Why Use Agents? Consumer Reference Manipulation in Life Insurance Market

Abstract

We explore agents' role in life insurance on the premise that consumers are of limited rationality and their life insurance purchase decisions are reference-dependent and regret-induced. Our analysis provides a unified interpretation for both the bright and dark sides of the agency system that is still a predominant form of distribution in life insurance markets. On the one hand, consumers are passive in life insurance purchase even with complete information of mortality risk and agents can increase their welfare by promoting a larger insurance amount, in the meanwhile increasing profits for the insurers. On the other hand, agents can manipulate consumers into buying an excessively high amount of insurance. With a commission-based compensation mechanism, agents have a strong incentive for overselling. As a response, insurers resort to a rate cutting strategy to help consumers maintain their reservation welfare, inducing more insurance purchase in market equilibrium. We further explore the welfare implications of agent selling and find that consistent with market realities, the agency system is often more welfare enhancing in whole life insurance than in term life insurance.

Keywords: Life insurance, Insurance Agency system, Limited rationality, Reference-dependent preference, Regret, Behavioral insurance

1 Introduction

As the age-old saying goes, life insurance is "sold, not bought" (e.g., Gravelee 1993, 1994; Bernheim et al. 2003). Anecdotal evidence and extant literature have documented that consumers are indeed passive, if not reluctant, in making life insurance purchase whereas agents have always played an indispensable role in this market. For example, Morrison (1939) argues that only a small number of people buy adequate amounts of life insurance without incentives, and agents' selling effort is helpful in improving consumers' welfare. Auerbach and Kotlikoff (1991) provide empirical evidence supporting that low life insurance purchase is prevalent. In an experimental setting, Braun et al. (2014) show that individuals' willingness to pay for term life insurance on average is low. Indirect evidence also arises from the supply side. Even as the information age has penetrated insurance businesses with abundant online resources and innovative forms of sales and marketing, life insurance agents are still the predominant distributional form widely used by world's largest insurers (cf., Regan and Tennyson 2000 and Hilliard et al. 2012).[1] With a low average 4-year retention rate of only 10-18% of new agents, insurance companies incur a substantial cost in recruiting and maintaining agents, in additional to the high commission costs (cf., Hoesly 1996; Trese 2011). This implies the benefits of an agency system must be so significant as to more than offset the associated costs. Despite its prevalence, there is overwhelming anecdotal evidence for the dark side of the agency system that agents may be stimulated to oversell to consumers (Diacon and Ennew, 1996, Gokhale and Kotlikoff 2002, Inderst and Ottaviani 2009).

Explanations offered within the framework of the standard Expected Utility Theory (EUT) are partial at best. To explain the prevalence of agency system, the EUT-based models often take as a primitive the existence of asymmetric information on insurance products and argue that agents play a positive role by offering valuable information and advice to consumers and can facilitate matching consumers with appropriate insurance products. (Hofmann and Nell 2011, Focht et al. 2013).[2] However, given that agents' contingent commissions are based on consumers' welfare valuation, this rational suggests that agents have no incentive to missell to consumers (Focht et al. 2013), contradicting much of empirical evidence on the negative role of the agency system. Moreover, while plausible before, this rational faces increasing challenge since information becomes abundantly available on the internet and from other less costly sources, both to the consumers and the insurers. Indeed, other insurance lines of business have successfully transformed into an online selling system while the life agency system

[1] According to LIMRA estimates, in 2011, independent agents, who represent several insurers, held 49 percent of the new individual life insurance sales market, followed by affiliated agents, who represent a single insurance company, with 40 percent, direct marketers with 4 percent and others accounting for the remaining 7 percent (2013 Financial Services Fact Book, P. 105).

[2] Another important rationale for the prevalence of agent system is that agents can help insurers make better risk classifications (Mathewson 1983, Regan and Tennyson 1996 2000, Eckardt and Räke-Dÿpner 2010).

remains a popular distributional form.

In this paper, we deviate from the EUT framework and explore formally agents' role in life insurance market on the premise that consumers are of limited rationality and their life insurance purchase decisions are reference-dependent and regret-induced. Life insurance agents are essential in facilitating (and manipulating) the formation of consumer reference in purchase decisions. In the recent decades, alternative theories have been developed to explain the observed consumers' insurance decisions that are not consistent with the predictions of EUT-based models. The influential Prospect Theory (Kahneman and Tversky 1979, 1991) suggests that when decision making involves uncertain outcomes, individuals do not just evaluate the absolute value of the outcomes, but rather those relative to some reference points. This theory provides explanatory power for puzzles on consumers' purchase of life insurance and annuities (Hu and Scott 2007; Gottlieb 2012) and the overinsurance of moderate risk (Sydnor 2010, Barseghyan et al. 2013).[3] In particular, the theory of Regret seems to be particularly relevant for insurance decisions, where the realized outcome of an actual decision is compared with the potential outcome of a counterfactual decision. Braun and Muermann (2004) argue that consumers who make insurance purchase decisions often try to avoid the emotional regret and show that their optimal decisions exhibit behavior consistent with the predictions of Regret Theory. Building on the insights of the Prospect Theory and the Regret Theory, we present a simple formal framework to simultaneously account for both the bright and the dark sides of agents' selling effort and provide insights into the enduring popularity of the agency system in the life insurance market.

Section 2 describes in detail consumers' reference-dependent preference and the formulation of regret-based comparisons. To decide whether or not to buy life insurance and how much insurance to buy, a consumer often balances the pros and cons of different decision options relative to some (sometimes implicit) reference. The trade-off exhibits a gain-loss comparison between the decision and the reference that is captured by the reference-dependent utility. The consumer's reference-dependent utility has two important components: a reference point that the consumer's gain-loss comparison is based upon and the concept of loss aversion where losses are more painful than equal-sized gains are pleasant. In her gain-loss comparison, the insurance consumer often experiences the emotional regret, or the feeling that "I should have bought more (or less) insurance" in the ex post assessment. As we make it clear below, this is specifically captured by the state-by-state comparison between the outcomes of the consumer' decision and of her reference.

Section 3 explores agents' role in determining the consumer's life insurance purchase. We consider both term life and whole life insurance. Given any reference, consumers anticipate

[3] Prospect Theory has been widely used in various risky decision contexts, see the review by Barberis (2013) for details.

two possible types of regret in buying life insurance: one is the regret of paying too much premium (buying more insurance) when death did not occur during a period of interest, and the other is paying too little premium (buying less insurance) when death did occur during this period. It is due to her *anticipatory regret* that the consumer prefers not to deviate her choice much from the reference and therefore her optimal choice is substantatively affected by the initial reference. This dependence on reference provides an opportunity for agents to significantly influence the consumer' purchase decision by manipulating her initial reference, even when the consumer's decision is fully rational in our framework.

We show both the positive and negative effects of agents' selling effort. Without insurance agents, a consumer naturally takes the initial state where she is without life insurance as her reference, i.e., the "status quo reference." With this reference, she ends up purchasing strictly less insurance than what maximizes her welfare.[4] Agents, by manipulating the consumer's reference, can help her choose an increased amount of coverage, making her better off. However, we also show that when the reference coverage is sufficiently large (i.e., when agents' selling effort is too aggressive), the consumer prefers to purchase strictly more insurance than what maximizes her welfare, making her worse off. In summary, our analysis suggests that while agents can always stimulate life insurance demand by manipulating consumers' reference, consumer welfare is enhanced only when agents' selling effort is moderate.

In section 4, we turn to exploring the effect of agent selling in the formation of market equilibrium. With a commission-based compensation mechanism, agents have a strong incentive to induce consumers to purchase insurance as much as possible. As a result, consumers purchase more insurance than what maximizes their welfare, become worse off ex post and consequently have a lower evaluation of insurers. To keep their reputation and attract consumers in the long run, insurers respond by decreasing the premium rate to help consumers maintain their reservation welfare. This rate cutting strategy induces more insurance purchase in market equilibrium. This resulting increased sales increase consumers' welfare because combined with the reduced premium rate from the insurers, they help bring consumers' welfare back to the reservation level.

We further explore the welfare implications of agent selling by comparing insurers' optimal pricing with and without agent selling. We find that agent selling is more beneficial to con-

[4]The standard reference-independent EUT is used to measure consumers' welfare, providing a benchmark to analyze the welfare effects of agents' selling efforts. Intuitively, consumers' welfare should only be affected by their actual choice on insurance purchase and not by their reference, and thus the welfare measure should be reference-independent. To the extent that consumers optimize regret-based decisions, under an appealing assumption that an experienced consumer should fully realize the effects of her anticipatory regret on the decision, we derive that this consumer should behave the same as a reference-independent decision maker, which also lends support to the above choice of the welfare measure.

sumers of whole life insurance than to consumers of term life insurance. Moreover, consumers' optimal whole life insurance coverage under the agency system increases substantially relative to that without the agency system, whereas the optimal coverage of term life insurance only increases slightly. This result predicts that the agency system accounts for a higher fraction in the whole life insurance market than in the term life insurance market, which is consistent with the observation that more than 90% of whole life insurance policies are sold by agents while less than 70% of term life insurance is sold by agents (Hilliard et al. 2012).

Our study contributes to the literature on insurance distribution system choice. In the asymmetric information framework, previous studies propose various explanations for insurance distribution choices, including potential incentive conflicts between the insurer and consumers (Kim et al. 1996, Carr et al. 1999), transaction cost theory (Regan 1997), and costly consumer search (Posey and Tennyson 1998, Eckardt 2007). These studies focus on exploring the relative efficiency of various forms of agency systems and not on analyzing the use of distribution system in response to consumers' shopping behavior (Hilliard et al. 2012).[5] Our paper complements prior research by studying the relationship between agents' selling behavior and consumers' purchase decisions while eliminating the confounding effects of asymmetric information, as we assume complete information between consumers and insurers (i.e., mortality is common knowledge to both consumers and insurers) and no other market frictions. Our analysis provides insights into the interactions between consumers and agents in life insurance markets, and are helpful to practitioners in optimizing their distribution system design and properly justifying their choice to investors and the general public.

Our analysis of the welfare effects of the agency system also has important implications for life insurance regulation. We characterize a type of agent behavior that may harm consumer welfare. In this case, consumers, equipped with complete information, can be manipulated by the agents despite of making rational optimal insurance purchase decision. This suggests potential deficiencies in current regulation of agent behavior, such as the required extensive documentation of agents' advice and the code of conduct. However, this does not necessarily mean that new agent compensation mechanisms, such as a fee-for-service compensation mechanism, is better than the traditional commission-based compensation. In our setting, in order to keep their reputation and attract consumers in the long run, insurers can choose to lower the price to counteract the negative impact of agent overselling on consumers' welfare. This implies that consumers can achieve the same level of welfare with and without agent selling. This result echoes the insight of Focht et al. (2013) that fee-based and commission-based compensation system are payoff equivalent for consumers' welfare. However, different from their

[5]After reviewing studies of the insurance distribution system, Hilliard et al. (2012) point out that "while existing academic studies of distribution system choice have focused primarily on the choice between an independent and a tied agency force, current market trends distinguish more clearly between fully integrated distribution without the use of professional agents versus the agency system of distribution itself."

prediction, our analysis allows the existence of agent overselling, i.e., in their terminology, when mismatching occurs.

Our analysis of the dark side of agent selling is reminiscent of Inderst and Ottaviani (2009) who argue that to accommodate the impact of agency problem, firms need to decrease products' suitability standard and thus make agent oversell. However, insures' overselling in our setting delivers welfare implications distinct from theirs. Inderst and Ottaviani (2009) argue that overselling with the lower suitability standard only makes consumers' welfare become worse on average because more consumers will now purchase the unsuitable products. We show that consumers' welfare is not necessarily worse since insurers can counteract the negative effect of agent overselling by setting lower premium rate in market equilibrium, which in turn induces more insurance sales. This distinction highlights one main insight from our analysis that positive and negative effects of agent selling coexist in the life insurance market.

2 Consumers' reference-dependent Expected Utility with Regret-based comparisons

The key assumption in our model of consumers' preference is that consumers' utility is reference-dependent and the calculation of their expected utility involves the regret-based comparisons between the random outcomes determined by their decisions and the reference. To decide whether or not to buy life insurance and how much insurance to buy, a consumer often balances the advantages and disadvantages of different decision options relative to some (maybe implicit) reference. For example, taking no life insurance (i.e., status quo) as a natural reference, a consumer deciding whether to buy insurance faces a gain-loss comparison: if death does not occur, she will feel loss for buying insurance (compared to not having bought insurance); or if death happens, there is a gain for buying insurance since the insurance repayment provides life protection for her heirs. The consumer will average both gains and losses in different realized states of nature (i.e., death or no death in the context of life insurance) and obtain the expected utility from the decision. In determining her optimal choice, the consumer anticipates two possible disutilities: one is the disutility of paying too much premium (buying more insurance) when death does not occur, and the other is the disutility of having too little insurance (buying less insurance) when death occurs. This type of disutility is exactly the regret defined by Zeelenberg et al. (2000), who writes "Regret is assumed to originate from comparisons between the factual decision outcome and a counterfactual outcome that might have been had one chosen differently" (p. 529). Thus we build our framework around this disutility of "the regret" and consumers' "regret-based comparison" driving the disutility.

2.1 Basic model

Our approach builds on the Prosect Theory proposed by Kahneman and Tversky (1979, 1991) and developed by Kőszegi and Rabin (2006, 2007). This theory suggests that a consumer maximizes the expected value of a reference-dependent utility of the form

$$V(x, r) = u(x) + g(u(x) - u(r)). \tag{1}$$

Given the reference r, the consumers' utility is determined by the monetary wealth x. To be specific, consumers' overall utility consists of two components: intrinsic utility and gain-loss utility. Intrinsic utility derived from the wealth x determined by the consumers' decision is denoted by $u(x)$ with the properties $u'(\cdot) > 0, u'(0) = +\infty$ and $u''(\cdot) \leq 0$.[6] $g : \mathbb{R} \to \mathbb{R}$ is a value function that depends on the difference between the realized utility of the chosen alternative x and that of the reference alternative r, capturing the consumer's gain-loss comparison. This value function $g(\cdot)$ satisfies the assumptions imposed by Tversky and Kahneman (1991), consisting of four ingredients: reference point, loss aversion that losses are more painful than equal-sized gains are pleasant, a diminishing sensitivity to changes in an outcome as it moves farther from the reference point, and nonlinear probability weighting. In this paper, we focus on exploring the impact of agents' selling effort on consumers' life insurance purchase through reference manipulation, and thus preclude diminishing sensitivity and nonlinear probability weighting, and assume that $g(\cdot)$ is a piecewise linear function throughout this paper, i.e.,

$$g(y) = \begin{cases} \eta y & y \geq 0 \\ \eta \lambda y & y < 0 \end{cases} \tag{2}$$

where $\eta \geq 0$ is the weight of gain-loss utility relative to the intrinsic utility, reflecting how consumers value the gain-loss comparison, and $\lambda > 1$ captures the consumers' loss aversion.

In the context of insurance, not only the outcome of consumers' choice but also that of their reference is stochastic. For example, when the reference is no life insurance, the outcome of this reference is random because consumers may or may not die in the next period. This is also true for consumers' insurance purchase decision. Therefore, given the choice $\tilde{x}$ and the reference $\tilde{r}$, consumers' expected reference-dependent utility is

$$E[V(\tilde{x}, \tilde{r})] = \int \int u(x) + g(u(x) - u(r)) dH(x, r) \tag{3}$$

where $H(x, r)$ denotes the joint cumulative distribution function of $\tilde{x}$ and $\tilde{r}$. Different forms of the joint cumulative distribution reflect different economic considerations. In our context, consumers try to compare the random outcomes of their life insurance decision with the

[6] As we shall see, this set of assumptions ensure that the internal solutions exist for the optimization problem of consumers' insurance purchase.

reference in the realized states of nature (death or not), i.e., they make comparisons between the realized outcome of $\tilde{x}$ and that of $\tilde{r}$ state by state, or the "regret-based" comparisons. Let $F(s)$ denote the cumulative distribution function of the state of nature variable s, then consumers' expected reference-dependent utility become

$$E[V(\tilde{x}, \tilde{r})] = \int u(x(s)) + g(u(x(s)) - u(r(s)))dF(s) \tag{4}$$

To capture that consumers try to make rational and responsible decisions, as we elaborate more below, we restrict their possible reference points in their choice set such that any reference point $\tilde{r}$ is one of their possible decisions.

2.2 Behavioral characteristics

Our model builds on the Prospect Theory by incorporating the reference point and loss aversion into the analysis. This is motivated by experimental and empirical evidence. When decision making involves uncertain outcomes, Kahneman and Tversky (1979) demonstrate in experimental settings that people normally perceive outcomes as gains and losses relative to some reference point, rather than as final states of wealth or welfare. These observations systematically violate the predictions of the standard expected utility theory. Instead, they propose an alternative theory and call it "Prospect Theory." Prospect Theory has been widely used in various risky decision contexts (Barberis 2013), including insurance decisions (Hu and Scott 2007, Sydnor 2010, Barseghyan et al. 2013). A notable feature of Prospect Theory is that consumers' choice is affected by their reference point. In the process of insurance purchase, consumers often collect insurance information and obtain advice from agents, friends or family, and hence their reference point can be reshaped by other people's suggestions. As we shall see, it is the updating of consumers' reference point that significantly affects their purchase of life insurance.

The introduction of reference point imposes an additional challenge: how consumers reconcile their actual decisions and their reference choice. Kőszegi and Rabin (2006, 2007) offer a solution to this problem. They argue that the reference-dependent consumers try to make a rational decision and have some ability to predict their own behavior. To characterize the above features, they define a "personal equilibrium," where the optimal decision conditional on the consumers' reference coincides with the reference. Under this mechanism, consumers achieve their optimal choice by adjusting their choice and reference until their choice becomes consistent with the reference. This characterization is exactly what we need for capturing consumers' desire to make a rational and responsible decision in their life insurance purchase.

We emphasize that consumers' decision in our context is "regret-based" rather than "disappointment-based." As commented by Zeelenberg et al. (2000), "disappointment is assumed to originate from a comparison between the factual decision outcome and a counterfactual outcome that might have been had another state of the world occurred" and "disappointment is typically experienced in response to unexpected negative events that were caused by uncontrolled circumstances, or by another person." In our context, these comments can be translated as that the "disappointment-based" comparison is the state-independent one and disappointment reflects the emotion that consumers wish loss never happened.[7] However, as argued by Braun and Muermann (2004), "the ex post assessment of consumers' insurance decision is 'I should have bought more (or less) insurance' and not 'I wish I hadn't incurred that loss.' " Clearly, regret rather than disappointment is more pertinent to consumers' insurance decisions.[8]

Moreover, for regret-based comparisons, we allow consumers to compare the outcome of their actual decision with that of the reference, which can be any reasonable alternative decision. Braun and Muermann (2004) examine optimal insurance purchase decisions of individuals that exhibit behavior consistent with Regret Theory. In their study, the regret mechanism entails the comparison between the outcome of a decision and the best possible outcome in any state of nature, leading to a conservative decision in which consumers neither purchase too much nor too little insurance. This regret mechanism is intuitively appealing but somewhat implausible since all of the best outcomes in any state in general cannot be induced by a consistent decision, as illustrated in Sugden (2003) model. Life insurance is a long-term and economically significant decision for an individual and she attempts to make a responsible and rational decision that she can commit to. In this context, comparisons based on inconsistent decisions do not seem appropriate and thus we replace the best outcome by the outcome of the reference to avoid the potential inconsistency problem.

[7] The disappointment theory is studied by Bell (1985), Loomes and Sugden (1986), Gul (1991), etc. Kőszegi and Rabin (2006, 2007) introduce disappointment-based comparison into the calculation of consumers' expected reference-dependent utility. Formally, assume $\tilde{x}$ and $\tilde{r}$ have marginal distribution function $F(x)$ and $G(r)$ respectively, The disappointment-based comparison implies that $\tilde{x}$ and $\tilde{r}$ are independent and consumers' expected utility writes: $E[V(\tilde{x}, \tilde{r})] = \int\int u(x) + g(u(x) - u(r))dF(x)dG(r)$.

[8] Abundant experimental and empirical evidence supporting the application of Regret Theory has been documented by Braun and Muermann (2004).

3 Consumers' life insurance purchase

To protect their loved ones upon death, people have a demand for life insurance.[9] We consider a life insurance market consisting of a continuum of households. A household consists of one head and at least one heir. Because household heads make insurance decisions while alive, we refer to them as "the consumers."

3.1 Term life insurance

We first consider consumers' term life insurance purchase in one-period framework. At the beginning of the period, an agent visits a consumer endowed with an initial wealth w_1 and offers him insurance policies, and the consumer decides which one to purchase (if any). During the period, the consumer dies with probability $p \in (0, 1)$ and he earns income $w > 0$ if alive. From life cycle perspective, we can treat this period as consumers' working period. Insurance company offers a term life policy (P, I) specifying that the consumer pays premium P, and his heir receives the repayment I if he dies. Note that the repayment I should be smaller than the income w, i.e. $I \in [0, w]$. We assume the policy has a premium rate α such that insurance premium $P = \alpha I$ where $\alpha \in [p, 1)$.

3.1.1 Optimal purchase under certain reference

The term life insurance coverage in the consumer's reference point is denoted by I_r. To guarantee the plausibility of the reference, we assume $I_r \in [0, w]$, reflecting that the consumer's reference point is one of his feasible insurance decisions. The stochastic outcome from the reference point is a lottery $\tilde{r}(I_r) = (w_1 + (1 - \alpha)I_r, p; w_1 + w - \alpha I_r, 1 - p)$, meaning that the consumer with initial wealth w_1 dies with probability p and his heirs receive the net repayment $(1 - \alpha)I_r$, otherwise the consumer obtains the net wealth $w - \alpha I_r$ with probability $1 - p$. In particular, consumers' status quo is to purchase no insurance, corresponding to the reference point $\tilde{r}(0) = (w_1, p; w_1 + w, 1 - p)$. If the consumer purchases the contract $(\alpha I, I)$, his expected

[9] The demand for life insurance may arise from a bequest motive (Bernheim 1991, Inkmann and Michaelides 2012), or from the desire to provide a life-cycle protection for financial vulnerability caused by the death of family members (Bernheim et al. 2003, Lin and Grace 2007).

utility is characterized as follows:

$$E[V(\tilde{x}(I),\tilde{r}(I_r))] = \begin{cases} pu(w_1+(1-\alpha)I)+(1-p)u(w_1+w-\alpha I) & \\ +p\eta[u(w_1+(1-\alpha)I)-u(w_1+(1-\alpha)I_r)] & \\ +(1-p)\eta\lambda[u(w_1+w-\alpha I)-u(w_1+w-\alpha I_r)] & \text{if} \quad I \geq I_r \\ pu(w_1+(1-\alpha)I)+(1-p)u(w_1+w-\alpha I) & \\ +p\eta\lambda[u(w_1+(1-\alpha)I)-u(w_1+(1-\alpha)I_r)] & \\ +(1-p)\eta[u(w_1+w-\alpha I)-u(w_1+w-\alpha I_r)] & \text{if} \quad I < I_r \end{cases} \tag{5}$$

Equation (5) demonstrates the asymmetry of consumers' regret-based comparison between decision outcomes and reference levels. If the consumer would like to increase his coverage ($I \geq I_r$), he treats the case as a loss that he pays higher insurance premium relative to reference level when he is alive in period 1 ($u(w_1+w-\alpha I)-u(w_1+w-\alpha I_r) \leq 0$), whereas the case as a gain that death occurs since his heir obtains more insurance repayment than reference level ($u(w_1+(1-\alpha)I)-u(w_1+(1-\alpha)I_r \geq 0$). Conversely, were the consumer to decrease his coverage ($I < I_r$), a loss occurs when the death happens and his heir receives less insurance repayment than reference level ($u(w_1+(1-\alpha)I)-u(w_1+(1-\alpha)I_r < 0$), and the case of no death becomes a gain since the consumer pays less premium than reference level ($u(w_1+w-\alpha I)-u(w_1+w-\alpha I_r) > 0$). Due to loss aversion, the consumer' regret-based gain-loss comparison is asymmetric and captures his two anticipatory regret in the decision: one is the regret paying too much premium (due to buying more insurance compared to the reference) in case of no death occurring, and the other is the regret having too little insurance (due to less purchase relative to the reference) in case of death happening. As a result, the consumer's choice is, as we shall see, not to deviate much from his reference. The first-order derivative of (5) with respect to I equals

$$\frac{d[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI} = \begin{cases} p(1-\alpha)(1+\eta)u'(w_1+(1-\alpha)I) & \\ -\alpha(1-p)(1+\eta\lambda)u'(w_1+w-\alpha I) & \text{if} \quad I \geq I_r \\ p(1-\alpha)(1+\eta\lambda)u'(w_1+(1-\alpha)I) & \\ -\alpha(1-p)(1+\eta)u'(w_1+w-\alpha I) & \text{if} \quad I < I_r \end{cases} \tag{6}$$

It follows that

$$\frac{d[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI} = \begin{cases} p(1-\alpha)(1+\eta)u'(w_1+w-\alpha I)\left[k(I)-\frac{\alpha(1-p)(1+\eta\lambda)}{p(1-\alpha)(1+\eta)}\right] & \text{if} \quad I \geq I_r \\ p(1-\alpha)(1+\eta\lambda)u'(w_1+w-\alpha I)\left[k(I)-\frac{\alpha(1-p)(1+\eta)}{p(1-\alpha)(1+\eta\lambda)}\right] & \text{if} \quad I < I_r \end{cases} \tag{7}$$

where $k(I) \equiv u'(w_1+(1-\alpha)I)/u'(w_1+w-\alpha I)$. Define $\underline{I}$ and $\hat{I}$ satisfying

$$k(\underline{I}) = \frac{\alpha(1-p)(1+\eta\lambda)}{p(1-\alpha)(1+\eta)}, \quad k(\hat{I}) = \frac{\alpha(1-p)(1+\eta)}{p(1-\alpha)(1+\eta\lambda)}. \tag{8}$$

With the assumptions $u'(\cdot) > 0, u'(0) = \infty$ and $u''(\cdot) \leq 0$, we easily derive that $0 < \underline{I} \leq \hat{I}$ and $\underline{I} < w$ (see Lemma A.1 in Appendix). Note that $\hat{I}$ may excess w when the premium rate α is low.[10] However, in our context, the repayment I for the consumer should be smaller than his total wage income w. We thus define $\overline{I} = \min\{\hat{I}, w\}$ (note $\underline{I} < \overline{I}$), and obtain the following proposition.

Proposition 1. *Given the reference coverage $I_r \in [0, w]$, the optimal coverage is*

$$I^* = \begin{cases} \underline{I}, & \text{if} \quad I_r < \underline{I} \\ I_r, & \text{if} \quad \underline{I} \leq I_r \leq \overline{I} \\ \overline{I}, & \text{if} \quad \overline{I} < I_r \leq w \end{cases}$$

Proposition 1 demonstrates that consumers' reference plays a significant role in determining their optimal insurance purchase. When their reference coverage is small enough ($I_r < \underline{I}$), consumers prefer to purchase insurance coverage $\underline{I}$, whereas when their reference coverage is large enough ($\overline{I} \leq I_r \leq w$), their optimal purchase becomes $\overline{I}$. Moreover, when consumers' reference coverage is moderate ($\underline{I} \leq I_r \leq \overline{I}$), their optimal purchase turns out to be equal to their reference coverage. In contrast, when the reference coverage is not moderate, their optimal purchase is not consistent with their reference. In particular, consider a consumer taking the case of not insuring as his status quo reference. Proposition 1 says that the consumer will prefer to purchase insurance with coverage $\underline{I}$. Realizing the inconsistency between his preferred choice and his reference, a consumer trying to make a rational decision may reconsider and update his original reference and the corresponding choice.

3.1.2 Optimal purchase consistent with reference

In practice, life insurance purchase is an economically significant decision in one's life, and consumers take it serious and try to make a rational purchase. To characterize this rationality, we assume that consumers have ability to predict their own behavior and make a consistent decision. That is, consumers' optimal choice based on their reference yields the stochastic outcome that coincides with the reference. This situation is defined as "personal equilibrium" in Kőzegi and Rabin(2006). Formally, we have the following solution concept.

Definition 1. An insurance purchase I^{PE} is a personal equilibrium if for all $I \in [0, w]$, $E[V(\tilde{x}(I^{PE}), \tilde{r}(I^{PE}))] \geq E[V(\tilde{x}(I), \tilde{r}(I^{PE}))]$.

Definition 1 says that if the consumer expects to purchase the insurance coverage I^{PE} for all possible choice $I \in [0, w]$, she should indeed choose the coverage I^{PE}. From Proposition 1,

[10] As shown in Lemma A.1 of Appendix, $\hat{I} \geq w$ if $\alpha = p$.

with the reference coverage $I^r = 0$, the consumers' optimal choice is $\underline{I} > 0$. Therefore, $I = 0$ is not a personal equilibrium. However, the reference $I_r = 0$ can be updated to $I_r = \underline{I}$ that is a personal equilibrium. Indeed, combining Proposition 1 with Definition 1, we obtain the following corollary.

Corollary 1. *Any insurance purchase $I \in [\underline{I}, \overline{I}]$ is a personal-equilibrium choice, i.e.*

$$I^{PE} = \{I : I \in [\underline{I}, \overline{I}]\}.$$

Moreover, $\partial \underline{I}/\partial \eta < 0, \partial \underline{I}/\partial \lambda < 0$ whereas $\partial \overline{I}/\partial \eta \geq 0, \partial \overline{I}/\partial \lambda \geq 0$.

This corollary shows that on the contrary to the unique optimal choice in the framework of standard reference-independent utility theory, the reference-dependent consumer's rationally optimal choices satisfying the personal equilibrium are multiple. The multiplicity of the optimal choice comes from the asymmetry of the consumer's gain-loss comparison driven by the anticipatory regret: due to the consumer' loss aversion, he cares more about the less insurance repayment than the less premium cost if he would like to reduce his insurance purchase relative to reference, but lay more emphasis on the more premium payment than more insurance coverage if he would like to buy more insurance than reference. In other words, due to the anticipatory regret, the consumer cares more about the negative consequences of his decision than the positive ones. As a result, he prefers not to deviate his choice too much from his reference and his rational choice driven by the regret-based tradeoff turns out to be multiple and depends on the initial reference.

It is the dependence of rational choice on the reference that justifies the popularity of agents' selling behavior, which is our focus in this paper. That is, agents can influence consumers' insurance purchase through manipulating their reference even consumers try to make a rational purchase decision. The direct implication of this result is that agents' selling effort can make consumers buy the insurance coverage up to $\overline{I}$. In contrast, under the pattern of direct selling without agents, consumers's reference is to purchase no insurance ($I_r = 0$) and thus their insurance purchase is only $\underline{I}$. Compared with the case of direct selling, agents' selling effort significantly increases consumers' life insurance demand.

Moreover, as η or λ increases, $\underline{I}$ decreases while $\overline{I}$ increases, and then consumers' rational choice interval $[\underline{I}, \overline{I}]$ becomes larger. This result implies that as the consumer attaches the larger relative weight of gain-loss comparison or his loss aversion becomes higher, his regret-based trade-off imposes a more significant impact on his optimal insurance purchase in a way that his choices are more prone to reference manipulation. As a consequence, the impact of agents' selling effort on stimulating consumers' demand becomes larger.

3.1.3 Welfare analysis

Since consumers' rational choice varies with different references, an important question arises: what choice is best for consumers? In other words, what choice makes consumers achieve their highest welfare? To answer this question, we have to define what is consumers' welfare. Intuitively, consumers' welfare should only be affected by their actual choice on insurance purchase and not by their references, i.e., welfare measure is reference-independent. Thus a natural choice is to adopt the standard reference-independent utility to measure consumers' welfare. We next show that in the context consumers make regret-based decisions, it has an appealing economic justification for this choice.

Imagine that an agent herself is a consumer. Since the agent has substantial experience for life insurance purchase, she can realize the effects of her anticipatory regret on the decision. To avoid the negative role of regret, she would fully endogenize her choice and take her choice as reference. In other words, she would take her insurance purchase as a committed decision long before outcomes occur, and hence she affects her reference point by her choice. This case is defined as "choice-acclimating personal equilibrium" by Kőzegi and Rabin(2007). In the context of life insurance, we give the formal definition of this equilibrium.

Definition 2. An insurance purchase I^C is a choice-acclimating personal equilibrium if for all $I \in [0, w]$, $E[V(\tilde{x}(I^C), \tilde{r}(I^C))] \geq E[V(\tilde{x}(I), \tilde{r}(I))]$.

In this case, the gain-loss utility from regret-based comparison between the outcomes determined by the consumer's decision and the reference totally vanishes and the optimal choice thus becomes the one that maximizes the standard reference-independent utility. This definition is appealing because it grasps the intuition that an experienced consumer should not regret his decision. We thus can exactly employ the standard reference-independent utility to measure consumers' welfare, and call the consumer's optimal choice in this case achieves his highest welfare. The above arguments can be formalized as the following proposition.

Proposition 2. *A welfare-maximizing insurance purchase I^C satisfies*

$$I^C = \arg\max_{I \in [0,w]} E[u(\tilde{x}(I))]. \tag{9}$$

Moreover, for all $\eta > 0$ and $\lambda > 1$, $\underline{I} < I^C \leq \overline{I}$ and $I^C = \overline{I}$ occurs if and only if $\alpha = p$.

Proposition 2 demonstrates that for the consumer with the status-quo reference $I_r = 0$, the optimal purchase $\underline{I}$ is strictly smaller than the coverage I^C. This implies that with the status quo reference, the consumer is only willing to purchase less insurance than the one maximizing her welfare. As argued by Morrison (1939) and Gravelee (1993, 1994), consumers are passive in purchase of life insurance and hence life assurance products are "sold rather than bought". This result provides a formal explanation for the above argument and justifies the positive role of agents' selling behavior in the insurance markets, i.e., agents can help consumers choose proper insurance with more coverage, I^C, by manipulating their reference to be I^C and thus improve consumers' welfare.

However, agents' selling behavior also has the dark side. As we have analyzed, agent's selling effort can lead consumers to choose choice $\overline{I}$ (if consumers' reference is manipulated to be $I_r \geq \overline{I}$), strictly larger than I^C under the condition $\alpha > p$ which is commonplace for insurance pricing. In this situation, agents induce consumers to purchase more insurance than the one maximizing their welfare and thus harm them. Therefore, this inducing behavior is unethical. Our finding puts forward an challenge for the regulation of unethical agent behavior, as the inducing behavior can occur even in the case that consumers with complete information about life insurance products make rational decisions. As a result, it becomes difficult to identify and regulate this inducing behavior in practice. The related regulation on unethical agent behavior, such as the requirement of extensive documentation of agents' advice, and adhering to the codes of ethical conduct, seems not as powerful as expected. Instead, compensation design for agents can play an important role in mitigating this type of unethical behavior. Under all current distribution systems in life insurance, agent compensation is largely via commissions. However, the heavy reliance on commission compensation has recently come into question (Regan and Tennyson 2000, Hilliard et al. 2012). An often-suggested alternative to commission compensation is that consumers pay fees to the agent. Our analysis supports the use of the fee-for-service compensation because with this compensation, agents have no incentive to induce consumers buy too much insurance and thus can help consumers achieve their welfare-maximizing choice.

3.2 Whole life insurance

We now consider consumer's whole life insurance purchase in a two-period framework. Period 1 is the consumer's working period and he earns income $w > 0$ in this period if alive. Period 2 represents the retirement period. The consumer is endowed with initial wealth w_1 and w_2 in Period 1 and 2 respectively. In period 1, the consumer dies with probability $p \in (0, 1)$ and then he will survive to Period 2 with probability $1-p$. We assume that there is a risk-free rate r between period 1 and period 2 and utility discount factor δ. For simplicity, we assume that $\delta = 1/(1 + r)$. For a whole life policy (P, I), the consumer will pay premium P and her heir will receive the repayment I in Period 1 if he dies. Moreover, he or his heirs will be certain to receive the repayment I in Period 2. In our setting, the repayment in Period 1 captures the insuring feature of whole life insurance while the repayment in Period 2 captures the saving feature of whole life insurance.

Note that the whole-life insurance is expensive since it partly serves as a saving tool. Indeed, the actuarially fair premium rate for this policy is $p + (1-p)/(1+r)$, implying that the premium rate $\alpha \geq p+(1-p)/(1+r)$. Hence $\alpha \in [p+(1-p)/(1+r), 1)$. In this framework, the consumer's expected utility with the reference I_r becomes:

$$E[V(\tilde{x}(I), \tilde{r}(I_r))] = \begin{cases} pu(w_1 + (1-\alpha)I) + (1-p)u(w_1 + w - \alpha I) + (1-p)\delta u(w_2 + I) \\ +(1-p)\eta\delta[u(w_2 + I) - u(w_2 + I_r)] \\ +p\eta[u(w_1 + (1-\alpha)I) - u(w_1 + (1-\alpha)I_r)] \\ +(1-p)\eta\lambda[u(w_1 + w - \alpha I) - u(w_1 + w - \alpha I_r)] \quad \text{if} \quad I \geq I_r \\ pu(w_1 + (1-\alpha)I) + (1-p)u(w_1 + w - \alpha I) + (1-p)\delta u(w_2 + I) \\ +(1-p)\eta\lambda\delta[u(w_2 + I) - u(w_2 + I_r)] \\ +p\eta\lambda[u(w_1 + (1-\alpha)I) - u(w_1 + (1-\alpha)I_r)] \\ +(1-p)\eta[u(w_1 + w - \alpha I) - u(w_1 + w - \alpha I_r)] \quad \text{if} \quad I < I_r \end{cases} \tag{10}$$

Compared with the case of term-life insurance as illustrated in equation (5), a new term in gain-loss comparisons refers to the difference of monetary utility in retirement period ($u(w_2 + I) - u(w_2 + I_r)$), representing the regret-based saving effect.

The first-order derivative of (10) with respect to I equals

$$\frac{d[V(\tilde{x}(I), \tilde{r}(I_r))]}{dI} = \begin{cases} (1+\eta)[(1-p)\delta u'(w_2 + I) + p(1-\alpha)u'(w_1 + (1-\alpha)I)] \\ -\alpha(1-p)(1+\eta\lambda)u'(w_1 + w - \alpha I) \quad \text{if} \quad I \geq I_r \\ (1+\eta\lambda)[(1-p)\delta u'(w_2 + I) + p(1-\alpha)u'(w_1 + (1-\alpha)I)] \\ -\alpha(1-p)(1+\eta)u'(w_1 + w - \alpha I) \quad \text{if} \quad I < I_r \end{cases} \tag{11}$$

We define $\underline{I}_w$, $\hat{I}_w$ and I_w^C satisfying

$$(1+\eta)[(1-p)\delta u'(w_2 + \underline{I}_w) + p(1-\alpha)u'(w_1 + (1-\alpha)\underline{I}_w)] \\ - \alpha(1-p)(1+\eta\lambda)u'(w_1 + w - \alpha\underline{I}_w) = 0 \tag{12}$$

$$(1+\eta\lambda)[(1-p)\delta u'(w_2 + \hat{I}_w) + p(1-\alpha)u'(w_1 + (1-\alpha)\hat{I}_w)] \\ - \alpha(1-p)(1+\eta)u'(w_1 + w - \alpha\hat{I}_w) = 0 \tag{13}$$

$$(1-p)\delta u'(w_2 + I_w^C) + p(1-\alpha)u'(w_1 + (1-\alpha)I_w^C) - \alpha(1-p)u'(w_1 + w - \alpha I_w^C) = 0 \tag{14}$$

where I_w^C represents the optimal purchase of whole life insurance maximizing consumers' welfare as interpreted in the case of term life insurance purchase. We have the following lemma.

Lemma 1. *With actuarially fair premium rate, i.e.* $\alpha = p + \frac{1-p}{1+r}$, *when* $w_2 + (p + \frac{1-p}{1+r})w > w_1$, $\hat{I}_w^C < w$ *holds.*

Note that w_1 and w_2 represent consumers' initial wealth in Periods 1 and 2, respectively. w represents their total earning income. It is reasonable to expect that $w_2 + (p + \frac{1-p}{1+r})w > w_1$ holds for most consumers. As a result, Lemma 1 implies that for most consumers, different from the case of term life insurance, consumers' optimal whole life insurance purchase that maximizes their welfare is no longer full insurance. That is, consumers prefer to buy less insurance. This distinction reflects exactly the feature of whole life insurance in which the expensive saving part is introduced. We will return to this point in the next subsection. By defining $\overline{I}_w = \min\{\hat{I}_w, w\}$, with the analogous reasoning to the case of term-life insurance purchase, we obtain the following Proposition.

Proposition 3. *Any insurance purchase* $I \in [\underline{I}_w, \overline{I}_w]$ *is a personal-equilibrium choice, i.e.*

$$I^{PE} = \{I : I \in [\underline{I}_w, \overline{I}_w]\}.$$

Moreover,
(i) $\partial \underline{I}/\partial \eta < 0, \partial \underline{I}/\partial \lambda < 0$ *whereas* $\partial \overline{I}/\partial \eta \geq 0, \partial \overline{I}/\partial \lambda \geq 0$.
(ii) $\underline{I}_w < I_w^C \leq \overline{I}_w$ *for all* $\eta > 0$ *and* $\lambda > 1$. *When* $w_2 + (p + \frac{1-p}{1+r})w > w_1$, $I_w^C < \overline{I}_w$ *holds.*

This proposition says that consumers' purchase behavior for whole life insurance exhibits the same pattern as that for term life insurance and thus the similar economic implications are applicable. There exist multiple personal equilibria for their optimal choice, implying that agents have an opportunity to affect consumers' rational choice through manipulating their reference. This manipulation also has both positive and negative aspects for improving consumers' welfare. On the one hand, consumers consumer endowed with status quo as not insuring, prefer to buy less insurance than the amount making them achieve the highest welfare ($\underline{I}_w < I_w^C$), and hence agents are helpful for consumers by suggesting they should buy proper insurance with more coverage. On the other hand, as we shall see, agents may be stimulated by commission mechanism commonly used in life insurance to become substantially aggressive so that they sell more insurance than the one maximizing consumers' welfare ($\overline{I}_w \geq I_w^C$) and thus make consumers worse off.

3.3 The comparison between term life insurance and whole life insurance purchase

We have shown that consumers' purchase behavior for whole life insurance exhibits the same pattern as that for term life insurance. Despite the qualitative analogy between term life and whole life insurance purchase, the quantitative comparison between these two is ambiguous. Specifically, whole life insurance is much more expensive than term life due to its savings feature. That is, the premium rate α becomes much higher for whole life than that for term life. Roughly speaking, the higher α the lower optimal purchase $\underline{I}$ and $\overline{I}$ for term life insurance, as well as the lower $\underline{I}_w$ and $\overline{I}_w$ for whole life insurance.[11] However, with the same premium rate α, compared to Equation (6) in the case of term life insurance, adding the new term $u'(w_2+\underline{I}_w)$ of (10), which reflects the savings part in whole life insurance, increases the insurance optimal purchase. As a result, the aggregate quantitative effect of the savings feature in whole life insurance is theoretically undetermined. We thus explore this quantitative effect by use of numerical illustrations in this section.

In the household finance literature, a standard assumption is that consumers' preference is characterized by a constant relative risk aversion (CRRA) function $u(x) = x^{1-\theta}/(1-\theta)$ with a risk aversion parameter θ (Mitchell, Poterba, Warshawsky, and Brown 1999, Cocoo 2005, Einav et al. 2010, Hong and Ríos-Rull 2012). For estimating consumers' preference from their life insurance purchase, Hong and Ríos-Rull (2012) use $\theta = 3$ as the coefficient of relative risk aversion. Moreover, a long line of papers studying retirement planning and annuity purchase decisions use this value in their simulations (e.g., Hubbard, Skinner, and Zeldes 1995, Engen, Gale, and Uccello 1999, Mitchell, Poterba, Warshawsky, and Brown 1999, Scholz, Seshadri, and Khitatrakun 2006, Einav et al. 2010). We thus assume that consumers have the CRRA utility function and set $\theta = 3$.[12] The consumer's income is standardized to be $w = 100$. We follow Cocoo(2005) by setting per year's utility discount factor $\delta = 0.96$ and the real risk-free rate $r = 0.02$. Berkelaar et al. (2004) in the context of portfolio choice estimate $\eta = 1$ and $\lambda = 2.5$ where they use the reference-dependent utility model and capture the gain-loss utility with a piecewise linear function, and De Giorgi and Post (2011) use it to perform the numerical analysis for investors' portfolio choice problem.[13] We thus assume that $\eta = 1$ and $\lambda = 2.5$.

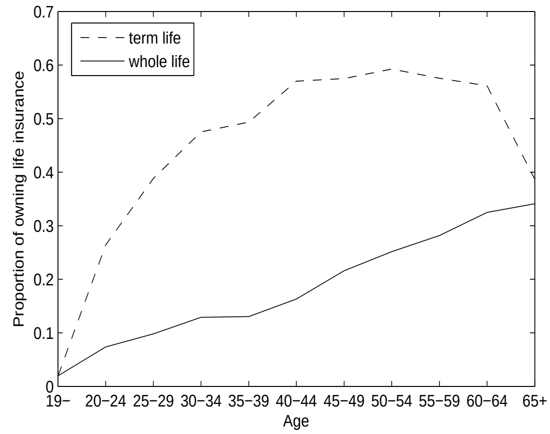
In order to fix the age of a typical household head who makes a decision on life insurance purchase, we first use the 2010 survey of consumer finances (SCF) to examine the age profile of the heads of households for term life and whole life insurance purchase and their wealth-income ratio. From this data set, we obtain the proportion of households owning term life insurance or whole life insurance over ages. As shown in Figure 1, the proportion of owning

[11] As we have shown in Lemma A.2 of Appendix, when consumers' absolute risk aversion parameter $\gamma(x)$ satisfies $\gamma'(x) < \frac{16}{w^2}$ for all $x \in [w_1, w_1 + w]$, this comparative static holds strictly.

[12] Although some papers found risk aversion coefficient close to 1, as in consumption studies summarized by Laibson, Repetto, and Tobacman (1998), other papers report higher levels of relative risk aversion (γ is estimated to be around 4 in Barsky, Kimball, Juster, and Shapiro 1997, and γ is set to be 5 in Cocoo 2005).

[13] Tversky and Kahneman (1992) estimated λ to be 2.25 and Benartzi and Thaler(1995) estimated λ to be 2.77 in the framework of Prospect Theory where there only exists gain-loss utility captured by a piecewise linear function.

Figure 1: Age Profile of Households holding term life insurance and whole life insurance in 2009



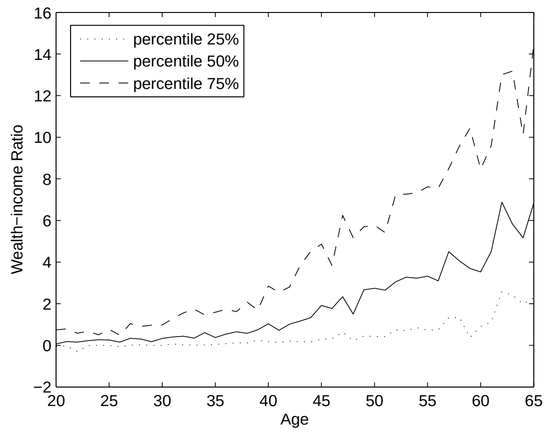
In this figure, the horizontal axis and vertical axis represent the age of the head of household and the proportion of owning life insurance respectively. The dashed and solid curves represent the proportion of owning term life insurance and whole life insurance, respectively.

term life insurance increases sharply from age 20 to age 34 while the proportion of owning whole life insurance increases most quickly from age 35 to age 49. To the extent that different cohorts of households in the data share the same life insurance purchase dynamics, the change of the proportion of owning life insurance thus implies that households are most likely to purchase term life insurance when their heads are 20-34 years old while they are most likely to purchase whole life insurance when their heads are 35-49 years old. Considering that most people start to work after 25 years old, we thus focus on the households whose heads are 25-35 years old for term life insurance while the age of the head is assumed to be from 35 to 50 for whole life insurance purchase. Notably, Figure 1 also suggests that households' purchase rate of term life insurance is significantly higher than that of whole life insurance, which has important implications that we will explore below in detail.

We further present households' wealth-income ratio. To do this, we follow Bricker et al. (2011) to calculate households' net wealth, or total assets minus total liabilities, and obtain the wealth-income ratio by dividing the net wealth by the total income directly from the SCF data.[14] As shown in Figure 5, the 25%, 50% and 75% percentiles of the ratio exhibits an

[14] As Bricker et al. (2011) stated, "Assets include a main residence, other real estate holdings, net business equity, vehicles, trusts in which the family has an equity interest, annuities with a cash value, other financial assets, pension accounts that the family can withdraw from or take loans against, and miscellaneous assets. Defined-benefit pensions and other assets where there is no equity interest are not included here as assets. Liabilities include mortgages on primary and secondary residences, lines of credit, credit card debt, loans for vehicles, other installment loans, loans against pension accounts or life insurance, and all other types of personal debt. Debt held by a family's business or nonresidential real estate is netted against the value of those assets."

Figure 2: Households' wealth-income ratio over age 20-65

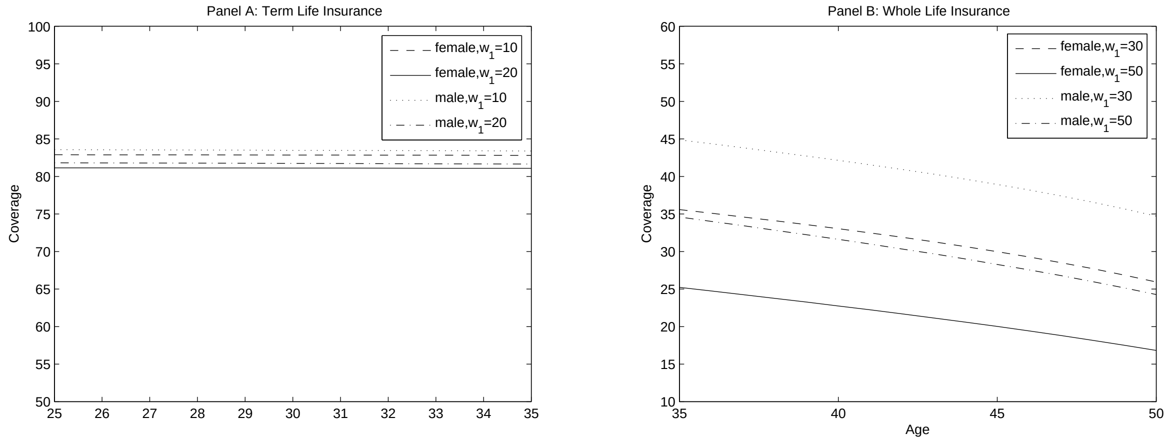


In this figure, the horizontal axis and vertical axis represent the age of the head of household and the wealth-income ratio respectively.

increasing trend over age 20-65, implying that households accumulate their wealth over time during the working period. For households whose heads are 25-35 years old, the median of wealth-income ratio lies between 15% and 61%, while for households whose heads are 35-50 years old, the median of wealth-income ratio lies between 37% and 274%. To measure the relative initial wealth to the total income over the working period, we need to calculate discounted sum of all income from the starting age of life insurance purchase to the retirement age. However, from Figure 1, we can see that the rate of owning term life insurance has been around 50% for the heads of 25-35 years old, while the rate of owning whole life insurance only lies between 10% and 25% for the heads of 35-50 years old. It is well recognized that households' wealth is a key determinant of their life insurance demand: the larger wealth a household owns, the more life insurance she purchases. We thus adopt the median of the wealth-income ratio to represent the relative wealth level for term insurance and the 75% percentage of the ratio for whole life insurance. Considering these factors, we assume $w_1 = 10, 20$ relative to $w = 100$ for term life insurance, whereas $w_1 = 30, 50$ for whole life insurance. We further assume the initial wealth in second period $w_2 = 100$.

In our two-period framework for whole life insurance, the timing of the repayment I need to be specified. For the head of each household, we can obtain her remaining life expectancy from the 2009 period life table, and take it as the realized time of repayment I in period 2. Under these settings, we present our illustrations on households' optimal life insurance purchase decisions. Here we consider the scenario without agents, i.e., consumers takes status quo reference of not having any insurance. As shown in Figure 3, neither households' optimal term life insurance purchase nor their optimal whole life insurance purchase is full coverage, even with the actuarially fair premium rate, reflecting that with status quo reference, consumers have a propensity to insure less. Note that the role of status quo reference is different for term

Figure 3: Households' life insurance purchase

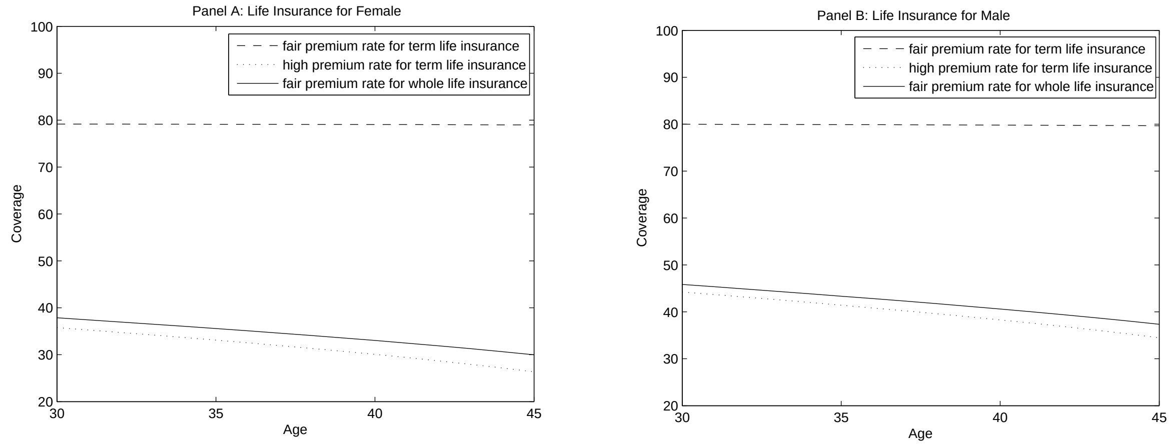


In this figure, the horizontal axis and vertical axis represent the age of the head of household and the optimal insurance coverage respectively.

life insurance and for whole life insurance. The optimal term life insurance for both males and females is over 80% of the full coverage with initial wealth level $w_1 = 10, 20$, as shown in Panel A of Figure 3. However, the optimal whole life insurance for both males and females is lower than 45% of the full coverage with initial wealth level $w_1 = 30, 50$, and the optimal purchase becomes lower for females and/or with higher initial wealth, as shown in Panel A of Figure 3. Moreover, the optimal whole life insurance decreases with increased age when purchasing the insurance. This is because that the later the purchase, the smaller proportion is insurance protection and the higher proportion is savings in the whole life insurance, and thus consumers are less willing to purchase it. An alternative explanation for the difference between optimal term life insurance and whole life insurance is that the difference may be due to the different specification of initial wealth between them. We show below that it is not the case.

To make a direct comparison between term life insurance purchase and whole life insurance purchase, and explore the impact of the savings feature of whole life insurance, we report results under the setting that the consumer's age at insurance purchase is from 30 to 45 and their initial wealth $w_1 = 30$ for both term life insurance and whole life insurance. To disentangle the aggregate quantitative effects of the savings feature of whole life insurance, we compare three cases: term life insurance with actuarially fair premium rate, term life insurance with (higher) premium rate that is actuarially fair for whole life insurance, and whole life insurance with actuarially fair premium rate. Therefore, the comparison between the first two cases captures the impact of the increased premium rate arising from the savings feature in whole life insurance, and the comparison between the latter two cases reflects the direct impact of having a savings component in whole life insurance. As shown in Figure 4, for both males and females, the increased premium rate leads to a significant reduction of

Figure 4: Comparison between term life insurance and whole life insurance purchase



In this figure, the horizontal axis and vertical axis represent the age of the head of household and the optimal insurance coverage respectively.

insurance purchase, whereas the savings component only increases the insurance purchase a bit. Consequently, the aggregate effect of the savings feature is much lower whole insurance purchase than term life, which is consistent with the direct comparison presented in Figure 3.

4 Agent selling and its welfare implications

We have shown in the previous section that consumers' regret-based decision making provides a chance for agents to sell more insurance by manipulating consumers' insurance purchase and thus increase insurers' revenue. However, agent selling may decrease consumers' welfare and in turn damage insurers' reputation and harm their competitiveness in the market (Inderst and Ottaviani 2009). In this section, we incorporate this tradeoff into the analysis of insurers' optimal pricing problem in an insurance market with monopolistic competition, and explore the resulting implications on market equilibrium. The feature that insurance markets are monopolistically-competitive is well documented in the insurance literature (Schlesinger and Schulenburg 1991).[15]

To facilitate the analysis, we first consider the benchmark case when consumers with complete information have no behavioral bias and make life insurance purchase to maximize their expected utility. In this benchmark case, agent selling provides no benefit and thus is not

[15]As summarized by Schlesinger and Schulenburg (1991), the idea is that consumers in insurance markets, may incur some nontrivial costs for switching from one insurer to another. These costs enable insurers to exert a certain degree of monopolistic power in the markets. Together with the assumption of product heterogeneity, the markets exhibit the feature of monopolistic competition even the number of insurers is quite large.

adopted by insurers. Focusing on this case thus allows us to isolate the impact of agent selling on the market equilibrium. In a market with monopolistic competition, profit-maximizing insurers face two constraints. The first one is the incentive compatibility constraint (IC): given a premium rate, consumers will purchase an optimal amount of insurance to maximize their expected utility. The second one is the individual rationality constraint (IR): consumers' welfare cannot be less than a certain reservation utility level. Specifically, denote consumers' welfare $U(\alpha, I) = pu(w_1 + (1-\alpha)I) + (1-p)u(w_1 + w - \alpha I)$ for a term life insurance contract (α, I), and $U_w(\alpha, I) = pu(w_1 + (1-\alpha)I) + (1-p)u(w_1 + w - \alpha I) + (1-p)\delta u(w_2 + I)$ for a whole life insurance contract. Insurers' optimization problem writes

$$
\begin{aligned}
&\max_{\alpha, I} \quad \pi \equiv (\alpha - p - c)I \\
&\text{s.t.} \\
&\text{(IC)} \quad I = I^C(\alpha) \text{ for term life insurance}, \quad I = I_w^C(\alpha) \text{ for whole life insurance}, \\
&\text{(IR)} \quad U(\alpha, I) \geq \underline{U} \text{ for term life insurance}, \quad U_w(\alpha, I) \geq \underline{U}_w \text{ for whole life insurance},
\end{aligned}
\tag{15}
$$

where p is the probability of death, and c represents the loading rate of the insurance contract. For simplicity, we assume the loading cost is proportional to the insurance coverage. (IC) constraint shows that given any insurance premium rate α, consumers will choose the optimal insurance purchase $I^C(\alpha)$ to maximize their reference-independent expected utility (which also represents their welfare). The level of consumers' reservation welfare in (IR) constraint reflects the fierceness of market competition: the higher the reservation welfare, the fiercer market competition is. (IR) constraint thus captures the feature of monopolistic competition in which, to attract consumers and maintain competitive, profit-maximizing insurers should make the welfare of their consumers no less than the reservation welfare. Define the utility level $\overline{U} \equiv U(p, I^C(p))$ and $\overline{U}_w \equiv U_w(p + (1-p)/(1+r), I_w^C(p + (1-p)/(1+r)))$, where $I^C(p)$ and $I_w^C(p + (1-p)/(1+r))$ denote consumers' welfare-maximizing insurance purchase with the actuarially fair premium rate p for term life insurance and $p + (1-p)/(1+r)$ for whole life insurance, respectively.

Assumption 1: $\underline{U} \leq \overline{U}$ holds for term life insurance and $\underline{U}_w \leq \overline{U}_w$ holds for whole life insurance.

Note that $\overline{U}$ and $\overline{U}_w$ represent the maximal welfare consumers can achieve given the two types of life insurance purchase. Assumption 1 implies that consumers' reservation welfare should be no more than their maximal welfare.

Lemma 2. *Under Assumption 1:*
(i) There exists a unique optimal solution to the optimization problem (15) for the two types of insurance. The solutions for term life insurance and whole life insurance are denoted by $(\alpha^*, I^C(\alpha^*))$ *and* $(\alpha_w^*, I_w^C(\alpha_w^*))$*, respectively;*

(ii) For the sets $\mathcal{C} = \{\alpha : U(\alpha, I^C(\alpha)) \geq \underline{U}\}$ and $\mathcal{C}_w = \{\alpha_w : U(\alpha_w, I^C(\alpha_w)) \geq \underline{U}_w\}$, $\alpha^* = \max_{\alpha \in \mathcal{C}}\{\alpha\}$ and $\alpha_w^* = \max_{\alpha_w \in \mathcal{C}_w}\{\alpha_w\}$.

Result (i) of Lemma 2 establishes the standard prediction that the insurers' optimization problem facing traditional expected utility maximizing consumers (i.e., those without a behavioral bias) is well defined and has a unique solution for both term life insurance and whole life insurance. Note that the sets $\mathcal{C}$ and $\mathcal{C}_w$ represent the feasible solutions under the (IC) and (IR) constraints for term life insurance and whole life insurance, respectively. Therefore, result (ii) of Lemma 2 demonstrates that in a market of monopolistic competition, life insurers' optimal pricing strategy is to set the maximal premium rate under the constraints (IC) and (IR).

When consumers' behavioral bias and agent selling are taken into account, the constraint (IC) of insurers' optimization problem will change. Since consumers have reference-dependent preference and make regret-based insurance purchase decision, agent may exploit consumers by manipulating consumers' reference. Indeed, since agent compensation is largely via commissions under any form of agent selling system in life insurance market (Regan and Tennyson 2000, Hilliard et al. 2013), agents always have an incentive to exploit consumers by inducing them to purchase as much insurance as possible. Consumers' insurance purchase induced by this type of agent selling behavior constitutes the new (IC) constraint for insurers' profit maximizing problem in this case. Formally, with agent selling, insurers' optimization problem becomes

$$
\begin{aligned}
&\max_{\alpha, I} \quad \pi \equiv (\alpha - p - c)I \\
&\text{(IC)} \quad I = \overline{I}(\alpha) \text{ for term life insurance}, \quad I = \overline{I}_w(\alpha) \text{ for whole life insurance}, \\
&\text{(IR)} \quad U(\alpha, I) \geq \underline{U} \text{ for term life insurance}, \quad U_w(\alpha, I) \geq \underline{U}_w \text{ for whole life insurance}.
\end{aligned}
\tag{16}
$$

The constraint (IC) refers to consumers' life insurance purchase choice determined by agents' selling behavior that is in turn the best response to insurers' premium rate choice. As we have shown in Section 3, agents' manipulation of consumers' reference leads consumers to purchase exactly $\overline{I}(\alpha) = \min\{\hat{I}, w\}$, where $\hat{I}$ is defined in Equation (8) for term life insurance and Equation (13) for whole life insurance.

Lemma 3. *If the utility level $\underline{U} \leq U(p, \overline{I}(p))$ for term life insurance and $\underline{U}_w \leq U_w(p + \frac{1-p}{1+r}, \overline{I}_w(p + \frac{1-p}{1+r}))$ for whole life insurance:*
(i) There exists an optimal solution to the optimization problem (16), i.e. $(\alpha^{**}, \overline{I}(\alpha^{**}))$ for term life insurance and $(\alpha_w^{**}, \overline{I}_w(\alpha_w^{**}))$ for whole life insurance;
(ii) For the set $\mathcal{C}^A = \{\alpha : U(\alpha, \overline{I}(\alpha)) \geq \underline{U}\}$, $\alpha^{**} = \max_{\alpha \in \mathcal{C}^A}\{\alpha\}$ when $\overline{I}(\alpha)_{\alpha \in \mathcal{C}^A} \geq \min\{\frac{\eta(\lambda-1)}{(1+\eta)\gamma_0}, w\}$, where $\gamma_0 = \min_{x \in [w_1, w_1+w]}\{-u''(x)/u'(x)\}$;

(iii) For the set $\mathcal{C}_w^A = \{\alpha : U(\alpha, \overline{I}_w(\alpha)) \geq \underline{U}_w\}$, $\alpha_w^{**} = \max_{\alpha \in \mathcal{C}_w^A}\{\alpha\}$ *when* $\overline{I}_w(\alpha)_{\alpha \in \mathcal{C}_w^A} \geq \min\{\frac{\eta(\lambda-1)}{(1+\eta)\gamma_1}, w\}$, *where* $\gamma_1 = \min_{x \in [w_1, \max\{w_1+w, w_2+w\}]}\{-u''(x)/u'(x)\}$.

Result (i) of Lemma 3 shows that when the competition in life insurance market is not very fierce such that consumers' reservation welfare is not too high, i.e., $\underline{U} \leq U(p, \overline{I}(p))$ and $\underline{U}_w \leq U_w(p + \frac{1-p}{1+r}, \overline{I}_w(p + \frac{1-p}{1+r}))$, there exists an optimal solution to this optimization problem. Moreover, as shown in (ii) of Lemma 3, when the insurance coverage with agent selling is reasonably high, insurers' optimal pricing strategy is still to set the highest premium rate under the constraints (IC) and (IR) if certain reasonable conditions are met.

When there is agent selling, the (IR) constraint deserves further discussions. This constraint is imposed on consumers' welfare, which, as we discussed in the previous section, is measured by the reference-independent expected utility. Note that this constraint is inconsistent with the (IC) constraint imposed on consumers' optimal insurance purchase that is affected by agent selling when consumers have regret-based behavioral bias. By imposing this welfare requirement, we capture an important concern of the insurers to maintain a certain level of reputation and attract consumers in the long run, even with agents' overselling behavior.

Proposition 4. *(i) If* $\overline{I}(\alpha)_{\alpha \in \mathcal{C}^A} \geq \min\{\frac{\eta(\lambda-1)}{(1+\eta)\gamma_0}, w\}$, *where* $\gamma_0 = \min_{x \in [w_1, w_1+w]}\{-u''(x)/u'(x)\}$, $\alpha^{**} \leq \alpha^*$ *and the inequality holds when premium rate is not actuarially fair.*
(ii) If $\overline{I}_w(\alpha)_{\alpha \in \mathcal{C}_w^A} \geq \min\{\frac{\eta(\lambda-1)}{(1+\eta)\gamma_1}, w\}$, *where* $\gamma_1 = \min_{x \in [w_1, \max\{w_1+w, w_2+w\}]}\{-u''(x)/u'(x)\}$, $\alpha_w^{**} \leq \alpha_w^*$ *and the inequality holds when* $w_2 + (p + \frac{1-p}{1+r})w > w_1$.
(iii) If consumers' absolute risk aversion parameter satisfies $-\gamma'(x) \leq \frac{16}{w^2}$ *for all* $x \in [w_1, w_1 + w]$, *there exist* $\overline{I}(\alpha^{**}) \geq I^C(\alpha^*)$ *and* $\overline{I}_w(\alpha_w^{**}) \geq I_w^C(\alpha_w^*)$. *For term life insurance, when premium rate is not actuarially fair, the inequality holds. For whole life insurance, when* $w_2 + (p + \frac{1-p}{1+r})w > w_1$, *the inequality holds.*

Results (i) and (ii) of Proposition 4 shows that when life insurance coverage with agent selling is reasonably high, insurers' optimal strategy is to choose a premium rate lower than that without agent selling.[16] Result (iii) of Proposition 4 implies that when consumers' risk aversion is not too high, the lower premium rate results in more insurance sold to consumers in market equilibrium.[17] To see this, consider the the case when consumers have a CRRA utility

[16]To see the conditions in (i) and (ii) of Proposition 4 are not too strict, consider consumers with CRRA utility with CRRA parameter $\theta = 3$. Let $\eta = 1$ and $\lambda = 2.5$. We obtain that the conditions of (i) and (ii) of Proposition 4 turn out to be $\overline{I}(\alpha)_{\alpha \in \mathcal{C}^A} \geq \min\{\frac{w_1}{4}, w\}$ and $\overline{I}_w(\alpha)_{\alpha \in \mathcal{C}_w^A} \geq \min\{\frac{\min\{w_1, w_2\}+w}{4}, w\}$, respectively. Since it is reasonable to expect that $w_1 \leq w$ and $w_2 \leq w$, the above conditions seem not too strict in real-life life insurance markets.

[17]Influential empirical evidence provided by Cawley and Philipson (1999) shows that unit price decreases step by step with insurance coverage and low-risk consumers hold more coverage. This indicates that our prediction is practically relevant.

with the constant risk aversion parameter θ. In this case, the condition in (iii) $-\gamma'(x) \leq \frac{16}{w^2}$ turns out to be $\theta \leq 16$, which is satisfied by most consumers in different decision scenarios including life insurance purchase (Hubbard, Skinner, and Zeldes 1995, Engen, Gale, and Uccello 1999, Mitchell, Poterba, Warshawsky, and Brown 1999, Scholz, Seshadri, and Khitatrakun 2006, Einav et al. 2010, Hong and Ríos-Rull 2012).

Proposition 4 demonstrates that to reduce the negative impact of agents' overselling on consumers' welfare, insurers' optimal strategy is to choose a lower premium rate rather than to limit agents' selling. As a result, more insurance is sold to consumers in market equilibrium. This compensated overselling is due to an inherent conflict arising from the gap between consumers' reference-dependent decision utility and their reference-independent welfare. At the time of insurance purchase, due to their reference-dependent preference, consumers' decision is regret-based and thus can be manipulated by agent selling. With commission-oriented compensation, agents have a strong incentive to induce consumers to purchase as much insurance as possible. As a result, consumers purchase more insurance than what maximizes their reference-independent welfare. Consumers then become worse off ex post and have a lower evaluation of insurers. As a response, to maintain their reputation and attract consumers in the long run, insurers choose a lower premium rate α to maintain consumers' welfare no less than a certain reservation level. This strategy leads to higher insurance purchase in market equilibrium.

Our prediction that the existence of agent selling induces more insurance sales echoes the finding of Inderst and Ottaviani (2009). Inderst and Ottaviani (2009) show that to accommodate the impact of agency problem, firms need to decrease products' suitability standard, leading to agents' over selling. In our setting, also due to agency problem, insurers decrease the insurance premium rate and induce more sales. However, agents' overselling in our setting predicts welfare implication distinct from that of Inderst and Ottaviani (2009). We show that the increased insurance sales induced by lower price actually increases consumers' welfare to the reservation level. In contrast, Inderst and Ottaviani (2009) argue that the overselling with the lower suitability standard only make consumers' welfare become worse on average because more consumers will purchase the unsuitable products. This distinction highlights our main insight that agent selling has both the dark and the bright sides rather than merely decreasing consumers' welfare.

Next, we further explore welfare implications of agent selling by comparing insurers' optimal pricing with and without agent selling. To do this, we let the given utility level in consumers' IR constraint equal consumers' reference-independent welfare without agent selling. As we have shown in Section 3, when there exists no agent selling, consumers who are offered insurance with premium rate α take no insurance as their reference and their optimal insurance purchase turns out to be $\underline{I}(\alpha)$. we assume that the premium rate is actuarially fair, i.e., $\alpha = p$ for term life insurance and $\alpha = p + \frac{1-p}{1+r}$ for whole life insurance. Under this assumption, consumers' reservation welfare represents the maximal welfare they can achieve without agent selling help in a competitive market. Formally, the reservation welfare with agent selling writes

$$\underline{U} = U(p, \underline{I}) \text{ for term life insurance,} \quad \underline{U}_w = U_w(p + \frac{1-p}{1+r}, \underline{I}) \text{ for whole life insurance.} \tag{17}$$

Under the (IR) constraint with the reservation welfare defined in Equation (17), we can obtain the solutions to insurers' optimization problem (16), α^{**} for term life insurance and α_w^{**} for whole life insurance. Next we define the implied loading rate of agent compensation

$$c^* = \alpha^{**} - p \text{ for term life insurance,} \quad c_w^* = \alpha_w^{**} - p - \frac{1-p}{1+r} \text{ for whole life insurance.}$$

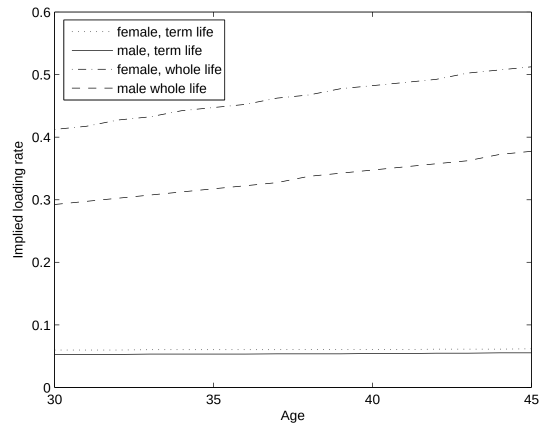
This indicator measures what is the maximal allowable cost of agent compensation by letting consumers' welfare under their optimal choice with agent-selling equal to that without agent selling. It allows us to compare the possible welfare improvement of agent selling: The larger the cost is, the higher welfare efficiency of agency selling is and the more possible that agent-selling is employed in a competitive life insurance market.

As shown in Figure 5, we calculate the implied loading rate c^* and c_w^*, which makes consumers' welfare indifferent between the case with agent selling and with direct selling. Here we set $w = 100, w_1 = 30, w_2 = 100$. Figure 5 shows that for whole life insurance, the implied loading rate lies in the interval [0.29,0.38] for males and [0.41,0.52] for females, while for term life insurance, the loading lies in the interval [0.05,0.06] for males and [0.06,0.07] for females. In practice, the commissions often account for 8-10% of the total premium.[18] Considering the actual cost of compensating agents, our quantitative results suggest that the agency system might be more beneficial to consumers of whole life insurance than to consumers of term life insurance. This result also predicts that term life insurance has more demand than whole life insurance through the direct selling channel, which is quite consistent with the observation that market share by agent selling is more than 90% for whole life while less than 70% for term life, (see Figures 25.2 and 25.3 provided by Hilliard et al. 2012).

The intuition behind the above results is simple. As shown in Figure 4, with the status quo reference, consumers prefer to purchase term life insurance with large coverage (around 80% of the full coverage) but are only willing to buy whole life insurance with low coverage (less than 45% of the full coverage). As a result, agency system can increase consumers'

[18]Based on the table "Life/Health Insurance Industry Income Statement, 2007-2011" on Page 103 of 2013 Financial Services Fact Book, we calculate the ratio of commissions over total premiums to approximate for the commission rate. The calculated rate lies in [8%, 10%] from 2007 to 2011.

Figure 5: The implied cost of agent selling relative to direct selling.



In this figure, the horizontal axis and vertical axis represent the consumer's age and implied loading rate, which makes consumers' welfare indifferent between the case with AS channel and the case with DS channel, respectively.

insurance purchase significantly of whole life insurance by manipulating their reference, but can only increase their purchase slightly of term life insurance. Note that compared to term life insurance, whole life insurance is not a pure insurance product since it involves a savings component. It is the savings feature of whole life insurance that decreases consumers' willingness to buy with the status quo reference and leaves a large room for agents to play a crucial role.

5 Conclusion

Building on Prospect Theory and Regret Theory, we show that consumers are passive in life insurance purchase and can only achieve a low level of welfare all by themselves. Agents can help them make a better (larger) purchase decision by providing them with an (increased) initial reference, in the meanwhile increasing profits for the insurers. On the other hand, we also shed light on the dark side of the agency system when agents lead consumers to buy more insurance than what maximizes their welfare, to generate more profits for insurance companies and themselves (under the popular commission-based agent compensation mechanism). This unethical behavior is difficult to identify and regulate in practice. However, this does not necessarily mean that new agent compensation mechanisms, such as a fee-for-service compensation mechanism, is better than the traditional commission-based compensation. Indeed, we show in market equilibrium that in order to keep their reputation and attract consumers in the long run, insurers can choose to lower the price to counteract the negative impact of agent overselling on consumers' welfare. This implies that consumers' welfare is not necessary to be harmed even with agent selling existing in the market. We further explore the welfare implications of agent selling by comparing insurers' optimal pricing with and without agent selling, and demonstrate that it is often more beneficial to consumers of whole life insurance than to consumers of term life insurance.

Our model proposes some testable predictions that are consistent with anecdotal evidence from real-world life insurance markets. First, the agency system on average should sell more life insurance than the direct channels. Second, agents may sell an excessive amount of insurance to consumers. Third, more term life insurance than whole life insurance is sold by the direct selling channels, which is consistent with the observation that market share through agents' selling is more than 90% for whole life while less than 70% for term life (for example, see Figures 25.2 and 25.3 provided by Hilliard et al. 2012).

As a first step toward explaining the implications of agent selling in life insurance markets, our current model is not readily applicable for the analysis of non-life insurance markets. First, the assumption that consumer is of limited rationality is not generally suitable for non-life insurance as commercial lines account for a large part of non-life insurance, where consumers are commercial companies who are commonly treated as rational decision makers.

Second, our statical framework does not account for consumers' repeated purchase behavior that is commonplace in non-life insurance markets where, unlike in the life insurance market, insurance policies are often short term in nature. The evolution of consumers' reference and decision has distinct features compared to the statical framework, as shown Kőszegi and Rabin (2010) in the context of consumers' consumption plan and in employment contract design.

Third, our model only focuses on the tradeoff between premium payment and insurance repayment and precludes the effects of factors in other dimensions, and thus inherits the "narrow bracketing" feature of Kőszegi and Rabin (2007) on risky choice analysis. This narrow bracketing seems proper for the analysis of life insurance but maybe not sufficient for non-life insurance because the latter often includes important multiple-dimensional factors, such as precautionary efforts, self-insurance, and other risk management considerations.

Last, our analysis assumes that consumers are homogeneous and abstracts from information asymmetry between consumers and insurers. This abstraction does not substantively affect our predictions for life insurance for at least two reasons. On the one hand, life insurers often have a large amount of insureds and thus the mortality distribution is stable and well known to insurers due to the law of large numbers. On the other hand, mortality distributions and other related information is increasingly accessible to consumers. However, this issue is much more essential to non-life insurance and the design of insurance products are consequently substantially more complicated. Future studies can explore how to adapt our model framework to explain consumer demand and agent selling in non-life insurance markets.

Appendix

Lemma A.1. *For $\eta > 0$ and $\lambda > 1$, we have:*
(i) $\underline{I} < \hat{I}$ and $\underline{I} < w$.
(ii)$\partial\underline{I}/\partial\eta < 0, \partial\underline{I}/\partial\lambda < 0$ whereas $\partial\hat{I}/\partial\eta > 0, \partial\hat{I}/\partial\lambda > 0$.
(iii) if the premium rate is actuarially fair, i.e. $\alpha = p$, we have $\hat{I} \geq w$.

Proof. (i) Given $k(I) \equiv \frac{u'(w_1+(1-\alpha)I)}{u'(w_1+w-\alpha I)}$, it is easy to see that $k(I)$ decreases as I increases since $u(\cdot) > 0$ and $u''(\cdot) \leq 0$. That is, $k'(I) < 0$. Because $k(\underline{I}) = \frac{\alpha(1-p)(1+\eta\lambda)}{p(1-\alpha)(1+\eta)} > k(\hat{I}) = \frac{\alpha(1-p)(1+\eta)}{p(1-\alpha)(1+\eta\lambda)}$ for $\eta > 0$ and $\lambda > 1$, it establishes that $\underline{I} < \hat{I}$. Moreover, due to $k(\underline{I}) \equiv \frac{u'(w_1+(1-\alpha)\underline{I})}{u'(w_1+w-\alpha\underline{I})} = \frac{\alpha(1-p)(1+\eta\lambda)}{p(1-\alpha)(1+\eta)} > 1$, it follows that $\underline{I} < w$.
(ii) From $k(\underline{I}) = \frac{\alpha(1-p)(1+\eta\lambda)}{p(1-\alpha)(1+\eta)}$ and $k(\hat{I}) = \frac{\alpha(1-p)(1+\eta)}{p(1-\alpha)(1+\eta\lambda)}$, it is straightforward to obtain that $\partial\underline{I}/\partial\eta < 0, \partial\underline{I}/\partial\lambda < 0$ whereas $\partial\hat{I}/\partial\eta > 0, \partial\hat{I}/\partial\lambda > 0$ since $k'(I) < 0$.
(iii) When $\alpha = p$, $k(\hat{I}) \equiv \frac{u'(w_1+(1-\alpha)\hat{I})}{u'(w_1+w-\alpha\hat{I})} = \frac{\alpha(1-p)(1+\eta)}{p(1-\alpha)(1+\eta\lambda)} = \frac{1+\eta}{1+\eta\lambda} < 1$ for $\eta > 0$ and $\lambda > 1$, it follows that $u'(w_1 + (1-\alpha)\hat{I}) < u'(w_1 + w - \alpha\hat{I})$, or $(1-\alpha)\hat{I} \geq w - \alpha\hat{I}$ since $u''(\cdot) \leq 0$. It establishes the result of (iii). ■

Proof of Proposition 1. Due to $u''(\cdot) \leq 0$, $\frac{d^2[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI^2} \leq 0$ and hence the first-order conditions are sufficient and necessary for the optimization of consume' life insurance purchase. We consider the following three cases to complete the proof:
(i) The case of $I_r < \underline{I}$. From equation (7), when the consumer tries to increase insurance purchase relative to the reference level ($I \geq I_r$), his optimal choice is $\underline{I}$ such that $\frac{d[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI} = 0$; When the consumer attempts to reduce insurance purchase relative to the reference level ($I < I_r$), it follows that $k(I) - \frac{\alpha(1-p)(1+\eta)}{p(1-\alpha)(1+\eta\lambda)} > 0$ since $k(I) > k(\underline{I})$ due to $I < \underline{I} < \hat{I}$, where the latter inequality holds due to (i) of Lemma A.1. Consequently $\frac{d[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI} > 0$ for all $I < I_r$, hence the consumer will not actually reduce his coverage relative to the reference level. Overall, for the case of $I_r < \underline{I}$, the consumer's optimal insurance overage $I^* = \underline{I}$.
(ii) The case of $\underline{I} \leq I_r \leq \overline{I}$. From equation (7), when the consumer attempts to purchase more insurance than the reference level ($I \geq I_r$), he finds that $\frac{d[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI} \leq 0$ due to the fact that $k(I) - \frac{\alpha(1-p)(1+\eta\lambda)}{p(1-\alpha)(1+\eta)} \leq k(\underline{I}) - \frac{\alpha(1-p)(1+\eta\lambda)}{p(1-\alpha)(1+\eta)} = 0$, and thus prefers not to do so. Alternatively, when the consumer considers purchase less insurance than the reference level ($I < I_r$), he finds that $\frac{d[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI} \geq 0$ and also prefer not to do so. In other words, for $\underline{I} \leq I_r \leq \overline{I}$, the consumer prefer to choose the optimal insurance purchase equaling to the reference level, and thus establishes $I^* = I_r$.

(iii) The case of $\overline{I} < I_r \leq w$. If $\overline{I} = w$ (i.e., $\hat{I} \geq w$), this case is ignorable. So we only need to consider the case $\overline{I} < w$ (i.e., $\overline{I} = \hat{I} < w$). When the consumer attempts to purchase more insurance than the reference level ($I \geq I_r$), he finds that $\frac{d[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI} \leq 0$ and thus prefers not to do so. However, when the consumer considers purchasing less insurance than the reference level ($I < I_r$), he optimal choice is $\overline{I}$ such that $\frac{d[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI} = 0$. Therefore, for $\overline{I} < I_r \leq w$, the consumer's optimal choice is $I^* = \overline{I}$. ■

Proof of Corollary 1. According to the definition of personal equilibrium, the choice must be consistent with the reference, i.e. the actual insurance coverage equals the reference one $I^* = I_r$. From Proposition 1, we directly establish that any insurance purchase $I \in [\underline{I}, \overline{I}]$ is a personal-equilibrium choice. Moreover, as proved in (ii) of Lemma A.1, $\partial \underline{I}/\partial \eta < 0, \partial \underline{I}/\partial \lambda < 0$. Moreover, $\partial \hat{I}/\partial \eta > 0, \partial \hat{I}/\partial \lambda > 0$, so we have $\partial \overline{I}/\partial \eta \geq 0, \partial \overline{I}/\partial \lambda \geq 0$ from the definition of $\overline{I} = \min\{\hat{I}, w\}$. ■

Proof of Proposition 2. For the reference-independent consumer's optimal insurance purchase problem, the first-order condition is

$$\frac{d[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI} = p(1-\alpha)u'(w-\alpha I)\left[k(I) - \frac{\alpha(1-p)}{p(1-\alpha)}\right],$$

and the second-order condition $\frac{d^2[V(\tilde{x}(I),\tilde{r}(I_r))]}{dI^2} \leq 0$ due to $u''(\cdot) \leq 0$. Thus $k(I) = \frac{\alpha(1-p)}{p(1-\alpha)}$ is sufficient and necessary for this optimization problem. In other words, the choice I^C is the optimal solution to the problem. Since $k'(I) < 0$ and $k(\underline{I}) = \frac{\alpha(1-p)(1+\eta\lambda)}{p(1-\alpha)(1+\eta)} > k(I^C) = \frac{\alpha(1-p)}{p(1-\alpha)} > k(\hat{I}) = \frac{\alpha(1-p)(1+\eta)}{p(1-\alpha)(1+\eta\lambda)}$ for $\eta > 0$ and $\lambda > 1$, we obtain $\underline{I} < I^C < \hat{I}$. Note that $I^C \leq w$ and the equity occurs if and only if $\alpha = p$. From the definition $\overline{I} = \min\{\hat{I}, w\}$, we thus establish $\underline{I} < I^C \leq \overline{I}$ and $I^C = \overline{I}$ if and only if $\alpha = p$ (note when $\alpha = p$, $\hat{I} \geq w$ from (iii) of Lemma 1, and thus $\overline{I} = w$.). ■

Proof of Lemma 1. Substituting the actuarially fair premium rate $\alpha = p + \frac{1-p}{1+r}$ and $\delta = \frac{1}{1+r}$ into the first-order condition (14), which maximizes consumers' reference-independent welfare for whole life insurance, yields

$$\begin{aligned}&\frac{1-p}{1+r}u'(w_2 + I_w^C) + \frac{p(1-p)r}{1+r}u'(w_1 + \frac{(1-p)r}{1+r}I_w^C)\\&\quad -(p + \frac{1-p}{1+r})(1-p)u'(w_1 + w - (p + \frac{1-p}{1+r})I_w^C) = 0\end{aligned} \tag{18}$$

Substituting $I_w^C = w$ into the left hand of Equation (18) yields by rearranging terms

$$\frac{1-p}{1+r}[u'(w_2 + w) - u'(w_1 + \frac{(1-p)r}{1+r}w)]$$

If $w_2 + (p + \frac{1-p}{1+r})w > w_1$, then $w_2 + w > w_1 + \frac{(1-p)r}{1+r}w$. So the above equation $\frac{1-p}{1+r}[u'(w_2 + w) - u'(w_1 + \frac{(1-p)r}{1+r}w)] < 0$ since $u''(\cdot) < 0$. Then there must be $I_w^C < w$. ■

Lemma A.2. *If consumers' absolute risk aversion parameter* $\gamma(x) \equiv -\frac{u''(x)}{u'(x)}$ *satisfies* $-\gamma'(x) < \frac{16}{w^2}$ *for all* $x \in [w_1, w_1 + w]$*, we have:*
(i) $\frac{\partial \underline{I}(\alpha)}{\partial \alpha} < 0, \frac{\partial I^C}{\partial \alpha} < 0, \frac{\partial \overline{I}(\alpha)}{\partial \alpha} \leq 0$ *with equality holds if* $\overline{I}(\alpha) = w$*;*
(ii) $\frac{\partial \underline{I}_w(\alpha)}{\partial \alpha} < 0, \frac{\partial I^C_w(\alpha)}{\partial \alpha} < 0, \frac{\partial \overline{I}_w(\alpha)}{\partial \alpha} \leq 0$ *with equality holds if* $\overline{I}_w(\alpha) = w$*.*

Proof. (i) We first prove that $\frac{\partial \overline{I}(\alpha)}{\partial \alpha} \leq 0$. Recall that $\overline{I} = \min\{\hat{I}, w\}$. We first consider the case that $\overline{I} = w$ holds for certain range of α. In this case, $\frac{d\overline{I}}{d\alpha} = 0$ and we are done. We next consider the case $\overline{I} = \hat{I}$. From Equation (6), we have

$$p(1-\alpha)(1+\eta\lambda)u'(w_1 + (1-\alpha)\hat{I}) - \alpha(1-p)(1+\eta)u'(w_1 + w - \alpha\hat{I}) = 0 \tag{A1}$$

Let $A = w_1 + (1-\alpha)\hat{I}$ and $B = w_1 + w - \alpha\hat{I}$ and then $A \leq B$. Differentiating (A1) with respect to α yields

$$\begin{aligned}&\left[p(1-\alpha)^2(1+\eta\lambda)u''(A) + \alpha^2(1-p)(1+\eta)u''(B)\right]\frac{\partial \hat{I}}{\partial \alpha}\\ &= p(1+\eta\lambda)u'(A) + (1-p)(1+\eta)u'(B) + \hat{I}p(1-\alpha)(1+\eta\lambda)u''(A) - \hat{I}\alpha(1-p)(1+\eta)u''(B)\end{aligned}$$

It is rewritten as

$$\begin{aligned}&\left[-p(1-\alpha)^2(1+\eta\lambda)u'(A)\gamma(A) - \alpha^2(1-p)(1+\eta)u'(B)\gamma(B)\right]\frac{\partial \hat{I}}{\partial \alpha}\\ &= p(1+\eta\lambda)u'(A)\left[1 - (1-\alpha)\hat{I}\gamma(A)\right] + (1-p)(1+\eta)u'(B)\left[1 + \alpha\hat{I}\gamma(B)\right].\end{aligned}$$

By using Equation (A1), we can substitute $u'(B) = \frac{p(1-\alpha)(1+\eta\lambda)}{\alpha(1-p)(1+\eta)}u'(A)$ into the above equation and obtain by rearranging terms

$$\frac{\partial \hat{I}}{\partial \alpha} = -\frac{\frac{1}{\alpha(1-\alpha)} + \hat{I}(\gamma(B) - \gamma(A))}{(1-\alpha)\gamma(A) + \alpha\gamma(B)} \tag{A2}$$

So to prove $\frac{\partial \hat{I}}{\partial \alpha} < 0$, we need to prove $\frac{1}{\alpha(1-\alpha)} + \hat{I}(\gamma(B) - \gamma(A)) > 0$, since $\gamma(A) > 0, \gamma(B) > 0$ and $\alpha \in (0,1)$. Note that $\gamma(B) - \gamma(A) = \gamma'(\xi)(B - A) = \gamma'(\xi)(W - \hat{I})$ where $\xi \in [A, B]$ (since $A \leq B$). We only need to prove $-\gamma'(\xi) < \frac{1}{\alpha(1-\alpha)\hat{I}(w-\hat{I})}$. If $\gamma(x)$ is increasing in x, this inequality holds automatically. If $\gamma(x)$ is decreasing in x, with the condition that $-\gamma'(x) < \frac{16}{w^2}$ for all $x \in [w_1, w]$. There must be $-\gamma'(\xi) < \frac{16}{w^2}$ since $\xi \in [A, B] \subset [w_1, w_1 + w]$. It follows that $-\gamma'(\xi) < \frac{1}{\alpha(1-\alpha)\hat{I}(w-\hat{I})}$ since $\alpha(1-\alpha) \leq \frac{1}{4}$ and $\hat{I}(w - \hat{I}) \leq \frac{w^2}{4}$. Therefore, when $-\gamma'(x) < \frac{16}{w^2}$, we establish $\frac{\partial \hat{I}}{\partial \alpha} < 0$. Combining two cases of proof, we obtain $\frac{\partial \overline{I}(\alpha)}{\partial \alpha} \leq 0$.

Next we prove $\frac{\partial \underline{I}(\alpha)}{\partial \alpha} < 0$. From Equation (6), we have

$$p(1-\alpha)(1+\eta)u'(w_1 + (1-\alpha)\underline{I}(\alpha)) - \alpha(1-p)(1+\eta\lambda)u'(w_1 + w - \alpha\underline{I}(\alpha)) = 0 \tag{A3}$$

Note that the only difference between (A1) and (A3) is the interchange of $(1+\eta)$ and $(1+\eta\lambda)$ in the two terms of the left hand of the equations, which has no essential influence on our above derivation. So we can establish $\frac{\partial I(\alpha)}{\partial \alpha} < 0$ by use of the similar derivation. Similarly, for the first-order condition with respect to I^C, we just eliminate $(1+\eta)$ and $(1+\eta\lambda)$ and keep other terms unchanged, we thus can derive $\frac{\partial I^C(\alpha)}{\partial \alpha} < 0$ in the similar way.

(ii) We first derive $\frac{\partial \overline{I}_w(\alpha)}{\partial \alpha} \le 0$. Recall that $\overline{I}_w = \min\{\hat{I}_w, w\}$. We first consider the case that $\overline{I}_w = w$ holds for certain range of α. In this case, $\frac{d\overline{I}_w}{d\alpha} = 0$ and we are done. We next consider the case $\overline{I}_w = \hat{I}_w$. Let $C = w_1 + (1-\alpha)\hat{I}_w$ and $D = w_1 + w - \alpha\hat{I}_w$ and then $C \le D$. We rewrite Equation (13) as

$$(1+\eta\lambda)[p(1-\alpha)u'(C) + (1-p)\delta u'(w_2+I)] - \alpha(1-p)(1+\eta)u'(D) = 0 \tag{A4}$$

Differentiating (A4) with respect to α yields

$$\begin{aligned}&\left[(1+\eta\lambda)[p(1-\alpha)^2u''(C) + (1-p)\delta u''(w+\hat{I}_w) + \alpha^2(1-p)(1+\eta)u''(D)\right]\frac{\partial \hat{I}_w}{\partial \alpha}\\ =&(1+\eta\lambda)pu'(C) + (1-p)(1+\eta)u'(D) + \hat{I}_w p(1-\alpha)(1+\eta\lambda)u''(C) - \hat{I}_w\alpha(1-p)(1+\eta)u''(D).\end{aligned}$$

It follows that

$$\begin{aligned}&-\big[(1+\eta\lambda)[p(1-\alpha)^2u'(C)\gamma(C) + (1-p)\delta u'(w+\hat{I}_w)\gamma(w+\hat{I}_w)\\ &+\alpha^2(1-p)(1+\eta)u'(D)\gamma(D)\big]\frac{\partial \hat{I}_w}{\partial \alpha}\\ =&(1+\eta\lambda)pu'(C) + (1-p)(1+\eta)u'(D)\\ &-\hat{I}_w p(1-\alpha)(1+\eta\lambda)u'(C)\gamma(C) + \hat{I}_w\alpha(1-p)(1+\eta)u'(D)\gamma(D).\end{aligned}$$

By using Equation (A4), we can substitute $u'(D) = \frac{(1+\eta\lambda)}{\alpha(1-p)(1+\eta)}[p(1-\alpha)u'(C) + (1-p)\delta u'(w_2+I)]$ into the above equation and obtain by rearranging terms

$$\begin{aligned}&-\big[(1+\eta\lambda)[p(1-\alpha)^2u'(C)\gamma(C) + (1-p)\delta u'(w+\hat{I}_w)\gamma(w+\hat{I}_w)\\ &+\alpha^2(1-p)(1+\eta)u'(D)\gamma(D)\big]\frac{\partial \hat{I}_w}{\partial \alpha}\\ =&(1+\eta\lambda)p(1-\alpha)u'(C)\left[\frac{1}{\alpha(1-\alpha)} + \hat{I}_w(\gamma(D)-\gamma(C))\right]\\ &+(1+\eta\lambda)(1-p)\delta u'(w_2+I)\left[\frac{1}{\alpha} + \hat{I}_w\gamma(D)\right].\end{aligned} \tag{A5}$$

From Equation (A5), it is easy to see that if $\frac{1}{\alpha(1-\alpha)} + \hat{I}_w(\gamma(D)-\gamma(C)) > 0$, there holds $\frac{\partial \hat{I}_w}{\partial \alpha} < 0$. With the same argument as in the part (i), when $-\gamma'(x) < \frac{16}{w^2}$, we establish $\frac{\partial \hat{I}_w}{\partial \alpha} < 0$. Combining two cases of proof, we obtain $\frac{\partial \overline{I}_w(\alpha)}{\partial \alpha} \le 0$.

Note that the only difference between (12) and (13) is the interchange of $(1+\eta)$ and $(1+\eta\lambda)$ in the two terms of the left hand of the equations, which has no essential influence on our above derivation. So we can establish $\frac{\partial \underline{I}_w(\alpha)}{\partial \alpha} < 0$ by use of the similar derivation. Similarly, for the first-order condition with respect to I_w^C, we just eliminate $(1+\eta)$ and $(1+\eta\lambda)$ and keep other terms unchanged, we thus can derive $\frac{\partial I_w^C(\alpha)}{\partial \alpha} < 0$ in the similar way. ■

Proof of Lemma 2. (i) Define the sets $\mathcal{A} \equiv \{(\alpha, I) : U(\alpha, I) \geq \underline{U}\}$ and $\mathcal{B} \equiv \{(\alpha, I) : U_w(\alpha, I) \geq \underline{U}_w\}$. Note that consumers' maximized expected utility with premium rate α are $U(\alpha, I^C(\alpha)) = pu(w_1 + (1-\alpha)I^C(\alpha)) + (1-p)u(w_1 + w - \alpha I^C(\alpha))$ for term life insurance and $U_w(\alpha, I_w^C(\alpha)) = pu(w_1 + (1-\alpha)I_w^C(\alpha)) + (1-p)u(w_1 + w - \alpha I_w^C(\alpha)) + (1-p)\delta u(w_2 + I_w^C(\alpha))$ for whole life insurance. By envelop theorem, it is easy to see that $dU(\alpha, I^C(\alpha))/d\alpha < 0$ and $dU_w(\alpha, I_w^C(\alpha))/d\alpha < 0$. Thus $U(\alpha, I) \leq \overline{U}$ and $U_w(\alpha, I) \leq \overline{U}_w$ since premium rate α must be larger than the actuarially fair one, p for term life insurance and $p + \frac{1-p}{1+r}$ for whole life insurance. That is, $\mathcal{A}$ and $\mathcal{B}$ are bounded. Since $U(\alpha, I)$ and $U_w(\alpha, I)$ are continuous at α and I, $\mathcal{A}$ and $\mathcal{B}$ must be closed. So both sets are compact. Moreover, since $\underline{U} \leq \overline{U}$ for term life insurance and $\underline{U}_w \leq \overline{U}_w$ for whole life insurance, $\mathcal{A}$ and $\mathcal{B}$ are no empty. Combining with the fact that the objective function is continuous, there must exist optimal solution to this optimization problem.

(ii) Note that with optimal solution, the (IR) constraint must be binding. we prove this by contradiction. If not, we can push the utility level up until it is binding. In this process, insurers in a market of monopolistic competition can attract more consumers and thus increase their profits. That is, the original solution is not optimal, leading to a contradiction. Combining with the fact that $dU(\alpha, I^C(\alpha))/d\alpha < 0$ and $dU_w(\alpha, I_w^C(\alpha))/d\alpha < 0$ shown in the part of (i), we establish that $\alpha^* = \max_{\alpha \in \mathcal{C}}\{\alpha\}$ and $\alpha_w^* = \max_{\alpha_w \in \mathcal{C}_w}\{\alpha_w\}$. ■

Proof of Lemma 3. (i) Note that with any premium rate α, consumers' welfare with the insurance coverage $\overline{I}(\alpha)$ for term insurance ($\overline{I}_w(\alpha)$ for whole life insurance) is always less than that with the insurance coverage $I^C(\alpha)$ ($I_w^C(\alpha)$). Therefore, $U(\alpha, I) \leq \overline{U}$ and $U_w(\alpha, I) \leq \overline{U}_w$ since premium rate α must be larger than the actuarially fair one. Under the assumption that $U(\alpha, I) \leq U(p, \overline{I}(p))$ for term life insurance and $U_w(\alpha, I) \leq U_w(p + \frac{1-p}{1+r}, \overline{I}_w(p + \frac{1-p}{1+r}))$, there must exist feasible solution to the optimization problem. Then with the similar argument to the part (i) of the proof of Lemma 2, we can prove the existence of optimal solutions to the optimization problem.

(ii) If $dU(\alpha, \overline{I}(\alpha))/d\alpha < 0$, it follows that $\alpha^* = \max_{\alpha \in \mathcal{C}}\{\alpha\}$. However, with agent selling, consumers' reference-dependent decision utility is distinct from their reference-independent welfare. As a result, to derive whether $dU(\alpha, \overline{I}(\alpha))/d\alpha < 0$ holds, we cannot apply the envelop theorem as we do for the part (ii) of Lemma 1 where agent selling is not allowed.

$$\begin{aligned}\frac{dU(\alpha, \overline{I}(\alpha))}{d\alpha} =& - \overline{I}[pu'(w_1 + (1-\alpha)\overline{I}(\alpha)) + (1-p)u'(w_1 + w - \alpha\overline{I}(\alpha))] \\ &+ (p(1-\alpha)u'(w_1 + (1-\alpha)\overline{I}(\alpha)) - \alpha(1-p)u'(w_1 + w - \alpha\overline{I}(\alpha)))\frac{d\overline{I}(\alpha)}{d\alpha}\end{aligned}$$

Recall that $\overline{I}(\alpha) = \min\{\hat{I}(\alpha), w\}$. We first consider the case that $\overline{I}(\alpha) = w$ holds for certain range of α. In this case, $\frac{d\overline{I}(\alpha)}{d\alpha} = 0$, and it follows that $dU(\alpha, \overline{I}(\alpha))/d\alpha < 0$. We next consider the case $\overline{I}(\alpha) = \hat{I}(\alpha)$. Recall that we let $A = w_1 + (1-\alpha)\hat{I}$ and $B = w_1 + w - \alpha\hat{I}$. By the first consider-condition (8), we can substitute $u'(B) = \frac{p(1-\alpha)(1+\eta\lambda)}{\alpha(1-p)(1+\eta)}u'(A)$ into the above equation and obtain

$$\frac{dU(\alpha, \hat{I}(\alpha))}{d\alpha} = -\frac{pu'(A)}{\alpha(1+\eta)}\left[[\alpha(1+\eta) + (1-\alpha)(1+\eta\lambda)]\hat{I} + \alpha(1-\alpha)\eta(\lambda-1)\frac{d\hat{I}(\alpha)}{d\alpha}\right]$$

By use of Equation (A2), we have

$$\begin{aligned}\frac{dU(\alpha, \hat{I}(\alpha))}{d\alpha} &= -\frac{pu'(A)}{\alpha(1+\eta)}[[1+\eta\lambda-\alpha\eta(\lambda-1)]\hat{I} - \frac{\eta(\lambda-1)[1+\alpha(1-\alpha)\hat{I}(\gamma(B)-\gamma(A))]}{(1-\alpha)\gamma(A)+\alpha\gamma(B)}] \\ &= -\frac{pu'(A)}{\alpha(1+\eta)}\left[[\frac{(1+\eta\lambda)(1-\alpha)\gamma(A) + (1+\eta)\alpha\gamma(B)]\hat{I} - \eta(\lambda-1)}{(1-\alpha)\gamma(A)+\alpha\gamma(B)}\right] \qquad \text{(A6)}\end{aligned}$$

So $\frac{dU(\alpha,\hat{I}(\alpha))}{d\alpha} < 0$ if and only if $[(1+\eta\lambda)(1-\alpha)\gamma(A) + (1+\eta)\alpha\gamma(B)]\hat{I} - \eta(\lambda-1) > 0$. Note that

$$(1+\eta\lambda)(1-\alpha)\gamma(A) + (1+\eta)\alpha\gamma(B) \geq (1+\eta)\gamma_0 + \eta(\lambda-1)(1-\alpha)\gamma_0 > (1+\eta)\gamma_0.$$

When $\hat{I} \geq \frac{\eta(\lambda-1)}{(1+\eta)\gamma_0}$, it follows that

$$[(1+\eta\lambda)(1-\alpha)\gamma(A) + (1+\eta)\alpha\gamma(B)]\hat{I} - \eta(\lambda-1) > 0.$$

Therefore, we establish $\frac{dU(\alpha,\hat{I}(\alpha))}{d\alpha} < 0$. Putting two cases together, we obtain that when $\overline{I} \geq \min\{\frac{\eta(\lambda-1)}{(1+\eta)\gamma_0}, w\}$, $\frac{dU(\alpha,\overline{I}(\alpha))}{d\alpha} < 0$ holds.

(iii) Similarly, We can establish $\alpha^{**} = \max_{\alpha\in\mathcal{C}^A}\{\alpha\}$ once $dU_w(\alpha, \overline{I}_w(\alpha))/d\alpha < 0$ holds. Now we need to prove $dU_w(\alpha, \overline{I}_w(\alpha))/d\alpha < 0$ holds under the given condition.

$$\begin{aligned}\frac{dU_w(\alpha, \overline{I}_w(\alpha))}{d\alpha} =& -\overline{I}_w[pu'(w_1 + (1-\alpha)\overline{I}_w(\alpha)) + (1-p)u'(w_1 + w - \alpha\overline{I}_w(\alpha))] \\ &+ [p(1-\alpha)u'(w_1 + (1-\alpha)\overline{I}_w(\alpha)) + (1-p)\delta u'(w_2 + \overline{I}_w) \\ &- \alpha(1-p)u'(w_1 + w - \alpha\overline{I}_w(\alpha))]\frac{d\overline{I}_w(\alpha)}{d\alpha}\end{aligned}$$

Recall that $\overline{I}_w(\alpha) = \min\{\hat{I}_w(\alpha), w\}$. We first consider the case that $\overline{I}_w(\alpha) = w$ holds for certain range of α. In this case, $\frac{d\overline{I}_w(\alpha)}{d\alpha} = 0$, and it follows that $dU(\alpha, \overline{I}_w(\alpha))/d\alpha < 0$. We next focus on the case $\overline{I}_w(\alpha) = \hat{I}_w(\alpha)$. Recall that we let $C = w_1 + (1-\alpha)\hat{I}_w$ and $D = w_1 + w - \alpha\hat{I}_w$. Let $J = p(1-\alpha)u'(C) + (1-p)\delta u'(w_2 + I)$ and $\beta = \frac{p(1-\alpha)u'(C)}{J}$. By the first consider-condition (13), we can substitute $u'(D) = \frac{1+\eta\lambda}{\alpha(1-p)(1+\eta)}J$ into the above equation and obtain by rearranging terms

$$\frac{dU_w(\alpha, \hat{I}_w(\alpha))}{d\alpha} = -\hat{I}_w J\left[\frac{\beta}{1-\alpha} + \frac{1+\eta\lambda}{\alpha(1+\eta)}\right] - J\frac{\eta(\lambda-1)}{1+\eta}\frac{d\hat{I}_w(\alpha)}{d\alpha} \qquad \text{(A7)}$$

Rearranging Equation (A5) yields

$$\frac{d\hat{I}_w(\alpha)}{d\alpha} = -\frac{\beta[\frac{1}{1-\alpha} - \hat{I}_w\gamma(C)] + [\frac{1}{\alpha} + \hat{I}_w\gamma(D)]}{(1-\alpha)\beta\gamma(C) + (1-\beta)\gamma(w_2+I) + \alpha\gamma(D)} \tag{A8}$$

Substituting (A8) into (A7) yields

$$\frac{dU_w(\alpha, \hat{I}_w(\alpha))}{d\alpha} = -\ J\left[\frac{K}{(1-\alpha)\beta\gamma(C) + (1-\beta)\gamma(w_2+I) + \alpha\gamma(D)}\right], \tag{A9}$$

where

$$\begin{aligned}
K = &\left[\beta - 1 + \frac{1+\eta\lambda}{\alpha(1+\eta)}\right]\beta\hat{I}_w\gamma(C) + \left[\frac{\alpha\beta}{1-\alpha} + 1\right]\hat{I}_w\gamma(D) \\
&+ \left[\frac{\beta}{1-\alpha} + \frac{1+\eta\lambda}{\alpha(1+\eta)}\right]\hat{I}_w(1-\beta)\gamma(w_2+I) - \frac{\eta(\lambda-1)}{1+\eta}\left[\frac{\beta}{1-\alpha} + \frac{1}{\alpha}\right] \\
\geq &\left[\beta - 1 + \frac{1+\eta\lambda}{\alpha(1+\eta)}\right]\beta\hat{I}_w\gamma_1 + \left[\frac{\alpha\beta}{1-\alpha} + 1\right]\hat{I}_w\gamma_1 \\
&+ \left[\frac{\beta}{1-\alpha} + \frac{1+\eta\lambda}{\alpha(1+\eta)}\right]\hat{I}_w(1-\beta)\gamma_1 - \frac{\eta(\lambda-1)}{1+\eta}\left[\frac{\beta}{1-\alpha} + \frac{1}{\alpha}\right] \\
= &\frac{\alpha}{1-\alpha}\beta(1-\beta)\hat{I}_w\gamma_1 + \left[\frac{1+\eta\lambda}{\alpha(1+\eta)}\hat{I}_w\gamma_1 - \frac{\eta(\lambda-1)}{\alpha(1+\eta)}\right] \\
&+ \left[\frac{\alpha}{1-\alpha}\hat{I}_w\gamma_1 - \frac{\eta(\lambda-1)}{(1-\alpha)(1+\eta)}\right]\beta + \hat{I}_w\gamma_1 \\
> &\frac{\alpha}{1-\alpha}\beta(1-\beta)\hat{I}_w\gamma_1 + \left[\frac{(1+\eta\lambda)\hat{I}_w\gamma_1 - \eta(\lambda-1)}{\alpha(1+\eta)}\right] + \left[\frac{(1+\eta)\hat{I}_w\gamma_1 - \eta(\lambda-1)}{(1-\alpha)(1+\eta)}\right]\beta.
\end{aligned}$$

When $\hat{I}_w > \frac{\eta(\lambda-1)}{(1+\eta)\gamma_1}$, we obtain $(1+\eta\lambda)\hat{I}_w\gamma_1 - \eta(\lambda-1) > 0$ and $(1+\eta)\hat{I}_w\gamma_1 - \eta(\lambda-1) > 0$, and then $K > 0$. We thus establish $\frac{dU_w(\alpha,\hat{I}_w(\alpha))}{d\alpha} < 0$ from Equation (A9). Putting two cases together, we obtain that when $\overline{I}_w \geq \min\{\frac{\eta(\lambda-1)}{(1+\eta)\gamma_1}, w\}$, $\frac{dU_w(\alpha,\overline{I}_w(\alpha))}{d\alpha} < 0$ holds. ■

Proof of Proposition 4. We now prove that $\alpha^{**} < \alpha^*$. Given the premium rate $\alpha^* > p$, which is the optimal solution to consumers' optimization problem (15), agent selling induces consumers to purchase $\overline{I}(\alpha^*) > I^C(\alpha^*)$ by Proposition 2. Then $U(\alpha^*, \overline{I}(\alpha^*)) < U(\alpha^*, I^C(\alpha^*)) = \overline{U}$. That is, the (IR) constraint is no longer satisfied. When the premium rate satisfies $\alpha < \frac{1+\eta\lambda}{2\eta(\lambda-1)}$, to increase consumers' welfare, insurers have to decrease the premium rate since $dU(\alpha, \overline{I}(\alpha))/d\alpha < 0$ by Lemma 3. As a result, there must be the optimal premium rate $\alpha^{**} < \alpha^*$. If the premium rate $\alpha^* = p$, we obtain $\overline{I}(\alpha^*) = I^C(\alpha^*)$ by Proposition 2. Then $U(\alpha^*, \overline{I}(\alpha^*)) = U(\alpha^*, I^C(\alpha^*)) = \overline{U}$. That is, the (IR) constraint is still binding. That is, $\alpha^{**} = \alpha^*$. Combining these two results together yields that when $\alpha < \frac{1+\eta\lambda}{2\eta(\lambda-1)}$, $\alpha^{**} \leq \alpha^*$ and the inequality holds when $\alpha^* > p$. With analogue argument, we can obtain that when $\alpha < \frac{1+\eta\lambda}{1+\eta\lambda+\eta(\lambda-1)}$ $\alpha_w^{**} \leq \alpha_w^*$. When $w_2 + (p + \frac{1-p}{1+r})w > w_1$, by Lemma 1, we obtain $\overline{I}_w(\alpha) > I_w^C(\alpha)$ for any $\alpha \geq p + \frac{1-p}{1+r}$, and thus $\alpha_w^{**} < \alpha_w^*$ holds .

When $-\gamma'(x) \leq \frac{16}{w^2}$ for all $x \in [w_1, w_1 + w]$, by Lemma A.2, we obtain $\frac{\partial I^C}{\partial \alpha} < 0, \frac{\partial \overline{I}}{\partial \alpha} \leq 0$ and $\frac{\partial I^C_w}{\partial \alpha} < 0, \frac{\partial \overline{I}_w}{\partial \alpha} \leq 0$. We thus establish that $\overline{I}(\alpha^{**}) \geq I^C(\alpha^*)$ and $\overline{I}_w(\alpha^{**}_w) \geq I^C_w(\alpha^*_w)$ because $\alpha^{**} \leq \alpha^*$, $\alpha^{**}_w \leq \alpha^*_w$ from the above conclusions, and $\overline{I}(\alpha) \geq I^C(\alpha)$ by Propositions 2 and $\overline{I}_w(\alpha) \geq I^C_w(\alpha)$ by Proposition 3. Moreover, $\overline{I}(\alpha) > I^C(\alpha)$ holds when premium rate is not actuarially fair, and $\overline{I}_w(\alpha) > I^C_w(\alpha)$ holds when $w_2 + (p + \frac{1-p}{1+r})w > w_1$. ■

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