(K, \kappa) = 0 \tag{37}$$

where $F(K, \kappa)$ is the excess demand function defined as (16), and C_s^* and C_d^* have to satisfy (8) and (14) separately.

In addition, to ensure the solution, κ, to equation (37) to be the equilibrium insurance price, it also needs to satisfy

$$pC^* \ge (K + \kappa^* C^*)R_L \tag{38}$$

where C^* is the face value of equilibrium insurance coverage such that $C_s^* = C_d^* = C^*$.

We next show that, given any K less or equal to $\overline{K}_1$, there exists an equilibrium insurance contract which includes the equilibrium insurance price κ^* and equilibrium face value of insurance coverage C^*.

(14) implies that the equilibrium insurance price, κ^*, need to lie in the interval $\left(\frac{p}{R_H}, \frac{p}{R_L}\right)$ because the insurer would like to supply either zero or infinite amount of insurance coverage for any price outside this region. Further, from (38) and (14), it is easy to show that the equilibrium insurance price κ^* also has to satisfy $\kappa^* \le \frac{p}{ER}$, where $ER = (1-q)R_H + qR_L$; otherwise, the conjecture will be violated due to the violation of (14).

To show the existence of κ^* that satisfies (37), we check the boundary conditions for $\kappa \in \left(\frac{p}{R_H}, \frac{p}{ER}\right]$.

The derivative of $F(K, \kappa)$ with respect to κ for any K less or equal to $\overline{K}_1$; that is,

$$\frac{\partial F(\kappa^*|K)}{\partial \kappa^*} = \frac{\partial C_d^*(\kappa^*, C_s^*(\kappa^*)|K)}{\partial \kappa^*} + \frac{\partial C_d^*(\kappa^*, C_s^*(\kappa^*)|K)}{\partial C_s^*}\frac{\partial C_s^*(\kappa^*|K)}{\partial \kappa^*} - \frac{\partial C_s^*(\kappa^*|K)}{\partial \kappa^*} \tag{39}$$

According to the proof of Lemma 2, it is easy to show

$$\frac{\partial F(\kappa)}{\partial \kappa} = \underbrace{\frac{\partial C_d^*(\kappa, C_s^*(\kappa))}{\partial \kappa}}_{<0} + \underbrace{\frac{\partial C_d^*(\kappa, C_s^*(\kappa))}{\partial C_s^*}}_{>0} \underbrace{\frac{\partial C_s^*(\kappa)}{\partial \kappa}}_{<0} - \underbrace{\frac{\partial C_s^*(\kappa)}{\partial \kappa}}_{<0} \tag{40}$$

Howver, the sign of $\frac{\partial F(\kappa)}{\partial \kappa}$ is indeterminate.

We then check the sign of $F(\kappa)$ at the lower boundary of κ.

$$\lim_{\kappa \to \frac{p}{R_H}} F(\kappa|K) = \lim_{\kappa \to \frac{p}{R_H}} C_d^*(\kappa, C_s^*(\kappa)|K) - \lim_{\kappa \to \frac{p}{R_H}} C_s^*(\kappa|K)$$

It is obvious that $\lim_{\kappa \to \frac{p}{R_H}} C_s^*(\kappa|K) \to +\infty$ for any K such that $0 < K \leq \overline{K}_1$ because insurers have to sell a very large finite amount so that condition (13)is binding. In addition, $C_d^*|_{\kappa \to \frac{p}{R_H}} = C_d^*(\kappa \to \frac{p}{R_H}|K) < \frac{R_H}{p} < +\infty$. It follows that $\lim_{\kappa \to \frac{p}{R_H}} F(\kappa|K) < 0$

To ensure the existence of equilibrium insurance price κ^*, a necessary condition is that

$$F(\kappa|K)|_{\kappa = \frac{p}{ER}} = \lim_{\kappa = \frac{p}{ER}} C_d^*(\kappa, C_s^*(\kappa)|K) - \lim_{\kappa = \frac{p}{ER}} C_s^*(\kappa|K) \geq 0 \tag{41}$$

We now examine that condition (41) is satisfied for any K, such that $0 < K \leq \overline{K}_1$. In other words, we have to show that $F(K|\kappa = \frac{p}{ER})$ is a decreasing function and $\overline{K}_1$ is the solution to (17).

It is obvious that

$$F(K \to 0|\kappa = \frac{p}{ER}) = \underbrace{C_d^*(K \to 0|\kappa = \frac{p}{ER})}_{>0} - \underbrace{C_s^*(K \to 0|\kappa = \frac{p}{ER})}_{\to 0} > 0 \tag{42}$$

When $\kappa = \frac{p}{ER}$, the insurance claims received by each insuree who incurs losses are equal to the insurance claims seld by each insurer. Thus, under the reasonable condition$R_f > \frac{R_L}{p}$

$$\frac{\partial F(K|\kappa = \frac{p}{ER})}{\partial K} = \frac{\partial C_d^*(K|\kappa = \frac{p}{ER})}{\partial C_s^*} \frac{\partial C_s^*}{\partial K} - \frac{\partial C_s^*}{\partial K} < 0 \tag{43}$$

Conditions (42) and (43) imply that there exists a solution $\overline{K}_1$ to the equation (17). Condition (43) also implies for any $K \leq \overline{K}_1$,$F(K|\kappa = \frac{p}{ER}) \geq 0$. Thus when $K < \overline{K}_1$,there exists at least one equilibrium insurance price. However, since $\frac{\partial EU(\kappa^*)}{\kappa^*} < 0$, we focus on the equilibrium with the smallest price κ; that is at which $\frac{\partial F(\kappa)}{\partial \kappa}|_{\kappa^*} > 0$ and social welfare are maximized.

2. Suppose $K > \overline{K}_1$. As we shown in previous case, when $K > \overline{K}_1$, (41) will be violated. It follows that the solution to equation (37) will be greater than $\frac{p}{ER}$. Consequently, the conjecture that equilibrium where the insurer will default in its "bad" state cannot be maintained. We now conjecture that in equilibrium insurers will not default in its "bad" state. According to previous argument, insuree's demand for insurance coverage is not binding. In this case, the insurers face no opportunity cost and earns zero profit at the actuarially fair price $\frac{p}{ER}$, at which the insurer is indifferent between selling insurance and no insurance. The equilibrium insurance coverage is, then determined by insurance demands, which satisfies (10), where $\kappa = \frac{p}{ER}$. It is easy to see the solution to (10) is $C^* = \left(p - (1-p)\frac{R_f - l}{\frac{ER}{p} - R_f}\right)\frac{ER}{p}$.

We next check that the insurance contracts $\kappa^* = \frac{p}{ER}$ and C^* are the equilibrium contracts. In other words, we have to check whether (38) holds. (38) implies that the insurer's internal capital has to satisfy $K > C^*(\frac{p}{R_L} - \frac{p}{ER})$; that is,

$$K > \underbrace{\left(p - (1-p)\frac{R_f - l}{\frac{ER}{p} - R_f}\right)\frac{ER}{p}(\frac{p}{R_L} - \frac{p}{ER})}_{let=\overline{K}_2}$$

We next show that $\overline{K}_1 = \overline{K}_2$. According to condition(17), we have $C_d^*(\overline{K}_1|\kappa \to \frac{p}{ER}) - \frac{q\overline{K}_1 R_L}{(1-q)(\frac{p}{ER}R_H - p)} = 0$. Thus $\overline{K}_1 = C_d^*(\overline{K}_1|\kappa \to \frac{p}{ER})\frac{(1-q)(\frac{p}{ER}R_H - p)}{qR_L}$. Since the equilibrium insurance demand $C_d^*(K = \overline{K}_1)$ is equal to $C^*(K > \overline{K}_2)$. In other words, $C_d^*(K = \overline{K}_1) = C^* = \left(p - (1-p)\frac{R_f - l}{\frac{ER}{p} - R_f}\right)\frac{ER}{p}$. Now we have $\overline{K}_1 = \left(p - (1-p)\frac{R_f - l}{\frac{ER}{p} - R_f}\right)\frac{ER}{p}\frac{(1-q)(\frac{p}{ER}R_H - p)}{qR_L}$. It is easy to show that $\frac{(1-q)(\frac{p}{ER}R_H - p)}{qR_L} = \frac{p}{R_L} - \frac{p}{ER}$; therefore, $\overline{K}_1 = \overline{K}_2$. Q.E.D. ■

Proof of Proposition 2

Proof. To examine the effects of internal capital of the insurer on the insurance price, we integrate its effects on both the competitive supply of insurance coverage and the demand for insurance coverage. In other word, we need to determine the sign of $\frac{\partial \kappa^*}{\partial K} = -\frac{\frac{\partial F(\kappa^*, K)}{\partial K}}{\frac{F(\kappa^*, K)}{\partial \kappa^*}}$. From (37), we have

$$\frac{\partial F(\kappa^*)}{\partial K} = \underbrace{\frac{\partial C_d(\kappa^*)}{\partial K}}_{\text{direct effect on demands}} + \underbrace{\frac{\partial C_d(\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*(\kappa^*)}{\partial K}}_{\text{indirect effect on demands}} - \underbrace{\frac{\partial C_s(\kappa^*)}{\partial K}}_{\text{direct effect on zero-expected-profit supply}}$$

Following the results of Lemma 2 and 3, it is easy to show the following

$$\frac{\partial C_d(\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*}{\partial K} = \frac{pq\kappa^2 R_f \frac{R_L}{p}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}\frac{qR_L}{(1-q)(\kappa^* R_H - p)} > 0$$

$$\frac{\partial C_s(\kappa^*)}{\partial K} = \frac{qR_L}{(1-q)(\kappa^* R_H - p)} > 0$$

$$\frac{\partial C_d^*}{\partial K} = -\frac{\frac{\partial \mathcal{G}(C_d^*)}{\partial K}}{\frac{\partial \mathcal{G}(C_d^*)}{\partial C_d}} = \frac{\frac{pq\kappa R_f \frac{R_L}{p}}{W_2^2}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} + \frac{pq\kappa^2 R_f^2}{W_2^2} + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}} > 0$$

Thus we have

$$\frac{\partial F(\kappa^*, K)}{\partial K} = \underbrace{\frac{\partial C_d(\kappa^*, K)}{\partial K}}_{>0} + \underbrace{\frac{\partial C_d(\kappa^*, K)}{\partial C_s^*}\frac{\partial C_s^*(\kappa^*, K)}{\partial K}}_{>0} - \underbrace{\frac{\partial C_s(\kappa^*, K)}{\partial K}}_{>0}$$

Further, we know $\frac{\partial F(\kappa^*, K)}{\partial \kappa^*} > 0$. Thus the sign of $\frac{\partial F(\kappa^*, K)}{\partial K}$ is indeterminate, and the effects of internal capital on the equilibrium price is non-monotonic. However, the excess insurance demand function is concave because

$$\begin{aligned}
\frac{\partial^2 F(\kappa^*, K)}{\partial K^2} =& \frac{\partial^2 C_d(\kappa^*, K)}{\partial K^2} + \frac{\partial\left(\frac{\partial C_d(\kappa^*,K)}{\partial C_s^*}\frac{\partial C_s^*}{\partial K}\right)}{\partial K} \\
=& \underbrace{-\frac{2W_2 pq\kappa^* R_f \frac{R_L}{p}\left[\left(\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2} + \frac{(1-p)\kappa^{*2} R_f^2}{W_3^2}\right)\frac{\partial W_2}{\partial K}\right]}{\left[\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2}W_2^2 + pq\kappa^{*2}R_f^2 + \frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}W_2^2\right]^2}}_{\text{direct effect part}<0} \\
& \underbrace{-\frac{2W_2 pq\kappa^{*2} R_f \frac{R_L}{p}\left[\left(\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2} + \frac{(1-p)\kappa^{*2} R_f^2}{W_3^2}\right)\frac{\partial W_2}{\partial K}\right]\frac{\partial C_s^*}{\partial K}}{\left[\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2}W_2^2 + pq\kappa^{*2}R_f^2 + \frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}W_2^2\right]^2}}_{\text{indirect effect part}<0} < 0
\end{aligned}$$

Thus the marginal effect of internal capital K on insurance demand C_d is decreasing, while the marginal effect on competitive insurance supply is constant. Consequently, the overall effects are decreasing.

Therefore, suppose $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} > 0$, there may exist a threshold level of $\widetilde{K}$ and the corresponding insurance price $\widetilde{\kappa}$ such that the equilibrium price κ^* decrease with the amount of internal capital when $K < \widetilde{K}$, while increase with an increase in the amount of internal capital when $K > \widetilde{K}$; suppose $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} < 0$, the equilibrium price increases with the amount of internal capital, where $\widetilde{K}$ and $\widetilde{\kappa}$ are jointly determined by the following two equations

$$\frac{\partial F(\kappa^*, K)}{\partial K}\left(\kappa^* = \widetilde{\kappa}; K = \widetilde{K}\right) = 0$$

$$C_d^*\left(\widetilde{\kappa}, \widetilde{K}, C_s^*(\widetilde{\kappa}, \widetilde{K})\right) = C_s^*(\widetilde{\kappa}, \widetilde{K})$$

■

Proof of Proposition 3

Proof. To examine the effects of default risk of insurer's risky assets on the equilibrium insurance price, we integrate its effects on both the competitive supply of insurance coverage and the demand for insurance coverage. In other word, we need to determine the sign of We check the sign of $\frac{\partial \kappa^*}{\partial q} = -\frac{\frac{\partial F(\kappa^*)}{\partial q}}{\frac{F(\kappa^*)}{\partial \kappa^*}}$. According to the proof of Lemma 2 and 3, it is easy to show the following:

$$
\begin{aligned}
\frac{\partial F(\kappa^*)}{\partial q} =& \frac{\partial C_d(\kappa^*)}{\partial q} + \frac{\partial C_d(\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*}{\partial q} - \frac{\partial C_s^*(\kappa^*)}{\partial q} \\
=& -\frac{\frac{p(1-\kappa R_f)}{W_1} + \frac{p\kappa R_f}{W_2}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} + \frac{pq\kappa^2 R_f^2}{W_2^2} + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}} \\
&+ \underbrace{\frac{pq\kappa^2 R_f \frac{R_L}{p} - pq\kappa^2 R_f^2}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}}_{<0 \quad \text{if} \quad \frac{R_L}{p}<R_f} \frac{JR_L}{(1-q)^2(\kappa R_H - p)} \\
&- \frac{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2} \frac{JR_L}{(1-q)^2(\kappa R_H - p)}
\end{aligned}
$$

Given condition that $\frac{R_L}{p} < R_f$, we have $\frac{\partial F(\kappa^*)}{\partial q} < 0$. It follows that $\frac{\partial \kappa^*}{\partial q} > 0$. The equilibrium insurance price, thus, increases with an increase in the asset risk. In other words, when $\frac{R_L}{p} < R_f$, the indirect effects of asset risk on insurance demand is offset by the direct effects on competitive insurance supply. Consequently, the demand effects dominates so that the equilibrium price goes up and the equilibrium coverage shrinks. ■

Proof of Proposition 4

Proof. We show the Pareto optimal allocation planning problem is maximizing (23) subject to (24),(25),(26),(27) and (28).

We substitute D^L and D^H with C^H and C^H using the relationships implied by(25) and (26). (28) can be omitted if $C^H \geq C^L$. Let λ and μ are the Lagrangian multiplier associate with (24) and (27), respectively. Thus

$$
\begin{aligned}
\mathcal{L} =& q\ln(\beta R_f + W^L - C^L) + (1-q)\ln(\beta R_f + W^H - C^H) \\
&+ \lambda\{\alpha K R_f + [(1-q)C^H + qC^L] - K[(1-q)R_H + qR_L]\} + \mu\{\alpha K R_f + C^L\}
\end{aligned}
$$

The first order condition with respect to C^H and C^L are, respectively:

$$
\begin{aligned}
\partial C^L &: -\frac{q}{\beta R_f + W^L - C^L} + \lambda q + \mu = 0 \\
\partial C^H &: -\frac{1-q}{\beta R_f + W^H - C^H} + \lambda(1-q) = 0
\end{aligned} \tag{44}
$$

We first suppose $\mu = 0$. Equations (44) imply $W^H - C^H = W^L - C^L$, and the relationship between C^H and C^L is

$$C^H = C^L + [(1-\beta) + (1-\alpha)K]\tau(R_H - R_L)$$

Plugging above relation into equation (24), we have

$$
\begin{aligned}
C^{L*} &= K\big(ER - \alpha R_f\big) - (1-q)[(1-\beta) + (1-\alpha)K]\tau(R_H - R_L) \\
C^{H*} &= K\big(ER - \alpha R_f\big) + q[(1-\beta) + (1-\alpha)K]\tau(R_H - R_L) \\
D^* = D^{H*} = D^{L*} &= (1-\beta)ER - \alpha K\big(ER - R_f\big) - pl
\end{aligned}
$$

where $ER = (1-q)R_H + qR_L$ as defined in Section 3.1. The insuree is fully insured, and its utility is:

$$\begin{aligned} EU_{\text{insuree}} &= \ln\big(\beta R_f + D^*\big) \\ &= \ln\big(-\beta(ER - R_f) - \alpha K(ER - R_f) + ER - pl\big) \end{aligned}$$

We now derive the optimal level of investment in safe assets.

$$\max_{\alpha,\beta} \ln\big(-(\beta + \alpha K)(ER - R_f) + ER - pl\big) \tag{45}$$

subject to

$$\begin{aligned} \alpha K R_f + C^{L*} &\geq 0 \\ 0 \leq \beta + \alpha K &\leq 1 + K \end{aligned} \tag{46}$$

Since the objective function, (45), is a decreasing function of $(\beta + \alpha K)$, thus $\beta^* + \alpha^* K = 0$. The above constraint (46) can be simplified as follows:

$$\beta + \alpha K \leq \frac{KER}{(ER - R_L)\tau} - (1 + K)$$

Suppose $K \geq \frac{ER - R_L}{R_L}$, $\frac{KER}{(ER - R_L)\tau} - (1 + K) \leq 0$ for any value of τ. In other words, (46) can be omitted, and the optimal level of investment in safe assets is zero. The Part 1 of Proposition 4, thus holds.

Suppose $K < \frac{ER - R_L}{R_L}$, $\frac{KER}{(ER - R_L)\tau} - (1 + K) \leq 0$ still holds for any τ such that $\tau \leq \tau_1$ where $\tau_1 = \frac{K \cdot ER}{(1+K)(ER - R_L)}$. Similarly, (46) can also be omitted, and the optimal level of investment in safe assets is zero. The first case of Part 2 of Proposition 4 holds.

However, if $\tau > \tau_1$, then the optimal level of investment in safe assets is determined by (46) when it binds. which contradicts with $\mu = 0$. Consequently, there does not exist the case where insurees are fully when $\tau > \tau_1$.

Now we suppose $\mu > 0$, and limited liability constraint of insurers in “bad" aggregate state, (27), binds; that is $\alpha K R_f + C^L = 0$. Thus $C^L = -\alpha K R_f$ and $C_H = \frac{ER - (1-q)\alpha R_f}{1-q} K$. It is easy to show

$$\begin{aligned} D^L &= W^L - C^L = [1 + K - (\beta + \alpha K)]M^L - pl + \alpha K R_f \\ D^H &= W^H - C^H = [1 + K - (\beta + \alpha K)]M^H - pl - \frac{ER}{1-q}K + \alpha K R_f \end{aligned}$$

Thus insurees’ total capital in “good" and “bad" aggregate states, receptively, are

$$\begin{aligned} N^L &= \beta R_f + D^L = (1 + K)M^L + (\beta + \alpha K)(R_f - M^L) - pl \\ N^H &= \beta R_f + D^H = (1 + K)M^H + (\beta + \alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K \end{aligned}$$

We now solve for the optimal level of investment in safe assets

$$\begin{aligned} \max\nolimits_{\alpha,\beta}\, q \ln\big((1 + K)M^L + (\beta + \alpha K)(R_f - M^L) - pl\big) + (1 - q)\ln((1 + K)M^H \\ +(\beta + \alpha K)(R_f - M^H) - pl - \tfrac{ER}{1-q}K) \end{aligned}$$

subject to

$$0 \leq \beta + \alpha K \quad \leq \quad 1 + K$$

The Lagrangian function is

$$\begin{aligned}\mathcal{L} = q \ln\big((1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl\big) + (1-q)\ln((1+K)M^H \\ + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K) - \lambda_1(1+K-(\beta+\alpha K)) - \lambda_2(\beta+\alpha K)\end{aligned}$$

The first order condition with respect to $(\beta + \alpha K)$ that is

$$\frac{q(R_f - M^L)}{N^L} - \frac{(1-q)(M^H - R_f)}{N^H} + \lambda_1 - \lambda_2 = 0$$

Suppose $\lambda_1 = \lambda_2 = 0$, then $\frac{q(R_f - M^L)}{N^L} = \frac{(1-q)(M^H - R_f)}{N^H}$ That is

$$\begin{aligned} & q(R_f - M^L)\big((1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K\big) \\ = \quad & (1-q)(M^H - R_f)\big((1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl\big)\end{aligned}$$

Rearrange the above equations, we have

$$\beta + \alpha K = \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)}$$

Now we have to check $0 < \beta + \alpha K < 1 + K$.
We first whether $\beta + \alpha K > 0$ holds, that is

$$\begin{aligned}(1-q)(1+K)(R_H - ER)(ER - R_L)\tau^2 - \Bigg((1-q)(1+K)(R_H + R_L - 2ER)ER \\ + q \cdot ER \cdot K(ER - R_L)\Bigg)\tau + \Bigg(\big(qK - (1-q)(1+K)\big) \cdot ER + pl(1-q)\Bigg)(ER - R_f) > 0\end{aligned}$$

We need $\tau \leq \tau_2'$ or $\tau \geq \tau_2$ to make above inequality hold, where

$$\begin{aligned}\tau_2' \quad = \quad & \frac{(1-q)(1+K)(R_H + R_L - 2ER)ER + q \cdot ER \cdot K(ER - R_L)}{2(1-q)(1+K)(R_H - ER)(ER - R_L)} \\ - \quad & \frac{\sqrt{\begin{array}{c}\Big((1-q)(1+K)(R_H + R_L - 2ER)ER + q \cdot ER \cdot K(ER - R_L)\Big)^2 \\ -4\Big((1-q)(1+K)(R_H - ER)(ER - R_L)\Big)\Big(\big(qK \\ -(1-q)(1+K)\big) \cdot ER + pl(1-q)\Big)(ER - R_f)\end{array}}}{2(1-q)(1+K)(R_H - ER)(ER - R_L)}\end{aligned}$$

$$\begin{aligned}\tau_2 &= \frac{(1-q)(1+K)(R_H+R_L-2ER)ER+q\cdot ER\cdot K(ER-R_L)}{2(1-q)(1+K)(R_H-ER)(ER-R_L)}\\ &+ \frac{\sqrt{\begin{array}{c}\Big((1-q)(1+K)(R_H+R_L-2ER)ER+q\cdot ER\cdot K(ER-R_L)\Big)^2\\ -4\Big((1-q)(1+K)(R_H-ER)(ER-R_L)\Big)\Big(\big(qK\\ -(1-q)(1+K)\big)\cdot ER+pl(1-q)\Big)(ER-R_f)\end{array}}}{2(1-q)(1+K)(R_H-ER)(ER-R_L)}\end{aligned}$$

We now compare τ_1 and τ_2'. When $\tau=\tau_1$, we check the value of $\beta+\alpha K|_{\tau=\tau_1}$, that is,

$$\beta+\alpha K|_{\tau=\tau_1}=\frac{(1+K)(ER\cdot R_f-M^HM^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}}{(M^H-R_f)(R_f-M^L)}$$

that is whether

$$(1+K)(ER\cdot R_f-M^HM^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}<0$$

The above inequality is equivalent to

$$\big(1+K\frac{1-2q}{1-q}\big)\cdot ER\cdot R_f<ER^2-pl(ER-R_f)$$

Therefore, when $q<0.5$, and $K<\min\Big(\frac{ER-R_L}{R_L},\frac{(ER-pl)(ER-R_f)}{ER\cdot R_f}\frac{1-q}{1-2q}\Big)$, or when $q>0.5$, and $K<\frac{ER-R_L}{R_L}$, we have $\beta+\alpha K|_{\tau=\left(\frac{K}{1+K}\right)\left(\frac{ER}{ER-R_L}\right)}<0$. In other words, $\tau_2'<\tau_1$.

We next check when $\tau=1$, whether $\beta+\alpha K<1+K$.

When $\tau=1$, we have $M^L=R_L$, and $M^H=R_H$.

$$\begin{aligned}\beta+\alpha K-(1+K) &= \frac{(1+K)(ER\cdot R_f-R_HR_L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-R_L)}{1-q}}{(R_H-R_f)(R_f-R_L)}-(1+K)\\ &= \frac{\begin{array}{c}(1+K)(ER\cdot R_f-R_HR_L)+pl(ER-R_f)\\ -\frac{qER\cdot K\cdot(R_f-R_L)}{1-q}+(1+K)(R_f-R_H)(R_f-R_L)\end{array}}{(R_H-R_f)(R_f-R_L)}\end{aligned}$$

To show $\beta+\alpha K-(1+K)>0$ which is equivalent to show

$$(1+K)(ER\cdot R_f-R_HR_L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-R_L)}{1-q}+(1+K)(R_f-R_H)(R_f-R_L)<0$$

that is,

$$pl(ER-R_f)+\big((ER+R_f-R_H-R_L)R_f\big)<K\big(\frac{q}{1-q}ER(R_f-R_L)-(ER+R_f-R_H-R_L)R_f\big)$$

Suppose

$$(ER+R_f-R_H-R_L)R_f+pl(ER-R_f)<0$$

$\beta+\alpha K-(1+K)|_{\tau=1}>0$ holds. In other words, $\tau_2<1$. Therefore, when $\tau>\tau_2$, the optimal level of investment in safe assets is

$$\beta^*+\alpha^*K=\frac{(1+K)(ER\cdot R_f-M_HM_L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M_L)}{1-q}}{(M_H-R_f)(R_f-M_L)}$$

Insuree and insurer both hold positive amount of safe assets, insuree and insurer share the aggregate shocks. However, when $\tau_1\leq\tau\leq\tau_2$, it is optimal that insurees and insurers still invest nothing in safe assets, that is, $\beta^*+\alpha^*K=0$, but they share the aggregate asset shocks. ■

Proof of Proposition 5

Proof. We first consider the case when $\tau<\tau_1$, the representative insuree is fully insured, we have the following system of equations for each state:

$$\begin{aligned}\beta R_f+d_{nl}^H(1-\beta)&=ER-(\beta+\alpha K)(ER-R_f)-pl\\ \beta R_f+d_{nl}^L(1-\beta)&=ER-(\beta+\alpha K)(ER-R_f)-pl\\ \beta R_f+d_{l}^H(1-\beta)-l&=ER-(\beta+\alpha K)(ER-R_f)-pl\\ \beta R_f+d_{l}^L(1-\beta)-l&=ER-(\beta+\alpha K)(ER-R_f)-pl\end{aligned}$$

Thus

$$\begin{aligned}d_{nl}^H=d_{nl}^L&=\tfrac{(1-\beta)ER-(\beta+\alpha K)(ER-R_f)-pl}{1-\beta}\\ d_{l}^H=d_{l}^L&=\tfrac{(1-\beta)ER-(\beta+\alpha K)(ER-R_f)+(1-p)l}{1-\beta}\end{aligned}$$

So insuree's utility is

$$\max_{\beta}\ln\big(ER-(\beta+\alpha K)(ER-R_f)-pl\big)$$

subject to

$$0\leq\beta\leq1$$

Thus

$$\beta^*=0$$

Regulator's problem is

$$\max_{\alpha}\ln\big(ER-\alpha K(ER-R_f)-pl\big)$$

subject to

$$0\leq\alpha\leq1$$

Thus

$$\alpha^*=0$$

Therefore, the optimal insurance contract is

$$d_{nl}^{L*}=d_{nl}^{H*}=ER-pl\qquad d_{nl}^{L*}=d_{nl}^{H*}=ER+(1-p)l$$

Now we solve for the optimal tax/subsidy depends on the realized aggregate states. In bad aggregate state, the successful insurer's payoff is $\big(1+K\big)R_H+T_S^L-D^L$, while failed insurer's payoff is

$\big(1+K\big)R_L + T_F^L - D^L$. And each insuree does not bear idiosyncratic risk

$$
\begin{aligned}
& \big(1+K\big)R_H + T_S^L - D^L = \big(1+K\big)R_L + T_F^L - D^L \\
= \quad & C^{L*} = K \cdot ER - (1-q)(M^H - M^L)(1+K) \\
\Rightarrow \quad & \begin{cases} T_S^L & = C^{L*} + d^L - (1+K)\cdot R_H = (1+K)(M^L - R_H) < 0 \\ T_F^L & = C^{L*} + d^L - (1+K)\cdot R_L = (1+K)(M^L - R_L) > 0 \end{cases}
\end{aligned}
$$

Thus the tax/subsidy for insurers whose assets success or fail, respectively, are:

$$
\begin{cases} T_S^L & = (1+K)(M^L - R_H) \\ T_F^L & = (1+K)(M^L - R_L) \end{cases}
$$

The tax budget is balance neutral because $\big(q(1-\tau)+\tau\big)\cdot T_F^L + (1-q)(1-\tau)\cdot T_S^L = (1+K)(M^L - M^L) = 0$.

Similarly, if in the good aggregate state, successful insurer's payoff is $\big(1+K\big)R_H + T_S^H - d^H$, while failed insurer's payoff is $\big(1+K\big)R_L + T_F^H - d^H$. each insurer does not bear idiosyncratic shocks, and the following equation holds.

$$
\begin{aligned}
& \big((1+K-t\cdot K\big)R_H + T_S^H - d^H = \big(1+K-t\cdot K\big)R_L + T_F^H - d^H \\
= \quad & C^{H*} = K \cdot ER + q(M^H - M^L)(1+K) \\
\Rightarrow \quad & \begin{cases} T_S^H & = C^{H*} + d^H - (1+K)\cdot R_H = (1+K)(M^H - R_H) < 0 \\ T_F^H & = C_H^* + d^H - (1+K)\cdot R_L = (1+K)(M^H - R_L) > 0 \end{cases}
\end{aligned}
$$

The taxes/subsidies for insurers whose assets succeed or fail are

$$
\begin{cases} T_S^H & = (1+K)(M^H - R_H) \\ T_F^H & = (1+K)(M^H - R_L) \end{cases}
$$

In good aggregate state, the taxation is also budget neutral since $\big((1-q)(1-\tau)+\tau\big)\cdot T_S^H + q(1-\tau)\cdot T_F^H = (1+K)(M^H - M^H) = 0$ Therefore, the taxation schem is

$$
\begin{cases} T_S^L & = (1+K)(M^L - R_H) \\ T_F^L & = (1+K)(M^L - R_L) \\ T_S^H & = (1+K)(M^H - R_H) \\ T_F^H & = (1+K)(M^H - R_L) \end{cases}
$$

We now consider the second case where $\tau_1 \le \tau \le \tau_2$, insurees cannot be perfectly insured, the insuree's payoffs in the two aggregate states are:

$$
\beta R_f + d^L(1-\beta) - pl = (1+K)M^L + (\beta + \alpha K)(R_f - M^L) - pl
$$
$$
\beta R_f + d^H(1-\beta) - pl = (1+K)M^H + (\beta + \alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K
$$

The insuree's problem is:

$$\max_{\beta}(1-q)\ln\big((1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K\big)$$
$$+\,q\ln\big((1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl\big)$$

subject to

$$0\le\beta\le1$$

Thus

$$\begin{aligned}\mathcal{L} &= (1-q)\ln\big((1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K\big)\\ &\quad +q\ln\big((1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl\big)+\lambda_1\beta+\lambda_2(1-\beta)\end{aligned}$$

It follow that the first order condition is:

$$\frac{(1-q)(R_f-M^H)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}$$
$$+\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}+\lambda_1-\lambda_2=0$$

that is $\lambda_2=\lambda_1=0$, that is $0<\beta<1$, However, we can solve the solution to function

$$\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}=\frac{(1-q)(M^H-R_f)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}$$

such that $\beta*<0$, which violates $0<\beta<1$.

Since $\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}-\frac{(1-q)(M^H-R_f)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}$ is a decreasing function of β, thus we need $\lambda_2=0$, and $\lambda_1>0$, that is

$$\beta^*=0$$

Similarly, the optimal investment of insurer in safe asset is as follows:

$$\max_{\alpha}(1-q)\ln\big((1+K)M_H+\alpha K(R_f-M_H)-pl-\frac{ER}{1-q}K\big)+q\ln\big((1+K)M_L+\alpha K(R_f-M_L)-pl\big)$$

subject to

$$0\le\alpha\le1$$

In the similar way, we can solve the optimal α^*, that is $\alpha^*=0$.
Therefore the optimal insurance contracts are:

$$d_{nl}^L=(1+K)M^L-pl \qquad d_{nl}^H=(1+K)M^H-\frac{ER}{1-q}K-pl$$
$$d_l^L=(1+K)M^L+(1-p)l \qquad d_l^H=(1+K)M^H-\frac{ER}{1-q}K+(1-p)l$$

$d^L = (1+K)M^L$

Now we derive the optimal taxation scheme: In the bad aggregate state, the payoff of the insurer whose assets succeed is $\big(1+K\big)R_H + T_S^L - d^L = (1+K)(R_H - M_L) + T_S^L$, while the payoff of the insurer whose assets fail is $\big(1+K\big)R_L + T_F^L - d^L = (1+K)(R_L - M_L) + T_F^L$. Since regulator can reinsure the idiosyncratic shocks to insuerers through tax, each insurer does not bear idiosyncratic risk. In other words,

$$(1+K)R_H + T_S^L - d^L = (1+K)R_L + T_F^L - d^L = C^{L*} = 0$$

Thus the taxation/subsidy among insurees are:

$$\begin{cases} T_S^L & = d^L - (1+K)R_H = (1+K)(M^L - R_H) < 0 \\ T_F^L & = (1+K)M^L - (1+K)R_L = (1+K)(M^L - R_L) > 0 \end{cases}$$

In bad aggregate state, the tax transfers satisfy the following budge neutral constraint:

$$\big(q(1-\tau)+\tau\big)\cdot T_S^L + (1-q)(1-\tau)\cdot T_F^L = (1+K)(M^L - M^L) = 0$$

Similarly, if in the good aggregate state, the payoff of insurers whose assets succeed is $\big((1+K\big)R_H + T_S^H - d_H$, while the payoff of the insurers whose assets fail is $\big(1+K\big)R_L + T_F^H - d_H$.Each insurer do not bear idiosyncratic risk, then

$$\big((1+K\big)R_H + T_S^H - d^H = \big(1+K\big)R_L + T_F^H - d^H = C^{H*} = \frac{ER\cdot K}{1-q}$$

$$\begin{cases} T_S^H & = d^H + \frac{ER\cdot K}{1-q} - (1+K)R_H = (1+K)(M^H - R_H) < 0 \\ T_F^H & = d^H + \frac{ER\cdot K}{1-q} - (1+K)R_L = (1+K)(M^H - R_L) > 0 \end{cases}$$

In good aggregate state, the taxation is budget budget neutral since

$$\big((1-q)(1-\tau)+\tau\big)\cdot T_S^H + q(1-\tau)\cdot T_F^H = (1+K)(M^H - M^H) = 0$$

Therefore, the optimal tax scheme is:

$$\begin{cases} T_S^L & = (1+K)(M^L - R_H) \\ T_F^L & = (1+K)(M^L - R_L) \\ T_S^H & = (1+K)(M^H - R_H) \\ T_F^H & = (1+K)(M^H - R_L) \end{cases}$$

In this case,it is still optimal for insurees invest all their capital in buying risky insurance contracts, insurers invest all their capital in risky assets, and regulators's taxes transfers are given as above.

We now proceed to the third case when $\tau > \tau_2$, insurees cannot be perfectly insured, thus the insurees' payoff in two states are: $\beta R_f + d^L(1-\beta) - pl = (1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl$ in the bad aggregate state and $\beta R_f + d^H(1-\beta) - pl = (1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K$

in good aggregate state, respectively. Thus insuree's problem is

$$\max_{\beta}(1-q)\ln\big((1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K\big)$$
$$+q\ln\big((1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl\big)$$

subject to

$$0\le\beta\le 1$$

Thus

$$\begin{aligned}\mathcal{L} &= (1-q)\ln\big((1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K\big)\\ &\quad +q\ln\big((1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl\big)+\lambda_1\beta+\lambda_2(1-\beta)\end{aligned}$$

The first order condition is

$$\frac{(1-q)(R_f-M^H)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}$$
$$+\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}+\lambda_1-\lambda_2=0$$

Suppose $\lambda_2=\lambda_1=0$, that is $0<\beta<1$ Thus the opitmal $\beta^*$is

$$\beta^*=\frac{(1+K)(ER\cdot R_f-M^HM^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}}{(M^H-R_f)(R_f-M^L)}-\alpha K$$

Thus insuree' utility is given by

$$(1-q)\ln\Bigg((1+K)M^H-\frac{(1+K)(ER\cdot R_f-M^HM^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}}{(R_f-M^L)}$$
$$-pl-\frac{ER}{1-q}K\Bigg)+q\ln\Bigg((1+K)M^L$$
$$+\frac{(1+K)(ER\cdot R_f-M^HM^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}}{(M^H-R_f)}-pl\Bigg)$$

for any α, $\beta=\beta^*(\alpha)$ such that insuree's welfare will not change. Thus we need

$$\beta^*=\frac{(1+K)(ER\cdot R_f-M^HM^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}}{(M^H-R_f)(R_f-M^L)}-\alpha K<1$$

If $\frac{(1+K)(ER\cdot R_f-M^HM^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}}{(M^H-R_f)(R_f-M^L)}\le 1$, α^*can be any number between 0 and 1.

If $1<\frac{(1+K)(ER\cdot R_f-M^HM^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}}{(M^H-R_f)(R_f-M^L)}<1+K$, then α^*has to be greater than $\frac{(1+K)(ER\cdot R_f-M^HM^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}}{K(M^H-R_f)(R_f-M^L)}-\frac{1}{K}$.

Regulator impose the minimum requirement of liquidity buffer α^* such that any

$$\alpha^* \in \left(\max\left\{ \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{K(M^H - R_f)(R_f - M^L)} - \frac{1}{K},\ 0 \right\},\ 1 \right)$$

, in safe assets, and insurees invest $\beta^*$where

$$\beta^* = \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} - \alpha^* K$$

and the opitmal insurance contracts are

$$\begin{aligned}
d^L_{nl} &= \frac{(1+K)M^L + (\beta^* + \alpha^* K)(R_f - M^L) - \beta^* R_f - pl}{1-\beta^*} \\
d^H_{nl} &= \frac{(1+K)M^H + (\beta + \alpha K)(R_f - M^H) - \frac{ER}{1-q}K - \beta^* R_f + (1-p)l}{1-\beta^*} \\
d^L_{l} &= \frac{(1+K)M^L + (\beta^* + \alpha^* K)(R_f - M^L) - \beta^* R_f - pl}{1-\beta^*} \\
d^H_{l} &= \frac{(1+K)M^H + (\beta + \alpha K)(R_f - M^H) - \frac{ER}{1-q}K - \beta^* R_f + (1-p)l}{1-\beta^*}
\end{aligned}$$

Now we derive the optimal taxation scheme. In the bad aggregate state, the payoff of insurers whose assets success is $\big(1 + K - (\beta^* + \alpha^* K)\big)R_H + T^L_S - d^L(1-\beta^*)$, while the payoff of insurers whose assets fail is $\big(1 + K - (\beta^* + \alpha^* K)\big)R_L + T^L_F - d^L(1-\beta^*)$ Since each insurer does not bear idiosyncratic risk:

$$\begin{aligned}
& \big(1 + K - (\beta^* + \alpha^* K)\big)R_H + T^L_S - d_L(1-\beta^*) \\
= \quad & \big(1 + K - (\beta^* + \alpha^* K)\big)R_L + T^L_F - d_L(1-\beta^*) \\
& = C^{L*} = -\alpha^* K R_f
\end{aligned}$$

Thus the tax schemes are:

$$\begin{cases}
T^L_S & = d_L(1-\beta^*) - \alpha^* K R_f - \big(1 + K - (\beta^* + \alpha^* K)\big)R_H \\
 & = (1 + K - (\beta^* + \alpha^* K))(M^L - R_H) < 0 \\
T^L_F & = d_L(1-\beta^*) - \alpha^* K R_f - \big(1 + K - (\beta^* + \alpha^* K)\big)R_L \\
 & = (1 + K - (\beta^* + \alpha^* K))(M^L - R_L) > 0
\end{cases}$$

Similarly, in good aggregate state, the payoff of insurers whose assets succeed is $\big(1 + K - (\beta^* + \alpha^* K)\big)R_H + T^H_S - d^H(1-\beta^*)$, while the payoff of insurers whose assets fail is $\big(1 + K - (\beta^* + \alpha^* K)\big)R_L + T^H_F - d^H(1-\beta^*)$ Since each insurer does not bear idiosyncratic risk:

$$\begin{aligned}
& \big(1 + K - (\beta^* + \alpha^* K)\big)R_H + T^H_S - d_H(1-\beta^*) \\
&= \big(1 + K - (\beta^* + \alpha^* K)\big)R_L + T^H_F - d_H(1-\beta^*) \\
&= C^{H^*} = \tfrac{ER \cdot K}{1-q} - \alpha K \cdot R_f
\end{aligned}$$

Therefore, the optimal tax schemes are:

$$\begin{cases} T_S^H & = d^H(1-\beta^*) + \frac{ER\cdot K}{1-q} - \alpha K \cdot R_f - \big(1+K-(\beta^*+\alpha^* K)\big)R_H \\ & = \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_H) < 0 \\ T_F^H & = d^H(1-\beta^*) + \frac{ER\cdot K}{1-q} - \alpha K \cdot R_f - \big(1+K-(\beta^*+\alpha^* K)\big)R_L \\ & = \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_L) > 0 \end{cases}$$

The tax budget neutral constraints are also satisfied in good and bad aggregate states, respectively:

$$q(1-\tau)\cdot T_F^H + \big((1-q)(1-\tau)+\tau\big)\cdot T_S^H = (1+K-(\beta^*+\alpha^* K))(M^H - M^H) = 0$$
$$(1-q)(1-\tau)\cdot T_S^L + \big(q(1-\tau)+\tau\big)\cdot T_F^L = (1+K-(\beta^*+\alpha^* K))(M^L - M^L) = 0$$

Thus we have:

$$\begin{cases} T_S^L & = (1+K-(\beta^*+\alpha^* K))(M^L - R_H) \\ T_F^L & = (1+K-(\beta^*+\alpha^* K))(M^L - R_L) \\ T_S^H & = \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_H) \\ T_F^H & = \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_L) \end{cases}$$

■

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