Interconnectedness and systemic risk in the US CDS market

Author

Abstract

This study assesses systemic risk in the US credit default swap (CDS) market. First, we estimate the bilateral exposures matrix using aggregate fair value data on call reports by the Federal Deposit Insurance Corporation and theoretically analyze interconnectedness in the US CDS network using various network measures. The robustness of the estimated bilateral matrix is fully assured by sensitivity analysis using a core-periphery model and modified Jaccard index. Second, we theoretically analyze the contagious defaults introducing the Eisenberg and Noe framework. The network analysis shows that three to six banks were central in the network in the past. The default analysis shows the theoretical occurrence of many stand-alone defaults and a few contagious defaults during the global financial crisis. A stress test based on a hypothetical severe stress scenario predicts the occurrence of future contagious defaults. To conclude, the risk contagion via the CDS network is relatively restricted.

Keywords: credit default swap; systemic risk; contagious default; interconnectedness; network theory; centrality measure

JEL classification: G10; D85; L14; G28; F37
1. Introduction

There has been growing interest in the systemic importance of financial institutions stemming from credit default swap (CDS) contracts among US market regulators. Hence, the lack of both theoretical and empirical studies on the role of CDSs and their interconnectedness has become a major issue. The purpose of this study is to analyze the systemic importance of CDS market dealers in the US financial market. Network analysis plays an important role in the analysis of systemic importance.

During the global financial crisis, CDSs became known as derivatives that triggered the systemic contagion risks in the derivative market. A typical example of contagions related to CDS contracts is the management crisis of American International Group (AIG), which was a major seller of CDSs. However, the AIG crisis was actually caused by a London-based derivative subsidiary – AIG Financial Products (AIG–FP), most of whose counterparties were Western financial institutions, such as Société Générale, Goldman Sachs, Deutsche Bank, Merrill Lynch, Calyon, UBS, and Deutsche Zentral-Genossenschaftsbank (Coral Purchasing) (ECB, 2009).

As at the end of September 2008, the aggregate gross notional amount of credit derivatives sold by AIG was 493 billion US dollars or 372 billion US dollars on a net basis. This amount potentially could affect the whole financial network. The net notional amount was almost double the aggregate net notional amount sold by all Depository Trust and Clearing Corporation (DTCC) dealers combined at the end of October 2008. However, as at the end of 2006, AIG was not ranked among the largest CDS market dealers in a Fitch survey, because AIG mainly sold bespoke CDSs, which were not covered by the DTCC data. Finally, as federal assistance to AIG, almost 50 billion US dollars was paid to the CDS counterparties at the end of 2008 (Harrington, 2009).

First, we analyze interconnectedness in the US CDS market using network centrality measures (“network analysis”) (Jackson, 2010). Second, we conduct a model analysis of contagious defaults, applying the Eisenberg–Noe framework (Eisenberg and Noe, 2001) to the US CDS network and setting up a default mechanism (“default analysis”). In this framework, defaults are classified into stand-alone defaults and contagious defaults, which are defaults that trigger a domino effect. In the original Eisenberg–Noe framework, multi-step defaulting events are not expressed. Because contagious defaults can be caused in a specific setting, it is important to determine whether
this framework is suitable for the theoretical analysis using real-world data. Strictly speaking, this framework describes only simultaneous defaults for one period, not a dynamic setting for a multi-period context. We use the framework in the multi-period setting using continuously estimated market values of assets and theoretically analyze the US CDS network. In addition, it is important to validate whether many defaulting and non-defaulting banks suffered losses owing to the payment defaults by the defaulting counterparties. Furthermore, we conduct a stress test to assess the occurrence possibilities of contagious defaults in the future. An important preparation to conduct these analyses is the estimation of bilateral exposures in the US CDS market. To check the robustness of the estimated matrix, we conduct sensitivity analysis.

The US CDS market was composed of 10–20 major dealers (refer to Appendix A for details) during 2006–2015 and most contracts were executed among five dealers – JPMorgan Chase, Bank of America, Citibank, Goldman Sachs, and HSBC Bank USA. Our dataset covers most transactions in the market. This study contributes to the literature on systemic risk by setting up a contagious default model, theoretically assessing the systemic importance of the US CDS market dealers using network centrality measures, and conducting a systemic stress test in the CDS network. Our methodology could assist in the development of monitoring and early-warning indicators related to systemic risk in the CDS network by supervisory authorities. Furthermore, it could be used in the implementation of banks’ internal systemic stress tests of default contagion.

The rest of this paper is organized as follows. Section 2 reviews the extant literature on systemic risk in the CDS market and the interconnectedness in financial networks. Section 3 describes the mechanism of defaults in the CDS network, and Section 4 discusses the estimations of the bilateral exposures matrix and market value of assets. Section 5 describes the data used in this study. Section 6 presents the results of the risk analysis. Section 7 assesses the robustness of the estimated bilateral exposures matrix, and Section 8 concludes.

2. Literature review

Brunnermeier et al. (2013) assess the potential systemic and contagion risks arising from a credit event for a major CDS reference entity or from the default of a key player in the CDS market. In particular, they argue that the
multi-faceted nature of interconnectedness is difficult to capture in existing analytical frameworks. First, to understand risk transfer and risk-bearing capacity better, it would be necessary to know whether CDS exposures stem from proprietary trading, market making, or hedging. Second, counterparty credit risk is material in other over-the-counter (OTC) derivatives markets, in which the transaction volume cleared by central counterparties (CCPs) is still relatively small.

Markose et al. (2012) provide an empirical reconstruction of the US CDS network for the fourth quarter in 2007 and 2008. The propagation of financial contagion in networks with dense clustering that reflects high concentration or localization of exposures between few participants is identified as “too interconnected to fail.” Systemic risk management from bank failures in uncorrelated random networks is different from those with clustering. Because systemic risk of highly connected financial institutions in the CDS markets is not priced into their holding of capital and collateral, they design a super-spreader tax based on eigenvector centrality of the banks that can mitigate potential socialized losses. Eigenvector centrality is one of the centrality measures and expresses the influence of a node in a network.

Network analysis is a highly effective approach to examine interconnectedness in CDS markets, which represent complex contract networks, with a set of “nodes” connected by “edges.” In a CDS network, the nodes represent dealers and the edges represent the CDS contracts between the dealers.

An analysis of CDS networks would alert the supervisory authorities or individual financial institutions about “contagion risk” from the channels through which shocks propagate. Hence, the resilience of a network is tested in such analyses, and systemically significant nodes are identified. In addition, network analysis provides an empirical tool to test the effectiveness of macro-prudential policies.

An analytically tractable example in financial networks is the interbank network, which is characterized by bilateral exposures in the interbank market. In many countries, data pertaining to bilateral exposures are not published, and many researchers are unable to use these data. This difficulty is the same as that with CDS exposures data. Therefore, estimating the bilateral exposures matrix with the elements exposed from one bank to another is a significant challenge. Recently, some studies have adopted an information theory-based method that minimizes the amount of information required in the bilateral exposures matrix (e.g., Censor and Zenios, 1998; Sheldon and Maurer, 1998; Upper and Worms, 2002; Wells, 2004).
The extant literature on financial networks includes two approaches. The first describes the network structure using topological indicators. The literature often relates these indicators to model graphs based on network theory. This approach does not assume a mechanism by which shocks propagate within the network; therefore, it is referred to as “static network analysis” (Alves et al., 2013). Eisenberg and Noe (2001), Boss et al. (2004), Afonso et al. (2011), Puhr et al. (2012), Tirado (2012), and Kanno (2015, 2016) are examples of studies based on this approach. Using the Austrian central credit register, Boss et al. (2004) and Puhr et al. (2012) report that the Austrian interbank market is hierarchized, and banks within subsectors tend to cluster together. The hierarchization of core banks and peripheral banks is confirmed for several national interbank systems, such as in Belgium (Degryse and Nguyen, 2007), Germany (Craig and von Peter, 2014), Italy (Iori et al., 2008), the Netherlands (in ’t Veld and van Lelyveld, 2012), and the United Kingdom (Langfield et al., 2014).

The second approach assesses the strength of the contagion channels and the resilience of the network by observing the responses of financial network structures to shocks. The introduction of a shock assumes a specific transmission mechanism, such as defaults by market participants. This approach is referred to as “dynamic network analysis” in Alves et al. (2013). Some extant studies that focus on the analysis of contagion effects include Elsinger et al. (2006), Cocco et al. (2009), Haldane and May (2011), and Duan and Zhang (2013).

Eisenberg and Noe (2001) develop a fundamental framework for assessing contagious default. According to their theorem, under mild regularity conditions, a unique “clearing payment vector” exists that clears members’ obligations from the clearing system. However, because no closed-form solution exists for the distribution of the payment vector in this algorithm, a simulation approach based on hypothetical scenarios must be used. The model discussed in Section 3 provides details about this approach.

3. Default mechanism

In this section, we describe the mechanism of contagious defaults in the CDS market. To this end, we apply the fundamental framework proposed by Eisenberg and Noe (2001) for risk analysis of the CDS market. Following their framework, we simultaneously solve CDS payments of all the dealers in the market. The monthly solutions are obtained using the market value-
based parameters of assets and volatilities estimated from a continuous-time model detailed in Appendix C.

We assume that the market value-based balance sheet is composed of the market value-based asset and equity values. This sheet is divided into an item in response to CDS payments and the other items.

Consider a set of CDS market dealers $\mathcal{N} = \{1, \ldots, N\}$ at time $t \in [0, T]$. The CDS network structure is represented as $(L, e)$, where $L = (l_{ij})_{1 \leq i, j \leq N}$ is an $(N \times N)$ CDS bilateral exposures matrix, and $e$ is the exogenous net claims cash flow vector, which is calculated as the difference between the market value of assets and the book value of liabilities.

If the total value of a CDS player becomes negative for a pair $(L, e)$, the player becomes insolvent. Let $d_i = \sum_{j=1}^{N} l_{ij}$ represent the total obligations of player $i$ to all dealers $j$ (any $j \in \mathcal{N}$) of the network. In addition, we consider a matrix $\Pi \in [0, 1]^{N \times N}$, which is derived by normalizing the entries with the total obligations:

$$
\Pi_{ij} = \begin{cases} 
\frac{l_{ij}}{d_i}, & \text{if } d_i > 0 \\
0, & \text{otherwise.}
\end{cases}
$$

For any $d_i > 0$, an expected CDS payment claims row vector $\bar{d}' \Pi$ is as follows:

$$
\bar{d}' \Pi = \begin{bmatrix} 
\frac{l_{11}}{d_1} & \ldots & \frac{l_{1N}}{d_1} \\
\vdots & \ddots & \vdots \\
\frac{l_{N1}}{d_N} & \ldots & \frac{l_{NN}}{d_N}
\end{bmatrix} \begin{bmatrix} 
\frac{l_{11}}{d_i} & \ldots & \frac{l_{1N}}{d_i} \\
\vdots & \ddots & \vdots \\
\frac{l_{N1}}{d_N} & \ldots & \frac{l_{NN}}{d_N}
\end{bmatrix}
\begin{bmatrix} 
\frac{l_{11}}{d_i} & \ldots & \frac{l_{1N}}{d_i} \\
\vdots & \ddots & \vdots \\
\frac{l_{N1}}{d_N} & \ldots & \frac{l_{NN}}{d_N}
\end{bmatrix}
= \begin{bmatrix} 
\sum_{i=1}^{N} l_{1i} & \sum_{i=1}^{N} l_{2i} & \ldots & \sum_{i=1}^{N} l_{Ni}
\end{bmatrix}.
$$

A CDS network is described as a 3-tuple $(\Pi, e, d)$, for which we define a clearing payment vector $p^*$. The clearing payment vector represents the limited liabilities of the dealers and the proportional sharing in the event of a default.

A payment vector $p^* \in [0, d]$ is a clearing payment vector if and only if
the following condition holds:

\[ p_i^* = \begin{cases} 
  d_i & \text{, case 1: if } \sum_{j=1}^{N} \Pi'_{ij} p_j^* + e_i \geq d_i \\
  \sum_{j=1}^{N} \Pi'_{ij} p_j^* + e_i & \text{, case 2: if } 0 \leq \sum_{j=1}^{N} \Pi'_{ij} p_j^* + e_i < d_i \\
  0 & \text{, case 3: if } \sum_{j=1}^{N} \Pi'_{ij} p_j^* + e_i < 0 
\end{cases} \quad (3) \]

The condition for case 1 is a solvent case for bank \( i \). By contrast, the conditions for cases 2 and 3 are defaulting cases.

The loss vector for the dealers is calculated as follows:

\[ \text{loss} := \Pi' d - \Pi'_{\text{new}} p^*. \quad (4) \]

The matrix \( \Pi_{\text{new}} \) is defined in the following equation (6).

We adopt the default algorithm developed by Eisenberg and Noe (2001) to find a clearing payment vector. They prove that a unique clearing payment vector always exists for \((\Pi, e, d)\) under mild regularity conditions. These results apply to our multi-period framework.

The number of defaulting dealers is calculated by comparing the clearing payment vector with the current payment vector. A theoretical default algorithm is implemented to calculate the clearing payment vector, which is summarized as follows:

**Step 1:** Initialize \( p_j = d_j \) and calculate the net claim value of bank \( j \) as

\[ v_j := \sum_{m=1}^{N} \Pi'_{jm} p_m + e_j - d_j. \]

If \( v_j > 0 \), its bank does not default, the clearing payment vector is \( p_j = d_j \), and this algorithm ends. Otherwise, proceed to Step 2.

**Step 2:** Find banks with net value \( v_j < 0 \) that can pay only a part of their liabilities to other banks. The ratio is defined as follows:

\[ \theta_j := \frac{\sum_{m=1}^{N} \Pi'_{jm} p_m + e_j}{d_j} \in (0, 1). \quad (5) \]

Under this assumption, only the banks identified in Step 2 would default. We replace \( l_{ij} \) with \( \theta_j l_{ij} \) to ensure the limited liability criterion is met. Thus, we obtain new \( l_{ij}, \Pi_{ij}, d_i, \) and \( v_i \). For example, when \( l_{ij} \) is replaced with \( l_{ij}^{\text{new}} := \theta_j l_{ij} \) \((i = 1, \ldots, N; j = \text{a fixed number})\), the
new $\Pi$ is as follows:

$$
\Pi_{\text{new}} = \begin{bmatrix}
\frac{l_{11}}{d_i^{\text{new}}} & \cdots & \theta_j \frac{l_{ij}}{d_i^{\text{new}}} & \cdots & \frac{l_{1N}}{d_i^{\text{new}}} \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\frac{l_{N1}}{d_N^{\text{new}}} & \cdots & \theta_j \frac{l_{Nj}}{d_N^{\text{new}}} & \cdots & \frac{l_{NN}}{d_N^{\text{new}}}
\end{bmatrix}.
$$

(6)

where the new $d_i$ is $d_i^{\text{new}} = l_{i1} + \cdots + (\theta_j l_{ij}) + \cdots + l_{iN}$. Step 2 is repeated for all defaulting banks.

This procedure provides a clearing payment vector for the CDS network that satisfies equation (3). Next, we distinguish between a stand-alone default caused by a guarantor’s insolvent situation and a contagious default caused by the defaults of other banks. These two types of defaults are described as follows:

**Type 1: Stand-alone default:**

$$
V_i(T) - D_i(T) := \left( \sum_{j=1}^{N} \Pi_{ij} d_j - d_i \right) + e_i \leq 0.
$$

(7)

where $V_i(T)$ is the market value of the total assets of bank $i$ at time $t = T$ ($T = 1$ month, 2 months, . . .), and $D_i(T)$ is the total face value of the interest-bearing liabilities with regard to bank $i$ at time $t = T$ and is fixed quarterly. The net claims of bank $i$ are composed of CDS net positive exposures ($\sum_{j=1}^{N} \Pi_{ij} d_j - d_i$) and non-CDS net exposures ($e_i$). Bank $i$ falls into stand-alone default if it cannot honor its payments, under the assumption that all of the other banks honor their promises.

**Type 2: Contagious default:**

$$
\left( \sum_{j=1}^{N} \Pi_{ij} d_j - d_i \right) + e_i > 0,
$$

(8)

and

$$
\left( \sum_{j=1}^{N} \Pi_{ij} p_j^* - d_i \right) + e_i \leq 0.
$$

(9)

A contagious default occurs when bank $i$’s net claims are positive (equa-
tion (8)) but other banks cannot fulfill their promises to the bank (equation (9)).

The estimation for the non-CDS net exposures in equation (7) is provided as follows:

\[ e_i = V_i(T) - D_i(T) - \left( \sum_{j=1}^{n} \Pi'_{ij} d_j - d_i \right). \] (10)

Because both net CDS exposures \( \sum_{j=1}^{n} \Pi'_{ij} d_j - d_i \) and total liabilities \( D_i(T) \) are constant, and total asset value \( V_i(T) \) is a random variable, net non-CDS exposures \( e_i \) is a random variable as well. Therefore, we first estimate the market value of \( V_i(T) \) \( (T = 1 \text{ month}, 2 \text{ months}, \ldots) \) in a multi-period setting using equity data.

4. Estimations

4.1. Bilateral exposures matrix

If all dealers in the US CDS network are included in the estimation of the bilateral exposures matrix, the bilateral positive fair value matrix would be in perfect accordance with the bilateral negative fair value matrix. According to our data resource – call reports by the Federal Deposit Insurance Corporation (FDIC) – the average difference between both matrixes from March 2006 to September 2015 is only 3%. Hence, in the US CDS market assumed in our research, almost all market dealers are covered. Thus, we estimate the bilateral exposures matrix using two aggregated fair value data. Refer to Appendix B for details.

4.2. Market value of assets

We need to consider the market value of assets, volatilities, and drifts in order to calculate the probabilities of default in the context of the structural model approach of credit risk. Because the asset value is a latent variable, it cannot be calculated directly. Hence, we estimate the market value-based parameters of assets using the estimation procedure applied by Duan (1994, 2000), Crosbie and Bohn (2003), and Duan et al. (2004). Refer to Appendix C for the details.
5. Data

The financial data used in our research, including market capitalization data, are obtained from Bankscope database provided by Bureau van Dijk. In addition, the data for the CDS contracts come from call reports, as collected by the FDIC under Section 1817(a)(1) of the Federal Deposit Insurance Act. All regulated financial institutions in the United States are required to file periodic financial and other information with regulators and other parties. Each national bank, state member bank, and insured non-member bank is required by the Federal Financial Institutions Examination Council (FFIEC) to file a call report.

In the call reports, we use two items: gross positive fair value (GPFV) and gross negative fair value (GNFV). GPFV is the sum total of fair values of contracts in which an institution is owed money by its counterparties, without taking into account netting. This represents the maximum losses an institution could incur if all its counterparties default and there is no netting of contracts, and the institution holds no counterparty collateral. GNFV is the sum total of fair values of an institution’s contracts in which the institution currently has a balance outstanding to the counterparty.

The share for the top 22 US banks ranked in terms of fair value amounts is more than 99% of the amounts listed in call reports. Nonetheless, AIG is not included in this report because it is not a bank.

6. Results

6.1. Estimation of bilateral CDS exposures matrix and network analysis

We estimate the bilateral CDS exposures matrix $X$ expressed in equation (B.1) before we conduct the various systemic risk analyses. A bank’s “gross negative fair value” is allocated as the CDS exposures among the counterparties. The descriptive statistics of the estimated matrixes are shown in Table 1, and the percentile distribution by quarter is shown in Figure 1. Table 1 shows the quartile at any year-end. All of the exposure sizes are quite small below the 75th percentile; they are near zero at the 75th percentile. However, the sizes increase sharply from the 95th percentile to the maximum, and they range from 2.7 billion to 88.7 billion US dollars at maximum.

Next, we analyze the US CDS network using network centrality measures. In order to consider the applicability of the various centrality measures to
Table 1: Descriptive statistics of bilateral exposures (in 1,000 US dollars)

<table>
<thead>
<tr>
<th>percentile</th>
<th>Dec-07</th>
<th>Dec-08</th>
<th>Dec-09</th>
<th>Dec-10</th>
<th>Dec-11</th>
<th>Dec-12</th>
<th>Dec-13</th>
<th>Dec-14</th>
</tr>
</thead>
<tbody>
<tr>
<td>25th</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Median</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>75th</td>
<td>1</td>
<td>1</td>
<td>168</td>
<td>23</td>
<td>10</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>95th</td>
<td>5,821</td>
<td>16,491</td>
<td>1,432,475</td>
<td>345,933</td>
<td>310,242</td>
<td>157,072</td>
<td>79,884</td>
<td>54,622</td>
</tr>
<tr>
<td>Maximum</td>
<td>2,724,729</td>
<td>9,980,327</td>
<td>64,926,936</td>
<td>69,409,331</td>
<td>88,656,443</td>
<td>44,843,249</td>
<td>37,849,167</td>
<td>38,213,304</td>
</tr>
<tr>
<td>Mean</td>
<td>16,992</td>
<td>44,626</td>
<td>844,792</td>
<td>639,638</td>
<td>834,794</td>
<td>474,054</td>
<td>377,071</td>
<td>349,276</td>
</tr>
<tr>
<td>S.D.</td>
<td>164,975</td>
<td>518,288</td>
<td>5,742,442</td>
<td>5,219,586</td>
<td>6,948,632</td>
<td>4,802,691</td>
<td>3,323,983</td>
<td>3,131,354</td>
</tr>
</tbody>
</table>

Note: The sum of the bilateral exposures each year is 484 (= 22^2). S.D. stands for standard deviation.

Figure 1: Bilateral exposure size distribution

Note: The distribution is shown in the range from the 90th percentile to the 100th percentile.
the interconnectedness in the US CDS network, we calculate the correlation among eight selected centrality measures: degree, weighted degree, eccentricity, closeness centrality, eigenvector centrality, betweenness centrality, hyperlink-induced topic search (HITS) hub centrality, and PageRank (Figure 2). The centrality measures shown for illustrative purposes are calculated based on the data as at the end of 2007 to 2014 and September 2015, and some are substantially different from one another. The Pearson correlation is reported in the upper part, and each line shows the linear regression. Eccentricity and closeness centrality represent a bank’s closeness, whereas betweenness centrality measures a bank’s substitutability and shows the bank’s central role in the network. The HITS hub centrality, eigenvector centrality, and PageRank (a variation of eigenvector centrality) capture the magnitude of the network relationships. The first measure relates to the obligation payments and shows the systemic importance in the network, whereas the second and third measures relate to the claims and exhibit no systemic importance. From Figure 2, some strong correlation relationships are observable, such as HITS hub centrality versus degree and betweenness centrality versus weighted degree. In the following part of this subsection, in light of each centrality’s availability in terms of systemic importance, we analyze the interconnectedness, focusing on degree, betweenness centrality, and HITS hub centrality in more detail.

First, the “degree” of a node is the number of edges connected to the node. The “out-degree” is the number of outgoing edges emanating from a node, and the “in-degree” is the number of incoming edges onto a node. In a directed graph, which is defined as a set of nodes in which all the edges are directed from one node to another, each node has a maximum of two degrees for each edge. The total degree of a node is the sum of its in- and out-degrees. The degree of a node can be considered as a proxy variable for interconnectedness.

The top five banks ranked according to interconnectedness, which is measured in terms of the degree of their nodes, are shown in the upper part of Table 2. They are large banks, such as JPMorgan Chase, Bank of America, Citibank, HSBC Bank USA, and Goldman Sachs. Their degrees reached a peak of 29–37 for the crisis period of 2007 to 2009 and thereafter, decreased. This trend is confirmed in Figure 3, which denotes the directed graphs of the
Figure 2: Correlation plot of network centrality measures
US CDS network as at the end of 2007 to 2014 and September 2015.\(^1\) The size of a node denotes the sum of its in- and out-degrees. The width of an edge denotes the size of its CDS exposures at the end of the quarter. The color is a mix of its source (start) node color and its target (end) node color. Certainly, the network is significantly dense for the period, and thereafter, is sparser. In addition, we can confirm that three to six large banks play a central role in each directed graph.

Second, we analyze the interconnectedness of the US CDS network in terms of “betweenness centrality” (Krause and Giansante, 2012; Kanno, 2015), which is a centrality measure of a node within a network graph. This measure quantifies the number of times a node acts as a bridge along the shortest path between two other nodes. Hence, betweenness centrality can also be considered as a measure of substitutability. A node with high betweenness centrality could potentially influence the spread of information through the network. If the normalized betweenness centrality, which is defined as \(\frac{bc - \min(bc)}{\max(bc) - \min(bc)}\) \((bc: \text{the betweenness centrality of a node})\), is close to one, a node (i.e., bank \(A\)) acts as a bridge along most of the shortest paths connecting two other nodes (i.e., banks \(B\) and \(C\)). If it is close to zero, bank \(A\) is less important to the two other banks (i.e., banks \(B\) and \(C\)) (Kanno, 2015).

The top five banks ranked according to interconnectedness as measured in terms of betweenness centrality are shown in the middle part of Table 2. There are about five banks a year with betweenness centrality of more than zero, and the original (not normalized) centrality measure level is around 5 on average and 103 at maximum. This is because of the structure that three to six banks play a dominant role in the US CDS network. If a bank with high betweenness centrality defaults, the clearing payment in the US CDS market may become non-functional.

Third, we analyze the interconnectedness of the US CDS network in terms of “HITS hub centrality.” HITS is known as “hubs” and “authorities.” HITS is proposed to find the main structures in the World Wide Web (WWW). Web pages are divided in two categories: hubs and authorities. By the creation of a hyperlink from page \(p\) to \(q\), the author of page \(p\) increases the authority of \(q\). The authority of a WWW site would be to consider its in-degree (i.e., the

\(^1\)For convenience drawing the graph, the exposure data are truncated to 10,000 US dollars.
Figure 3: Directed graphs based on degrees

Note 1: The figures are for 2007 to 2009 from the upper-left panel to the upper-right panel, for 2010 to 2012 from the middle-left panel to the middle-right panel, and for 2013 to 2015 from the lower-left panel to the lower-right panel.

Note 2: The graph is drawn in the Fruchterman–Reingold layout. The nodes are the mass particles, and the edges are the springs between the particles.
hyperlinks to return to the home page). Hence, HITS authority centrality is not suitable for measuring systemic importance of dealers in the CDS network. To overcome this problem, it is necessary to establish hubs as the counterpart of the authoritative sites. A hub is defined as a WWW site pointing to many authorities. Hence, HITS hub centrality is important in terms of systemic importance.

The top five banks ranked according to interconnectedness as measured in terms of HITS hub centrality are shown in the lower part of Table 2. The correlation between HITS hub centrality and degree centrality is 0.91 from Figure 2 and thus, is high. As a result, the top five banks list of the HITS hub centrality is similar to that of degree centrality.

6.2. Estimation of market variables

The market values of assets and the drifts and volatilities of the asset returns\(^2\) are estimated for each bank using the maximum likelihood estimation methodology detailed in Appendix C. All of the parameter estimations are conducted in local currency units. After final consolidation of the data, the values in local currency units are converted into US dollars using quarterly foreign exchange rates.

The six large banks designated as global systemically important banks (G-SIBs) are selected as graph samples from 22 banks. Their estimation results are shown in Figures 5, 6, and 7.

Because the asset value is a latent variable\(^3\) and is not observable in the financial market, it is important to check its level and time variation. Figure 5 shows that the asset values of previous investment banks, such as Goldman Sachs and Morgan Stanley, substantially decreased just after the bankruptcy of Lehman Brothers. By contrast, four large commercial banks increased their asset values.

As for the asset return volatilities, in general, the higher the asset volatility of a bank, the larger is its probability of default in our framework – the structural model approach of credit risk. The quarterly values of six selected banks are less than 10%, which is not very volatile. Finally, with respect to

\(^2\) Although the estimation frequency of assets and volatilities is monthly, one of the drifts is estimated yearly, given the estimation procedure. In the later default analysis in Subsection 6.3, only asset values are used.

\(^3\) The banks’ market capitalization data are not published in Bankscope for the period when asset values were zero.
Table 2: Top 5 banks ranked according to interconnectedness measured in terms of degree, betweenness centrality, and HITS hub centrality

<table>
<thead>
<tr>
<th>Centrality</th>
<th>R</th>
<th>Dec-07</th>
<th>Dec-08</th>
<th>Dec-09</th>
<th>Dec-10</th>
<th>Dec-11</th>
<th>Dec-12</th>
<th>Dec-13</th>
<th>Dec-14</th>
<th>Sep-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree</td>
<td>1</td>
<td>JPM (37)</td>
<td>JPM (39)</td>
<td>JPM (36)</td>
<td>JPM (28)</td>
<td>JPM (27)</td>
<td>JPM (26)</td>
<td>JPM (23)</td>
<td>JPM (24)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>BOA (37)</td>
<td>BOA (35)</td>
<td>BOA (28)</td>
<td>BOA (26)</td>
<td>BOA (25)</td>
<td>BOA (23)</td>
<td>BOA (23)</td>
<td>BOA (24)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>CITI (37)</td>
<td>CITI (35)</td>
<td>CITI (28)</td>
<td>CITI (26)</td>
<td>CITI (25)</td>
<td>CITI (23)</td>
<td>CITI (23)</td>
<td>CITI (24)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>HSBC (35)</td>
<td>HSBC (32)</td>
<td>HSBC (25)</td>
<td>HSBC (21)</td>
<td>HSBC (22)</td>
<td>HSBC (20)</td>
<td>HSBC (20)</td>
<td>HSBC (21)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>WB (35)</td>
<td>GS (32)</td>
<td>HSBC (29)</td>
<td>HSBC (25)</td>
<td>HSBC (21)</td>
<td>HSBC (22)</td>
<td>HSBC (20)</td>
<td>HSBC (20)</td>
<td>HSBC (21)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Betweenness Centrality (BC)</td>
<td>1</td>
<td>JPM (43)</td>
<td>JPM (105)</td>
<td>JPM (45)</td>
<td>JPM (29)</td>
<td>JPM (13)</td>
<td>JPM (45)</td>
<td>JPM (12)</td>
<td>JPM (33)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>BOA (43)</td>
<td>BOA (19)</td>
<td>BOA (28)</td>
<td>BOA (29)</td>
<td>BOA (16)</td>
<td>BOA (11)</td>
<td>BOA (12)</td>
<td>BOA (13)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>CITI (43)</td>
<td>CITI (19)</td>
<td>CITI (28)</td>
<td>CITI (23)</td>
<td>CITI (16)</td>
<td>CITI (11)</td>
<td>CITI (12)</td>
<td>CITI (13)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>HSBC (33)</td>
<td>HSBC (19)</td>
<td>GS (15)</td>
<td>GS (8)</td>
<td>GS (6)</td>
<td>GS (5)</td>
<td>n.a.</td>
<td>n.a.</td>
<td>HSBC (3)</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>WB (33)</td>
<td>GS (19)</td>
<td>HSBC (12)</td>
<td>HSBC (8)</td>
<td>n.a.</td>
<td>n.a.</td>
<td>HSBC (3)</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>HITS Hub Centrality (HITS)</td>
<td>1</td>
<td>JPM (0.1)</td>
<td>JPM (0.11)</td>
<td>JPM (0.1)</td>
<td>JPM (0.1)</td>
<td>JPM (0.1)</td>
<td>JPM (0.1)</td>
<td>JPM (0.1)</td>
<td>JPM (0.1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>BOA (0.1)</td>
<td>BOA (0.09)</td>
<td>BOA (0.1)</td>
<td>BOA (0.1)</td>
<td>BOA (0.1)</td>
<td>BOA (0.1)</td>
<td>BOA (0.1)</td>
<td>BOA (0.1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>CITI (0.1)</td>
<td>CITI (0.09)</td>
<td>CITI (0.1)</td>
<td>CITI (0.1)</td>
<td>CITI (0.1)</td>
<td>CITI (0.1)</td>
<td>CITI (0.1)</td>
<td>CITI (0.1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>HSBC (0.1)</td>
<td>HSBC (0.09)</td>
<td>GS (0)</td>
<td>GS (0.1)</td>
<td>GS (0.1)</td>
<td>GS (0.1)</td>
<td>GS (0.1)</td>
<td>GS (0.1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>WB (0.1)</td>
<td>GS (0.09)</td>
<td>HSBC (0.09)</td>
<td>HSBC (0.1)</td>
<td>HSBC (0.09)</td>
<td>HSBC (0.1)</td>
<td>HSBC (0.1)</td>
<td>HSBC (0.1)</td>
<td></td>
</tr>
</tbody>
</table>

Note 1: The figures in parentheses indicate degree centrality, betweenness centrality, or HITS hub centrality.

Note 2: Abbreviations: R: Ranking; BC: Betweenness centrality; HITS: HITS hub centrality; JPM: JPMorgan Chase; BOA: Bank of America; CITI: Citibank; GS: Goldman Sachs; WFB: Wells Fargo; WB: Wachovia.
the asset return drifts, the quarterly average of 22 banks for 2006 to 2015 is about 0.03%. The less the drift of a bank, the larger is its probability of default in our framework.

6.3. Contagious default analysis

The theoretical number of stand-alone defaulting banks and contagious defaulting banks was estimated. During the estimation period of 2006–2015, many stand-alone defaults occurred. In contrast, a few contagious default occurred. Figure 8 indicates the monthly variations of the total number of stand-alone defaulting banks (upper panel) and contagious defaulting banks (lower panel). Shortly after the bankruptcy of Lehman Brothers, the number of stand-alone defaults jumped significantly, peaking at 10 in March 2009. In addition, a second peak can be seen in August to September 2007 (the period of subprime mortgage crisis).

Figure 9 indicates the monthly variations of the loss amounts of banks listed in Table A.5. The banks that suffered losses included the defaulted banks as well as some banks that did not fall into a stand-alone default or a contagious default category. Each bank suffered loss for any period from 2006 to 2015, and during the subprime mortgage crisis and shortly after the bankruptcy of Lehman Brothers, five large banks – JPMorgan Chase, Bank of America, Citibank, Goldman Sachs, and HSBC Bank USA suffered losses as a result of the payments from defaulted counterparties. This result is in accordance with the reality. These five banks had credit exposure related to their derivatives trading that exceeds their capital, with four in particular – JPMorgan Chase, Goldman Sachs, HSBC Bank USA and Citibank taking especially large risks. According to Office of the Comptroller of the Currency (OCC), at the end of 2008, Bank of America’s total credit exposure to derivatives was 179 percent of its risk-based capital; Citibank’s was 278 percent; JPMorgan Chase’s was 382 percent; and HSBC Bank USA’s was 550 percent. In addition, in the fourth quarter of 2008, Goldman Sachs began reporting as a commercial bank, revealing an alarming total credit exposure of 1,056 percent, or more than ten times its capital.

6.4. Stress test

We conduct a systemic stress test to verify the resilience of the US CDS network at an evaluation point in the future. CDS market dealers are as-

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4Refer to Appendix D for the details.
sumed as systemic sources with the potential to trigger systemic contagious risk in the test. Hence, we examine whether any one or more of the listed banks trigger contagious defaults.

To examine the effect of contagious defaults, we stress any one or more banks in each test run. The evaluation time point is assumed as the end of 2016, at which point the market value of the asset of the selected bank is reduced by 30% from its value at the end of September 2015. In terms of the network structure, the exposures matrix at the end of either March 2009 (during the global financial crisis) or September 2015 is assumed. The liability value of each bank is assumed the same as that at September 2015.\(^5\) Hence, the stresses are imposed on the asset value of each bank and/or the CDS exposures network.

As a result, one contagious default is triggered by stand-alone defaults of four major banks, given the exposures matrix at the end of March 2009, whereas no default is triggered for the exposures matrix at the end of March 2015. In addition, no default is triggered by the stand-alone default of one major bank for each exposures matrix (Table 3). Judging from this result, unless many stand-alone defaults transpire simultaneously in a severe economic environment, such as the global financial crisis, contagious default is unlikely. In addition, CDS payment defaults directly would not trigger contagious defaults, given the current CDS network structure.

Table 3: Number of contagious defaulting banks based on the stress test

<table>
<thead>
<tr>
<th>Trigger bank(s) (no. of banks)</th>
<th>CDS exposures matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>March 2009</td>
</tr>
<tr>
<td>BOA &amp; CITI &amp; GS &amp; HSBC (4)</td>
<td>1</td>
</tr>
<tr>
<td>BOA / CITI / GS / HSBC / JPM (1)</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: Abbreviations: BOA: Bank of America; CITI: Citibank; GS: Goldman Sachs; HSBC: HSBC Bank USA; JPM: JPMorgan Chase.

\(^5\)Because a bank’s GNFV for the CDS outstanding to its asset value is so small, being 2.5% at maximum, its recovery payment defaults to its counterparties would not trigger a contagious default.
7. Sensitivity analysis

We assess the robustness of our analyses based on the estimated bilateral exposures matrix. In Subsection 4.1, we estimate the bilateral exposures matrix without any restriction as to the distribution for exposures. However, in this section, we consider the information pertaining to the core–periphery structure as additional information for more possible estimation.\(^6\)

Craig and von Peter (2014) present a core–periphery network model. They call the set of top-tier banks “the core,” and the set of lower-tier banks “the periphery.” In the interbank market, top-tier banks lend to each other \((CC):\) core to core), top-tier banks borrow from lower-tier banks \((PC):\) periphery to core), top-tier banks lend to lower-tier banks \((CP):\) core to periphery), and lower-tier banks do not lend to each other \((PP):\) periphery to periphery; a square matrix of zeros.). Thus, the bilateral exposures matrix \(Y\) with the core–periphery structure is constructed as follows:

\[
Y = \begin{bmatrix}
CC & CP \\
PC & PP
\end{bmatrix}.
\]

Core banks in the interbank market can be regarded as money center banks. They act as dealers in a broad range of markets, including the CDS market. According to Langfield et al. (2014), the strength of the core–periphery structure significantly varies depending on the asset class; hence, the core–periphery model fits more strongly for derivatives and marketable securities than for unsecured lending and repo agreements.

In addition, Langfield et al. (2014) argue that pure derivatives houses are at the core of the cluster related to “net CDS sold” as well as lending, marketable securities, securities lending and repo exposure, and derivatives exposure. This cluster comprises a group of dealers with significant exposures.

\(^6\)As a relevant study, Mistrulli (2011) argues that the comparison between the bilateral exposures matrix based on the estimation methodology detailed in Appendix B (i.e., maximum entropy method) and the observed interbank matrix can be interpreted as the theoretical comparison between complete and incomplete markets, which is proposed by Allen and Gale (2000). Paltalidis et al. (2015) also compare with maximum entropy method and the results obtained with the actual bilateral exposures for the German and French banking networks. They conclude that maximum entropy method neither over- nor under-estimates the bilateral exposures and is a suitable way to calibrate losses generated by systemic shocks.

\(^7\)Its symbol indicates a block in the following matrix \(Y\).
to securities holders, diversified banks, and other banks in their own cluster, mostly in the form of derivatives. These derivative houses are most likely exposed to securities holders, whereas they have exposures in both directions with more diversified banks partly due to market-making activities.

In order to obtain the optimal matrix with the core–periphery structure, the approach to minimize a distance measure of the total error score is proposed in Craig and von Peter (2014). As a result of the calculation by our optimization, five or six major banks\(^8\) are selected as core banks. Figure 4 denotes the core–periphery network structure as at the end of 2007 and 2008. However, compared with Figure 3, there is almost no difference between the core–periphery network structure and the estimated one.

We render the network with the core–periphery structure sparser than the true network because it is assumed no contracts exist between periphery banks. By contrast, the estimated network is denser than the true network because of the estimation algorithm detailed in Appendix B. We use a modified version of Jaccard index and network density as means for validating the structure in the network.

First, we introduce the so-called Jaccard index as a means to measure similarities between two network structures. The index counts the number of linkages that appear in two CDS networks (i.e., the network by an unconstrained estimation and the network with the core–periphery structure) and relates it to the total number of linkages in both networks. We make one minor modification to the index to compensate for the deficiency – the modified Jaccard index ranges from 0 to 1. Refer to Appendix D for details. Table 4 denotes the modified Jaccard index both prior to the bankruptcy of Lehman Brothers (as at December 2007) and shortly after the bankruptcy of Lehman Brothers (as at December 2008). Both networks are quite similar in that the index figures are 100\% for both periods. Hence, the network obtained from the unconstrained estimation has the same features and linkages as the network with the core–periphery structure.

Second, we calculate the network density, which is the ratio of actual to potential links between the nodes (Clerc et al., 2014). In a directed network,

---

\(^8\)As at the end of 2007 (prior to the bankruptcy of Lehman Brothers), JPMorgan Chase, Citibank, Bank of America, HSBC, and Wachovia are selected. As at the end of 2008 (shortly after the bankruptcy of Lehman Brothers), Goldman Sachs is selected in addition to the five banks.
Figure 4: Directed graphs with the core–periphery network structure as at the end of 2007–2008

Note: The left and right panels are as at the end of 2007 and 2008, respectively.

the ratio is defined as:

$$p = \frac{m}{n(n-1)}$$

(12)

where $n$ is the number of nodes and $m$ is the number of linkages connecting the nodes. This ratio ranges from 0 to 1, with higher values denoting “denser” networks.

As shown in Table 4, the network density for the network obtained from the unconstrained estimation in Subsection 6.1 is only a little larger than that for the network with the core–periphery structure as at both dates, and hence, the number of degrees of which is equal to or less than that of the true network.

Thus, in terms of similarities between the network obtained from the unconstrained estimation and the true network, we are convinced that our analyses are robust as a whole.
Table 4: Modified Jaccard index and network density

<table>
<thead>
<tr>
<th>Date</th>
<th>Mod. Jaccard Index</th>
<th>Network density</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unc.</td>
<td>CP</td>
</tr>
<tr>
<td>Prior to bk of LB (Dec-07)</td>
<td>100.00%</td>
<td>36.4%</td>
</tr>
<tr>
<td>Shortly after bk of LB (Dec-08)</td>
<td>100.00%</td>
<td>36.6%</td>
</tr>
</tbody>
</table>

Note: Abbreviations: Unc.: the density for the network obtained from the unconstrained estimation; CP: the network density for the network with the core–periphery structure; bk of LB: the bankruptcy of Lehman Brothers.

8. Conclusions

In this study, we analyzed the network structure of the US CDS market and assessed the systemic importance of each bank for the period of the global financial crisis and thereafter. During the crisis, AIG faced a management crisis owing to AIG-FP’s massive short position. Nonetheless, the interconnectedness in the US CDS network is not necessarily large compared to that of other financial networks.

First, we theoretically analyzed the CDS network structure using various network centrality measures, in terms of assessing systemic importance. Based on such measures as degree, betweenness centrality, and HITS hub centrality, one significant finding is that three to six major banks have played a central role in the network in the past.

Second, we modeled the mechanism of contagious defaults in the US CDS network and theoretically analyzed the contagious defaults conditional on a stand-alone default during and after the global financial crisis, using real contracts data on FDIC call reports. Our analysis theoretically shows a few contagious defaults triggered by stand-alone defaults during the global financial crisis.

Third, we conducted a stress test and analyzed the possibility of contagious defaults conditional on any one or more stand-alone defaults in the future. We proved that the possibility of contagious defaults triggered by the risk contagion via the CDS network is low.

As a complement to the first contribution, sensitivity analysis proved the robustness of the estimated bilateral exposures matrix. Because it is virtually impossible to obtain a complete dataset of the bilateral exposures for derivative contracts, such as CDSs, we estimated the bilateral exposures matrix.
Figure 5: Monthly variations in asset values for selected banks (in 1,000 of local currency units)

from the aggregated positions of each bank using an estimation algorithm (i.e., the RAS algorithm). To assure the robustness of the estimated matrix, we used the core–periphery model and the measures such as modified Jaccard index and network density.

Our methodology could assist in the development of monitoring and early-warning indicators related to systemic risk in the CDS network by supervisory authorities. In addition, it could be used in the implementation of banks’ internal systemic stress tests of contagion risk.

Acknowledgments

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

Appendix A. US banks list

The US banks for the analyses are listed in Tables A.5.
Figure 6: Quarterly variations in asset return volatilities for selected banks

Figure 7: Quarterly variations in asset return drifts for selected banks
Figure 8: Time variations in number of defaulting institutions

Table A.5: 22 US banks list

<table>
<thead>
<tr>
<th>No</th>
<th>Bank name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BB&amp;T</td>
</tr>
<tr>
<td>2</td>
<td>Bank of America</td>
</tr>
<tr>
<td>3</td>
<td>BONY</td>
</tr>
<tr>
<td>4</td>
<td>Citibank</td>
</tr>
<tr>
<td>5</td>
<td>Comerica</td>
</tr>
<tr>
<td>6</td>
<td>Commerce Kansas City</td>
</tr>
<tr>
<td>7</td>
<td>Goldman Sachs</td>
</tr>
<tr>
<td>8</td>
<td>HSBC Bank USA</td>
</tr>
<tr>
<td>9</td>
<td>JP Morgan Chase</td>
</tr>
<tr>
<td>10</td>
<td>Keybank</td>
</tr>
<tr>
<td>11</td>
<td>Merrill Lynch</td>
</tr>
<tr>
<td>12</td>
<td>Bank of Tokyo-Mitsubishi UFJ Trust</td>
</tr>
<tr>
<td>13</td>
<td>Morgan Stanley</td>
</tr>
<tr>
<td>14</td>
<td>National City</td>
</tr>
<tr>
<td>15</td>
<td>Northern Trust</td>
</tr>
<tr>
<td>16</td>
<td>PNC</td>
</tr>
<tr>
<td>17</td>
<td>Regions</td>
</tr>
<tr>
<td>18</td>
<td>State Street</td>
</tr>
<tr>
<td>19</td>
<td>Sun Trust</td>
</tr>
<tr>
<td>20</td>
<td>U.S. Bank</td>
</tr>
<tr>
<td>21</td>
<td>Wachovia</td>
</tr>
<tr>
<td>22</td>
<td>Wells Fargo</td>
</tr>
</tbody>
</table>
Figure 9: Time variations in loss amounts (in 1,000 of US dollars)

Note: The legend number in the vertical axis corresponds to the number in Table A.5.
Appendix B. Estimation methodology of bilateral exposures matrix

The contract relationship in the US CDS network is represented by the following \((N \times N)\) gross negative fair value matrix\(^9\) \(X\):

\[
X = \left[\begin{array}{cccc}
    x_{11} & \cdots & x_{1j} & \cdots & x_{1N} \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN}
\end{array}\right]
\]

\[
\sum_j x_{ij} = a_i
\]

\[
\sum_i l_i = 1
\]

where \(x_{ij}\) denotes the outstanding exposures payable of bank \(i\) to bank \(j\).

Summing across row \(i\) gives bank \(i\)'s total gross positive fair value receivable, and summing down column \(j\) gives bank \(j\)'s total gross negative fair value payable as follows:

\[
a_i = \sum_j x_{ij}, \quad l_j = \sum_i x_{ij}.
\]

Typically, a bank’s aggregated data on gross fair values are obtained only from the FFIEC Central Data Repository; hence, estimating matrix \(X\) without imposing further restrictions is not possible. If additional information is unavailable, one possible approach would be to choose a distribution that minimizes the uncertainty, such as the amount of information related to the distribution for these exposures. By following a normalization such that \(\sum_i a_i = \sum_j l_j = 1\), the solution \(x_{ij} = a_i \times l_j\) is yielded, which represents the normalized amount bank \(i\) received from bank \(j\). Thus, the exposures reflect the relative importance of each bank in the CDS network.

When calculating matrix \(X\), we consider the fact that an bank cannot

\(^9\) Matrix \(X\) is treated as not only the gross negative fair value matrix \(L\) in Section 3 but also as the gross positive fair value matrix.
have exposure to itself. Therefore, we populate the values into $x^0_{ij}$ as follows:

$$x^0_{ij} = \begin{cases} 0 & \text{for any } i = j \\ a_i l_j & \text{otherwise.} \end{cases} \quad (B.3)$$

Matrix $X^0 = (x^0_{ij})$ violates the summing constraints expressed in equation (B.2). Hence, a new matrix $X$ must be found to satisfy the constraints. Some possible methodologies are presented by Upper (2011), Elsinger et al. (2002), and Wells (2004). The solution is provided by solving the optimization problem as follows:

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij} \ln \left( \frac{x_{ij}}{x^0_{ij}} \right)$$

subject to $\sum_{j=1}^{N} x_{ij} = a_i$, $\sum_{i=1}^{N} x_{ij} = l_j$, $x_{ij} \geq 0$. \quad (B.4)

The RAS algorithm is used to solve this type of problem. For further details, refer to Censor and Zenios (1998).

**Appendix C. Estimation methodology of market value based parameters of assets**

We consider a probability space $(\Omega, \mathcal{F}, P)$ in which the generated filtration $\mathbb{F} = (\mathcal{F}_t)_{t \in [0,T]}$ satisfies the usual conditions. The dynamics of asset value $V(t)$ follows a geometric Brownian motion:

$$\frac{dV(t)}{dt} = \mu_V dt + \sigma_V dW(t), \ t \in [0,T], \quad (C.1)$$

where $\mu_V$ is the constant drift of asset returns, $\sigma_V$ is the constant volatility of asset returns, and $W(t)$ is the standard Brownian motion. The solution to this equation is obtained as:

$$V(T) = V(t) e^{\left( \mu_V - \frac{\sigma_V^2}{2} \right) (T-t) + \sigma_V \sqrt{T-t} z}, \ t \in [0,T], \quad (C.2)$$
where $z$ is a standard normal random variable. The market value of equity at time $T$ is given as:

$$E(T) = \max[V(T) - D(T), 0], \quad (C.3)$$

where $D(T)$ indicates the default threshold expressed by the constant value of the interest-bearing debt for risk horizon $T$. Under certain assumptions, the solution to equation $(C.3)$ for equity values in $t$ is given by the Black-Scholes model.

We estimate the market value of assets using the methodology proposed by Duan (1994), which was later augmented by Duan (2000) and Duan et al. (2004). This methodology is based on a maximum likelihood estimation. Duan et al. (2004) introduce the log-likelihood equation for the estimation of $\mu_V, \sigma_V$ and $V(th), th = (0, \ldots, kh, \ldots, nh)$ using the observed market values of equity as follows:

$$l(\hat{\theta}_V; \hat{V}(th)|E(th)) = -\frac{n}{2} \ln(2\pi \hat{\sigma}_V^2 h) - \frac{1}{2} \sum_{k=1}^{n-1} \frac{\left(\hat{R}(kh) - (\hat{\mu}_V - \frac{\hat{\sigma}_V^2}{2})h\right)^2}{\hat{\sigma}_V^2 h} \quad (C.4)$$

$$- \sum_{k=1}^{n} \ln(\hat{V}(kh)) - \sum_{k=1}^{n} \ln(\Phi(d_1)),$$

where $\hat{\theta}_V := (\hat{\mu}_V, \hat{\sigma}_V)$, $h = \frac{1}{12}$ year, and $\Phi$ is the cumulative distribution function of the standard normal variables. $\hat{V}(u)$ ($u = kh \leq nh = T, 1 \leq k \leq n$) is estimated as the solution to equation $(C.3)$ using the Black-Scholes model as follows:

$$E(u) = V(u)\Phi(d_1) - D(u)e^{-r(T-u)}\Phi(d_2), \quad (C.5)$$

where

$$d_1 = \frac{\ln \frac{V(u)}{D(u)} + (r + \frac{\sigma_V^2}{2})(T - u)}{\sigma_V \sqrt{T-u}}, d_2 = \frac{\ln \frac{V(u)}{D(u)} + (r - \frac{\sigma_V^2}{2})(T - u)}{\sigma_V \sqrt{T-u}},$$

and $r$ is the risk-free interest rate.

Equity and asset volatilities are related in the following equation provided
by Crosbie and Bohn (2003):

\[ \sigma_E = \frac{V(u)}{E(u)} \Phi(d_2) \sigma_V, \quad (C.6) \]

where \( \sigma_E \) is the constant volatility of equity returns.

The time series of the monthly market value of equity from which the parameter is estimated equals \( th = (0, h, 2h, \ldots, nh) \), where \( n = 12 \) months and \( h = 1/12 \). Each iteration of the optimization calculation produces a time series of monthly values \( \hat{V}(th) \), where the maturity of the liability ranges over \( th = (0, h, 2h, \ldots, nh) \).

The initial values of \( \hat{V}^{(m)}(kh) \) and \( \hat{\sigma}_V^{(m)}(kh) \) (\( m \): the \( m \)-th iteration of the optimization calculations; \( 1 \leq k \leq n \)) are chosen arbitrarily. However, we set \( \hat{V}^{(0)}(kh) \) as \( E(kh) \) plus \( D(kh) \) using data from the balance sheet of bank \( i \) and \( \hat{\sigma}_V^{(0)}(kh) \) as \( \hat{\sigma}_E E(kh)/\hat{V}^{(0)}(kh) \) from equation (C.6). \( E(kh) \) is the market capitalization of bank \( i \) at the end of the year. \( D(0) \) is the total face value of the interest-bearing debt of bank \( i \) at the end of the year. \( \sigma_E \) is estimated from the time-series of the monthly natural logarithms of the returns on bank equity as follows:

\[ \hat{\sigma}_E = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (\hat{R}(kh) - \bar{R})^2 / 12}, \quad (C.7) \]

where

\[ \hat{R}(kh) = \ln \frac{\hat{V}(kh)}{\hat{V}((k-1)h)}, \quad (C.8) \]

\[ \bar{R} = \frac{1}{n} \sum_{k=1}^{n} \hat{R}(kh). \quad (C.9) \]

The estimation procedure is as follows:

**Step 1:** Estimate the monthly values of \( \hat{V}(kh) \) and \( \hat{\sigma}_V(kh) \) (\( 1 \leq k \leq n \)) using the monthly market capitalization data from equations (C.5) and (C.6).

**Step 2:** Calculate the average values of yearly volatilities using the monthly estimated volatilities \( \hat{\sigma}_V(kh) \) and set them as the initial values. Es-
mate the yearly values of $\mu_V, \sigma_V$ by finding the maximum of equation (C.4).

**Step 3:** Calculate the monthly values of $\hat{V}(kh)$ once again given $\hat{\sigma}_V$ from equation (C.5).

This procedure allows the estimation of parameters based on the methodology proposed by Duan (1994, 2000) and Duan et al. (2004) using the monthly dataset with a maximum of 12 data points.

**Appendix D. Modified Jaccard index**

The Jaccard index measures similarities between network structures and neglects the weight of the links. Hence, we modify the index to compensate for this deficiency. The weighted adjacency matrices $M^1$ and $M^2$ express the features of two given networks $N_1$ and $N_2$, respectively (Halaj and Koka, 2015). Both networks span on the same set of $N$ nodes as follows:

$$
M_{12} := \{(i, j) \in \tilde{N} \times \tilde{N} | (i, j) \in N_1 \land (i, j) \in N_2\}
$$

$$
M_{10} := \{(i, j) \in \tilde{N} \times \tilde{N} | (i, j) \in N_1 \land (i, j) \notin N_2\}
$$

$$
M_{02} := \{(i, j) \in \tilde{N} \times \tilde{N} | (i, j) \notin N_1 \land (i, j) \in N_2\}
$$

where $\tilde{N}$ stands for a set $\{1, 2, \ldots, N\}$. The set $M_{12}$ express the number of links overlapped among both network graphs (sets), the sets $M_{10}$ and $M_{02}$ are present in one graph but not in the other. The modified Jaccard index is defined as follows:

$$
J(N_1, N_2) = \frac{(\#M_{12}) \sum_{(i, j) \in M_{12}} (M^1_{i,j} + M^2_{i,j})}{\sum_{Z \in \{M_{12}, M_{10}, M_{02}\}} (\#Z) \sum_{(i, j) \in Z} (M^1_{i,j} + M^2_{i,j})} \quad (D.1)
$$

where $\#M_{12}$ and $\#Z$ express the number of entries in the set $M_{12}$ and one in any set of a group ($M_{12}$, $M_{10}$, or $M_{02}$), respectively and $J(N_1, N_2) \in [0, 1]$.

**References**


