

Experimental Estimation of the Preference Parameters in Almost Stochastic Dominance

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Abstract

Almost stochastic dominance (ASD) as proposed by Leshno and Levy (2002) has been widely applied in decision theory and in practice. It can help to identify the preferred distribution for the majority of decision makers when a small violation in stochastic dominance is involved. The purpose of this paper is to experimentally define the preference parameters in the set of most decision makers. By adopting almost Nth-degree risk defined by Tsetlin et al. (2015), the parameters for economically relevant risk-averse and prudent decision makers are estimated.

Keywords: almost stochastic dominance, almost Nth-degree risk, experiments.

JEL classification: D81

1 Introduction

Since Rothschild and Stiglitz (1970), stochastic dominance (SD) has long served as a fundamental measurement of an increase in risk in the literature. However, as pointed out by Leshno and Levy (2002), the stochastic dominance rules fail to rank some distributions when the majority of decision makers have a clear preference. This drawback mainly comes from the fact that these rules are related to all decision makers in a given class which includes some extreme preferences.

To overcome the drawback, Leshno and Levy (2002) innovated the concept of almost SD (ASD) and have shown that ASD is the distribution ranking criterion for most decision makers, who are economically relevant. Following Leshno and Levy's (2002) seminal contribution, the literature has further explored ASD both theoretically and empirically. For theoretical studies, Tzeng et al. (2013) provided the correct necessary and sufficient conditions for almost second-degree SD (ASSD) and extended their results to almost N th-degree dominance. Tsetlin et al. (2015) generalized the findings of Tzeng et al. (2013) and established new conditions such that the newly-defined concept of ASD satisfies a hierarchy property, i.e., lower-degree dominance implies higher-degree dominance. They also defined almost N th-degree risk (ANR) which plays a role in ASD that is similar to the role of N th-degree risk proposed by Ekern (1980) for SD. Furthermore, Denuit et al. (2014a) and (2014b) respectively extended the univariate to bivariate ASD, and applied the concept of ASD to define almost expectation dependence.

In empirical studies, ASD has been demonstrated to be useful in explaining some puzzles in practice and has been applied to evaluate financial decisions. For example, Bali et al. (2009) indicated that ASD rules support the common practice which advises a higher stock to bond ratio for long investment horizons. Bali et al. (2013) showed that some hedge funds almost stochastically dominate the U.S. equity market and/or the U.S. Treasury market. Denuit et al. (2014c) further demonstrated that a high proportion of hedge funds are efficient portfolios according to the rule of almost marginal conditional SD, which is established based on the concept of ASD.

While applying the rules of ASD in the empirical studies, the values of the parameters in ASD, which characterize the set of most decision makers, are critical to concluding the

findings. Estimating the parameters is important since these numbers serve as a threshold to justify the preferred distributions. Thus, the purpose of this paper is to experimentally determine the parameters in ASD.

The study most related to our paper is that by Levy et al. (2010). Using experimental data, they estimated the critical parameter $\varepsilon_1 \in (0, 0.5)$ to be almost first-degree SD (AFSD). AFSD is the rule for all insatiable decision makers whose marginal utility $u^{(1)}$ is positive¹ and is not greater than $\inf\{u^{(1)}\} \times \left(\frac{1}{\varepsilon_1} - 1\right)$. The parameter ε_1 could be treated as the allowed violation ratio, which is the ratio of the area that violates first-degree SD (FSD) to the area between two cumulative density functions (CDFs), for all decision makers in this set. According to their estimation, the inferior-allowed ε_1 for all insatiable subjects is 5.9%. In addition, they further estimate the critical parameter ε_2 in their asserted ASSD, where ASSD is the distribution ranking rule for all insatiable and risk-averse decision makers whose $-u^{(2)}$ is not greater than $\inf\{-u^{(2)}\} \times \left(\frac{1}{\varepsilon_2} - 1\right)$.

Our paper complements the literature in three ways. First, ε_2 in Levy et al. (2010) is not valid since the experiments are settled on an alleged definition of ASSD rather than on an accurate definition as indicated by Tzeng et al. (2013). Tzeng et al. (2013) have demonstrated that, similar to ε_1 , the parameter ε_2 should relate the allowed violation ratio which violates second-degree SD (SSD) to the area between two cumulative CDFs. Thus, it is necessary in the literature to examine the correct critical value of ε_2 in ASSD.

Second, we propose experiments to estimate the parameter for most prudent decision makers which has never been estimated in the literature. Since Kimball (1990), prudence ($u^{(3)} > 0$) has become a well-known notion used to explain precautionary saving and many other important decisions.² Thus, the value of ε_3 in almost third-degree SD (ATSD) deserves to be further explored. As shown in Tzeng et al. (2013), ATSD is the rule for all insatiable, risk-averse and prudent decision makers whose $u^{(3)}$ is not greater than $\inf\{u^{(3)}\} \times \left(\frac{1}{\varepsilon_3} - 1\right)$. They pointed out that ε_3 corresponds to the allowed violation area which violates third-degree SD (TSD) .

¹ $u^{(N)}$ denotes the N th derivative of u .

²For example, complementing Kimball (1990), who found that risk-averse and prudent decision makers will devote more of their current wealth to saving in the presence of an independent background risk in the future than in the absence of it. Crainich et al. (2013) showed that risk lovers who are prudent also have precautionary saving. Eeckhoudt et al. (2012) and Courbage and Rey (2012) demonstrated that prudence is crucial for the precautionary effort decision.

Third, in the design of experiments, we control the equality of the moments among alternative choices when measuring the values of ε_2 and ε_3 . This idea is borrowed from that of Deck and Schlesinger (2010 and 2014) who provide laboratory experiments to examine the higher order properties of preferences proposed by Eeckhoudt and Schlesinger (2006) and Crainich et al. (2013). In other words, we adopt the distribution definitions of ANR, $N = 1, 2, 3$, proposed by Tsetlin et al. (2015). ANR is similar to the N th-degree risk proposed by Ekern (1980) in that when the N th order property of preferences is examined, the n th-order ($n < N$) properties of preferences does not matter since the first $N - 1$ moments of two distributions are equal. Thus, when measuring ε_2 , the means of two alternative choices are set to be the same. Furthermore, when measuring ε_3 , we carefully design the experiment so that both the mean and variance of two alternative choices are the same.

The subjects are students from the College of Management at National Taiwan University and the Graduate Institute of Finance at National Taiwan University of Science and Technology. In total, we have 223 participants. We find that the inferior of the estimated allowed ε_1 for all insatiable subjects is less than 5.27%. Our results further suggest that the inferior of the allowed ε_1 for 99% of the insatiable subjects is equal to 11.12%, and that for 95% of the insatiable subjects is 14.29%. For ε_2 , the findings show that the inferior of the allowed ε_2 for all risk-averse subjects is 2.20% and 95% of the risk-averse subjects have their ε_2 greater than 5.74%. These numbers substantially increase when the experiments are re-stated in a negative tone. This finding suggests that decision makers view the domain of gains and losses differently. For ε_3 , we show that the inferior of the allowed ε_3 for all prudent subjects is smaller than 0.67%, which is the inferior for 97% of the prudent subjects.

The remainder of this paper is organized as follows. Section 2 briefly reviews the definition of ASD. Section 3 describes the experimental design and the procedure adopted. Section 4 presents the experimental results and Section 5 concludes the paper. The questionnaire and other supporting materials are provided in the Appendices.

2 Almost Nth-degree Risk

Let $\tilde{x} \in [\underline{x}, \bar{x}]$ denote a random wealth, where \underline{x} and \bar{x} are constants. F and G represent CDFs of \tilde{x} . Define $F^{(N)}(x) = \int_{\underline{x}}^x F^{(N-1)}(t)dt$ for $N \geq 2$ with $F^{(1)}(x) = F(x)$, and define

$G^{(N)}(x)$ similarly. Also define

$$S_N(F, G) = \{x \in [\underline{x}, \bar{x}] | F^{(N)}(x) \geq G^{(N)}(x)\}, \quad N = 1, 2, 3. \quad (1)$$

In other words, $S_N(F, G)$ is the set of \tilde{x} which violates the rule of Nth-degree SD (NSD).

Let u denote a decision maker's utility function. The decision makers could be classed in the following sets:

$$V_N(\varepsilon_N) = \left\{ u \left| \begin{array}{l} (-1)^{N+1}u^{(N)} > 0, \text{ and} \\ \sup \{(-1)^{N+1}u^{(N)}(x)\} \leq \inf \{(-1)^{N+1}u^{(N)}(x)\} \left(\frac{1}{\varepsilon_N} - 1\right) \end{array} \right. \right\}, \quad (2)$$

where $\varepsilon_N \in (0, 0.5)$, $N = 1, 2, 3$. The condition

$$\sup \{(-1)^{N+1}u^{(N)}(x)\} \leq \inf \{(-1)^{N+1}u^{(N)}(x)\} \left(\frac{1}{\varepsilon_N} - 1\right) \quad (3)$$

places the restriction on the value of $u^{(N)}(x)$. Leshno and Levy (2002) argue that the preferences of most decision makers are not extreme such that the ratio of $\sup \{(-1)^{N+1}u^{(N)}(x)\}$ to $\inf \{(-1)^{N+1}u^{(N)}(x)\} \left(\frac{1}{\varepsilon_N} - 1\right)$ is limited. For each set of preferences, the number of decision makers increases when the corresponding ε_N decreases. For example, when ε_1 approaches 0, the set $U_1(\varepsilon_1)$ contains all decision makers with $u^{(1)} > 0$. When ε_1 approaches 0.5, only linear utility functions with positive slopes are included in $U_1(\varepsilon_1)$.

Tsetlin et al. (2015) define ANR, $N = 1, 2, 3$, as follows:

Definition 1 (Tsetlin et al., 2014) For $0 < \varepsilon_N < \frac{1}{2}$, $N = 1, 2, 3$, G has more ε_N -ANR than F if $F^{(n)}(\bar{x}) - G^{(n)}(\bar{x}) = 0$, $n = 1, \dots, N$, and

$$\int_{S_N(F, G)} [F^{(N)}(x) - G^{(N)}(x)] dx \leq \varepsilon_N \int_{\underline{x}}^{\bar{x}} |F^{(N)}(x) - G^{(N)}(x)| dx. \quad (4)$$

They also show the corresponding set of decision makers for each of the distribution ranking criteria as follows:

Theorem 1 (Tsetlin et al., 2014) G has more ε_N -ANR than F if and only if $E_F(u) \geq$

$E_G(u)$ for all $u \in V_N(\varepsilon_N)$, $N = 1, 2, 3$, where

$$V_N(\varepsilon_N) = \left\{ u \left| \begin{array}{l} (-1)^{N+1}u^{(N)} > 0, \text{ and} \\ \sup \{(-1)^{N+1}u^{(N)}(x)\} \leq \inf \{(-1)^{N+1}u^{(N)}(x)\} \left(\frac{1}{\varepsilon_N} - 1\right) \end{array} \right. \right\}. \quad (5)$$

In this paper, ε_N in $V_N(\varepsilon_N)$ is estimated, $N = 1, 2, 3$.

From the integration condition in Definition 1, we could define the *actual* area violation for any given distributions F and G as

$$\hat{\varepsilon}_N = \frac{\int_{S_N(F,G)} [F^{(N)}(x) - G^{(N)}(x)] dx}{\int_{\underline{x}}^{\bar{x}} |F^{(N)}(x) - G^{(N)}(x)| dx}, \quad N = 1, 2, 3. \quad (6)$$

As pointed out by Levy et al. (2010), this ε_N is objective and could be obtained directly from the distributions.

Since decision makers have heterogenous preferences in reality, the value of

$$\sup \{(-1)^{N+1}u^{(N)}(x)\} / \inf \{(-1)^{N+1}u^{(N)}(x)\}$$

for each decision maker is different from that of the others. Thus, to estimate ε_N in the set of $V_N(\varepsilon_N)$, we calculate the *inferior* of the value of the *allowed* area violation among all participants and denote it as ε_N^* . Thus, Theorem 1 could be modified as follows:

Theorem 2 For ε_N and ε_N^* in $(0, \frac{1}{2})$, $E_F(u) \geq E_G(u)$ for all $u \in V_N(\varepsilon_N^*)$, $N = 1, 2, 3$, if and only if

$$\hat{\varepsilon}_N \leq \varepsilon_N^* \text{ and } F^{(n)}(\bar{x}) = G^{(n)}(\bar{x}), \quad n = 1, \dots, N.$$

In our experiments, we design different pairs of F and G to obtain several $\hat{\varepsilon}_N$. ε_N^* is estimated such that the decision is consistent with the prediction of Theorem 2.

3 Experimental Design and Procedure

The experiments were conducted at National Taiwan University (NTU) and National Taiwan University of Science and Technology (NTUST). The subjects included undergraduate and graduate students from the College of Management in NTU and graduate students from the

Graduate Institute of Finance in NTUST. In total, we had 194 participants from NTU and 29 from NTUST.

Subjects were presented with a series of decision tasks in which they revealed their risk preferences. The experiment comprised four stages, and each stage contained 10 decision tasks. Each decision task was performed under independent scenarios. Subjects were presented with detailed descriptions of decision tasks, including verbal illustrations, payoff distribution graphs, a probability distribution table, and numerical examples.³ In every decision task, a subject chosen his/her preferred lottery option from two lottery options A and B. Lottery options A and B were both risky events with payoffs determined by the outcomes from tossing a coin.

After participants made all 40 decisions, one out of 40 decision tasks, with equal probabilities, was selected randomly. Thereafter, in this selected decision task, option A or B which had been chosen by the subject was implemented. That is, the subject tossed the coin and was paid in cash for the achieved lottery payoff. Under this experimental structure, because each task was equally likely to determine the subjects' actual payoffs, subjects were expected and instructed to regard every task as a real case and to decide carefully. The average cash payment made to 223 subjects was NTD\$293.⁴

The objective of the first stage was to estimate ε_1^* . In the first stage, we followed the structure of the experiment proposed by Levy et al. (2010). However, unlike the hypothetical scenarios in Levy et al. (2010), in our experiment, all subjects were actually paid in cash and their decisions directly determined the payment they received.

The objective of the second and third stages was to measure the allowed area violation ε_2^* . In these two stages, the experiment was carefully controlled by setting the same mean of the options A and B. Thus, the preference parameter ε_1^* did not play a role in making decisions. The structures of the second and third stages were basically the same, but were presented with positive and negative variable outcomes, respectively. In other words, these results could help analyze whether decision makers viewed the domain of gains and losses differently.

The objective of the fourth stage was to measure the allowed area violation ε_3^* . In the

³Please see the Appendix for the experiment's instructions.

⁴The exchange rate was approximately NTD\$30 to US\$1. All payoffs stated in the experiment were in terms of NTD.

fourth stage, the experiment was carefully controlled by setting the same mean and the same variance of options A and B so that the decisions made by the subjects were not affected by the preference parameters ε_1^* and ε_2^* , but were indeed affected by the sign of $u^{(3)}$ and the preference parameter ε_3^* .

3.1 First Stage: Measuring ε_1^*

In the first stage, by following the decision scenario proposed by Levy et al. (2010), we designed 10 similar tasks to measure the allowed area violation. The payoff was determined by tossing a coin. The payoff distribution of tasks $\#i$, $i = 1, 2, \dots, 10$, was as follows.

Option A : You will receive NT\$100 if the coin lands on Heads and NT\$200 otherwise;

Option B : You will receive NT\$50 if the coin lands on Heads and NT\$ x_i otherwise;

where x_i , $i = 1, 2, \dots, 10$, is respectively equal to 150, 250, 300, 400, 500, 600, 700, 800, 1000 and 1100.

Levy et al. (2010) asked subjects for what minimal value of x_i they would prefer B over A , whereas we set a range from 150 to 1100 and observed at which point subjects would switch their preference from A to B . Since the actual payoff is directly determined by the payoff in our experiment, by setting a range for x_i we could avoid rent seeking behavior.

In decision task $\#1$, it is obvious that A dominates B via FSD. Therefore, all subjects with $u^{(1)} > 0$ would choose A over B . The choice in decision task 1 is designed simply to check whether the subjects violated $u^{(1)} > 0$. From tasks $\#2$ to $\#10$, the actual area violation such that subjects would prefer B to A is

$$\varepsilon_1 = \frac{50}{50 + (x_i - 200)} \quad (7)$$

for task $\#i$.

Furthermore, if a subject chose B in task $\#j$, this subject should also have chosen B in any task $\#i$, $2 \leq i < j$, since option B in task $\#j$ dominates option B in any task $\#i$, $2 \leq i < j$ based on FSD. Hence, the subjects are categorized as being consistent if they either chose A for all 10 tasks in the first stage or chose A from task $\#1$ and then switched to B at any task. Once they switched to B , they would choose B for all the following tasks

in this stage. We only include the data for which the subjects behaved consistently.

To obtain ε_1^* , we first assumed that $\varepsilon_1^* \geq 5.27\%$, which was obtained according to the actual area violation in task #10. Theorem 2 predicts that all subjects should prefer option B to A in task #10.⁵ If this is not true, then we can only suggest that $\varepsilon_1^* < 5.27\%$. If it is true, then we increase the assumed ε_1^* to 5.89%, which is obtained from the actual area violation in task #9. When $\varepsilon_1^* = 5.89\%$, Theorem 2 indicates that all subjects should prefer option B to A in both tasks #9 and #10. If this is not true, then the suggested ε_1^* is 5.27%. However, if it is true, then we repeat the above procedure by increasing the assumed ε_1^* .

Furthermore, to provide the threshold of ε_1^* for empirical studies, we defined $\varepsilon_1^*(p\%)$ as the preference parameter so that $p\%$ of the subjects had their ε_1 greater than $\varepsilon_1^*(p\%)$. For example, if $p\%$ of all consistent subjects preferred B to A for task #10 and $1 - p\%$ of the subjects preferred option A to B , then $\varepsilon_1^*(p\%) = 5.27\%$.

3.2 Second Stage: Measuring ε_2^* with positive expected variable payoffs

In the second stage, we designed the experiment to measure ε_2^* when positive variable payoffs were presented. For each task in stage 2, subjects received an endowment of NT\$150 plus the variable payoff from Lottery option A or B , depending on their decision, where the expected values of A and B were the same. The payoff distribution of tasks # i , $i = 11, 12, \dots, 20$, is shown as Figure 1. Option A is a compound lottery. To avoid the effect of aversion to compound lotteries on the decisions,⁶ the payoff of the corresponding reduced lottery of option A was also stated in the experiment.

In tasks #11 and #12, option A dominates B by SSD. In task #13, the actual area violation such that A is preferred to B is 25%. From tasks #14 to #19, the actual area violations such that B is preferred to A are respectively 47.06%, 20.00%, 11.59%, 5.73%, 2.20%, 0.47%. Finally, B dominates A by SSD in task #20.

To estimate ε_2^* , we first select risk-averse subjects with consistent behavior. According to the prediction of SD, the subjects who choose A in tasks #2, #11 and #12, and choose

⁵Note that if ε_1^* is exactly the same as the actual area violation in task #10, then the ASD rules predict that there may exist some individuals with $u' > 0$ who are indifferent between A and B in task #10. To simplify the analysis, we set the assumed ε_1^* by taking the ceiling of the actual area violation obtained from the tasks to the fourth decimal place. The estimated ε_2^* and ε_3^* are set similarly.

⁶The interested reader should refer to Camerer and Ho (1994) for a nice survey on violations of the reduction in the compound lotteries principle.

B in task #20 are risk-averse. In addition, option A in task # i dominates option A in task # j , for $i < j$, by SSD. Thus, for all risk-averse subjects with consistent behavior, if the option B is chosen for task # i , then option A in task # j , $i < j$, will not be chosen in the second stage. In other words, the risk-averse subjects will only switch their choice once in the second stage.

To find out the *inferior* of the allowed area violation for all economically relevant risk-averse decision makers, we first assume that $\varepsilon_2^* \geq 0.47\%$, which is obtained from task #19. If $\varepsilon_2^* \geq 0.47\%$, then all screened subjects would prefer option B in task #19 based on Theorem 2. If only $p\%$ of the screened subjects choose option B and $1 - p\%$ choose option A in task #19, we then define $\varepsilon_2^*(p\%) = 0.47\%$ which indicates that $p\%$ of the risk-averse subjects have an allowed area violation that is smaller than 0.47%. On the other hand, if all screened subjects prefer option B in task #19, then we suggest that $\varepsilon_2^* \geq 2.20\%$ and repeat the process. Panel A in Table 1 shows the correct choices from tasks #11 to #20 corresponding to the assumed ε_2^* .

3.3 Third Stage: Measuring ε_2^* with negative expected variable payoffs

In the third stage, the set-up is basically the same as in the second stage. The payoff from tasks #21 to #30 is respectively the same as that from tasks #11 to #20. The only difference between these two stages is that the expected variable payoffs are positive in the second stage, but negative in the third stage. In other words, the description of options in the third stage is made in terms of losses, whereas that in the second stage is made in terms of gains.

For each task in the third stage, subjects receive an endowment of NT\$400 plus the variable payoff from Lottery option A or B . Again, the expected values of A and B are the same. The payoff distribution of tasks # i , $i = 21, 22, \dots, 30$, is shown as Figure 2. The measuring process of actual and allowed area violation in the third stage is the same as that in the second stage.

3.4 Fourth Stage: Measuring ε_3^*

In the fourth stage, we designed the experiment to measure ε_3^* and $\varepsilon_3^*(p\%)$. For each task in this stage, subjects received an endowment of NT\$100 plus the variable payoff from Lottery option A or B , where the expected values and the variances of A and B were the same.

The payoff distribution of tasks $\#i$, $i = 31, 12, \dots, 40$, is shown as Figure 3. Again, the corresponding reduced lotteries of options A and B are stated in the experiment.

In task $\#31$, option B dominates A by TSD, whereas option A dominates B by TSD in task $\#40$. For tasks $\#32$ and $\#33$, the actual area violations such that B is preferred to A are respectively 1.53% and 30.18%. From tasks $\#34$ to $\#39$, the actual area violations such that A is preferred to B are respectively 44.30%, 18.86%, 11.53%, 6.22%, 2.71% and 0.66%.

Similar to the procedure in stage two, we first select prudent subjects, i.e., the subjects choose option B in task $\#31$ and option A in task $\#40$. To find out the inferior allowed area violation for all prudent decision makers, we examine whether $\varepsilon_3^* \geq 0.67\%$, which is the smallest $\hat{\varepsilon}_3$ in this part, by checking the choices from tasks $\#32$ to $\#39$.

When $\varepsilon_3^* \geq 0.67\%$, all screened subjects would prefer option A in task $\#39$. If only $p\%$ of the screened subjects choose option A and $1 - p\%$ choose option B in task $\#39$, we define $\varepsilon_3^*(p\%) = 0.67\%$ which indicates that $p\%$ of the prudent subjects have an allowed area violation that is smaller than 0.67%. On the other hand, if the prediction is confirmed, then we suggest that $\varepsilon_3^* \geq 0.67\%$ and repeat the process by assuming a larger value of ε_3^* . Panel B in Table 1 shows the correct choices from tasks $\#31$ to $\#40$ according to the prediction of Theorem 2.

4 Experimental Results

4.1 The Estimated ε_1^*

204 out of a total of 223 subjects (91.48%) exhibit consistent behavior in the first stage. Among those subjects who made inconsistent choices, two subjects (0.90%) chose B in task $\#1$ and 17 subjects (7.62%) switched their decision more than once. The inconsistent proportion 8.52% is close to the result of Levy et al. (2010) whose sample exhibits 8% inconsistent choices.

Table 2 shows the choices of all the participants exhibiting consistent behavior in the first stage. The first column is the estimated ε_1^* which is calculated from the task shown by the second column. The third and the fourth columns respectively indicate the number and the percentage of participants whose choices are consistent with the case where their allowed area violation is greater than the corresponding ε_1^* .

Table 2 indicates that one subject always chooses option A . Thus, ε_1^* is smaller than 5.27%. Our estimation is smaller than the one in Levy et al. (2010), which is 5.9%. Table 2 further suggests that $\varepsilon_1^*(99\%)$ is greater than 11.12%, $\varepsilon_1^*(95\%)$ is about 14.29% and $\varepsilon_1^*(90\%)$ is between 14.29% and 20.00%.

4.2 The Estimated ε_2^*

We first include all the risk-averse subjects with consistent behavior in the second stage. Among the 223 subjects, we exclude the subjects who choose option B in tasks #11, #12 and option A in task #20, which are dominated options in terms of SSD, and the subjects who switch their choices more than once in the second stage.⁷ After this process, 68 risk-averse subjects remain.⁸

Table 3 shows the choices of all the risk-averse participants exhibiting consistent behavior in the second stage. Each column is defined similarly to the corresponding columns in Table 2. The first column is the estimated ε_2^* which is basically obtained from the task shown in the second column. The third and the fourth columns respectively indicate the number and the percentage of participants whose choices are consistent with the case where their allowed area violation is greater than the corresponding ε_2^* .

Table 3 shows that ε_2^* is greater than 2.20%. Our estimation is smaller than the one in Levy et al. (2010), which is 3.2%. Note that the actual area violation $\hat{\varepsilon}_2$ in Levy et al. (2010) is calculated according to the definition provided by Leshno and Levy (2002), whereas we adopt the definition in Tsetlin et al. (2015). Thus, our estimation of ε_2^* is different from that in Levy et al. (2010). In addition, Table 3 further suggests that $\varepsilon_2^*(99\%)$ is smaller than 5.74%, which is equal to $\varepsilon_2^*(95\%)$, and $\varepsilon_2^*(90\%)$ is between 11.60% and 20.00%.

Furthermore, if preferences are consistent with the prediction under the expected utility framework, then whether the payoff is stated by a positive or negative tone will not have any impact on the decision. Thus, we further screen these 68 subjects according to their choices in the third stage. In other words, we include the subjects who choose the dominating options according to the SSD rule in the third stage, i.e., option A in tasks #21 and #22

⁷We first respectively delete 85 and 19 subjects who choose option B in #11 and #12. After that, 44 are deleted since they choose option A in #20. Finally, we delete 7 subjects who switch their choices more than once.

⁸There are only 41 subjects who could be identified as risk lovers. The remaining 114 subjects do not have globally concave or convex utility functions.

and option B in task #30. Among the 68 subjects, 25 are deleted in this screening process. 3 out of the remaining 43 subjects switched their choices more than once in the third stage. Thus, in considering the choices in the second and the third stages, we have 40 risk-averse subjects.

Table 4 shows that ε_2^* is obtained from the second and the third stages according to the choices among the 40 subjects. Panel A in Table 4 indicates that ε_2^* is 2.20%, which is the same as for the estimation based on the 68 subjects, when the statement of the variable payment is stated by a positive tone. However, when the statement of the variable payment is stated by a negative tone, then ε_2^* is 11.60% as indicated by Panel B in Table 4. The possible explanations include prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), which indicates that individuals are risk-averse over gains and risk loving over losses, and the framing effect (Tversky and Kahneman, 1981), which states that different choices are made when the description of options is provided in terms of gains (a positive frame) rather than losses (a negative frame).

4.3 The Estimated ε_3^*

In the fourth stage, we include all the prudent subjects. Since estimating prudent subjects alone does not require the information regarding the signs of $u^{(1)}$ and $u^{(2)}$, among the 223 subjects, we exclude the subjects who choose option A in task #31 and option B in task #40, which are dominated options in terms of 3rd-degree risk as defined by Ekern (1980). After this process, 106 prudent subjects remain. We find that there are only 27 subjects who could be identified as imprudent. The remaining 90 subjects do not have globally concave or convex marginal utility functions. Note that, in our sample, the number of prudent subjects is greater than the number of risk-averse subjects. This finding may imply that some risk lovers are also prudent as argued by Crainich et al. (2013).

The results are shown in Table 5. Since there are three subjects who always choose option A in this stage, we cannot suggest the value of ε_3^* for all prudent subjects. Table 5 suggests that ε_3^* is less than 0.67%. It further indicates that $\varepsilon_3^*(97\%)$ is around 0.67%.

5 Concluding Remarks

This paper experimentally analyzes the sets of preferences that are considered as economically relevant. While estimating the higher-order preference parameters, we follow the conditions of ANR, proposed by Tsetlin et al. (2015), to carefully design the experiments such that the lower-order parameters do not affect the results. We find that the parameter for all insatiable subjects is less than 5.27%, while it is 14.29% for 95% of the subjects. The corresponding ε_2^* for almost second-degree risk is 2.20% and 95% of the risk-averse subjects have a value of ε_2^* greater than 5.74%. For almost third-degree risk, we show that the inferior of the allowed ε_3^* is less than 0.67% across all subjects and $\varepsilon_3^*(97\%)$ is around 0.67%.

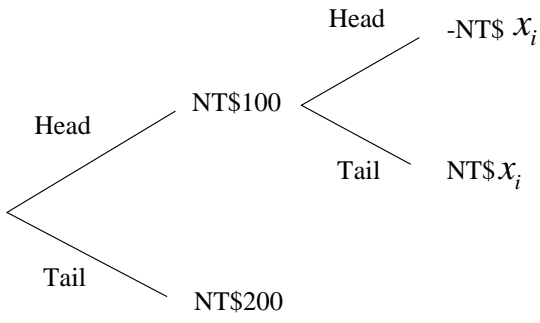
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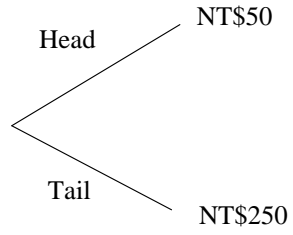
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Endowment: NT\$150

Option A:



Option B:

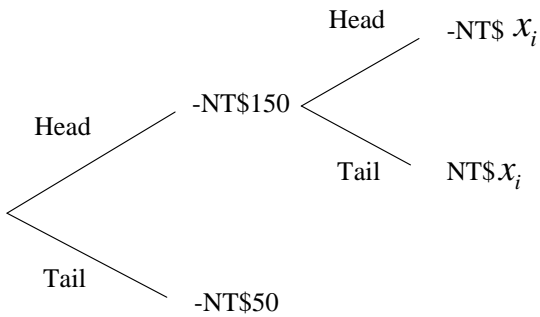


$x_i = 25, 50, 100, 125, 150, 160, 170, 180, 190, 200$ for $i = 11$ to 20 .

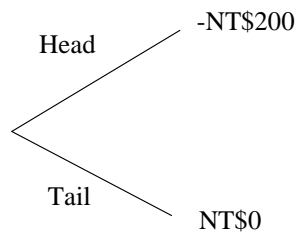
Figure 1: Payoffs of options A and B in the second stage.

Endowment: NT\$400

Option A:



Option B:

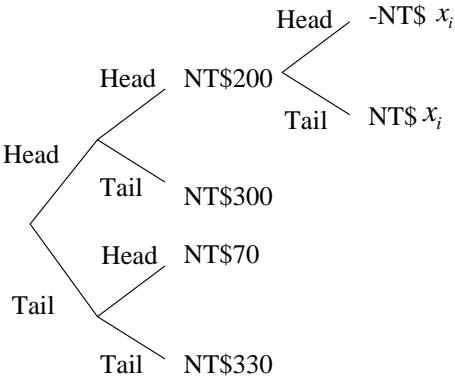


$x_i = 25, 50, 100, 125, 150, 160, 170, 180, 190, 200$ for $i = 21$ to 30 .

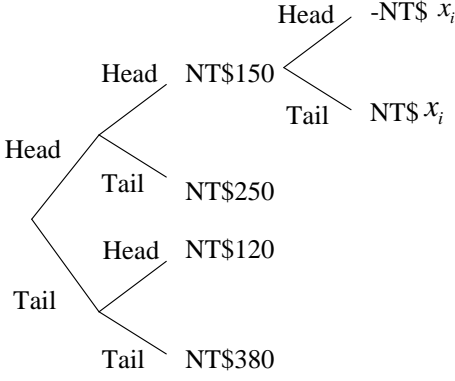
Figure 2: Payoffs of options A and B in the third stage.

Endowment: NT\$100

Option A:



Option B:



$x_i = 50, 100, 150, 175, 200, 210, 220, 230, 240, 250$ for $i = 31$ to 40 .

Figure 3: Payoffs of options A and B in the fourth stage.

Appendix A Experiment Instruction

Thank you very much for participating in this decision experiment!

In the following experiment, you will make a couple of decisions. By following the instructions and depending on your decisions, you can earn money. It is therefore very important that you understand how your decisions affect your payoff. If you have questions at any point, please let the experimenter know and someone will assist you. Otherwise, please do not talk during the experiment and please turn off your cell phone.

During the experiment **all amounts are stated in New Taiwan dollars (TWD)**. At the end of the experiment, your achieved payoff will be paid to you in cash.

Structure of the Experiment

The experiment is divided into four stages. The four stages comprise 40 individual decisions in total. These 40 decisions are independent and therefore unrelated to each other. You should make all 40 decisions.

In each and every decision, you will be given two options: Option A and Option B. Options A and B are both risky events with payoffs determined by the outcomes from tossing a coin. You will decide which of two options you prefer. The form of the options will be described when the stages are explained in-depth.

After you have made all your decisions, one of your 40 decisions from the four stages will be randomly selected to determine which decision is to be implemented. For this, you will draw one out of 40 cards, labeled with numbers from 1 to 40. Every number occurs only once so that the drawing of any particular number is equally likely.

Afterwards, based on Option A or B of your choice in that randomly selected decision, you will toss the coin to determine your payoff in the experiment.

To sum up, the experiment proceeds as follows:

1. **Make all 40 decisions**
2. **Draw the card to determine which decision is to be implemented**
3. **Toss the coin to determine your payoff, based on Option A or B of your choice in the randomly selected decision**

Please note:

- I. Only one of your 40 decisions will determine your payoff in the experiment and each of your 40 decisions can determine your entire payoff in the experiment, so please regard each decision as your final payoff decision and decide carefully.
- II. The experiment documents include the Experiment Instruction and Experiment Decision sheets. Please read the Experiment Instruction and listen to the experimenter carefully, and then record decisions on the Experiment Decision sheets.
- III. At the beginning of each of the four stages, the experimenter will explain the form for each stage. Start to make the decision only after the experimenter finishes explaining and tells you to start. Please make sure you fully understand the experiment before making any decision. If there are any questions, please raise your hand and the experimenter will assist you.
- IV. Please do not proceed to the next stage until the experimenter announces the start of the next stage.
- V. Payoffs may comprise an endowment for sure and a variable payoff from risky events. The variable payoff from risky events may be negative.

Coin Toss Outcomes

Variable payoffs for all decisions are determined by tossing the coin. Multiple coin tosses may be needed. The coin is a two-sided fair coin. Both sides of the outcomes

are equally likely, both with 50% probability. During the experiment, outcomes 'H' and 'T' are used to indicate two outcomes:

1. Outcome 'H': head of the coin
2. Outcome 'T': tail of the coin

Fill in Background Information

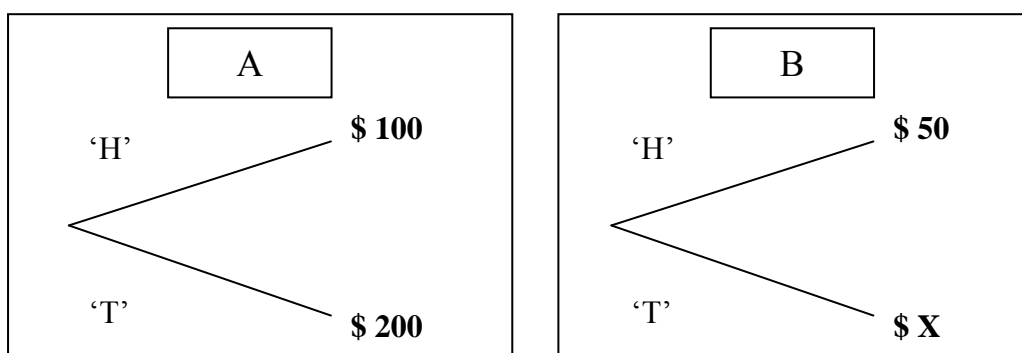
Please fill in the background information on the Experiment Decision sheets prior to the experiment.

I. First Stage

The payoff is determined by the coin toss outcome 'H' or 'T'. The payoffs of Options A and B are addressed as follows:

- Option A:
 1. If the coin toss outcome is 'H', the payoff will be **\$100**;
 2. If the coin toss outcome is 'T', the payoff will be **\$200**.
- Option B:
 1. If the coin toss outcome is 'H', the payoff will be **\$50**;
 2. If the coin toss outcome is 'T', the payoff will be **\$X**.

The following graphs and tables show the payoff distribution:



Option A		Option B	
Probability	Payoff	Probability	Payoff
50%	\$100	50%	\$50
50%	\$200	50%	\$X

For example:

1. *Example 1: when $X=300$, and the coin toss is 'T', then*
 - *Payoff of Option A is \$200*
 - *Payoff of Option B is \$300*
2. *Example 2: when $X=400$, and the coin toss is 'H', then*
 - *Payoff of Option A is \$100*
 - *Payoff of Option B is \$50*

II. Second Stage

You receive an endowment of \$150 plus an uncertain variable payoff. The variable payoff is determined by the coin toss outcomes. Tossing the coin one or two times may be needed. The payoffs of Options A and B are explained as follows:

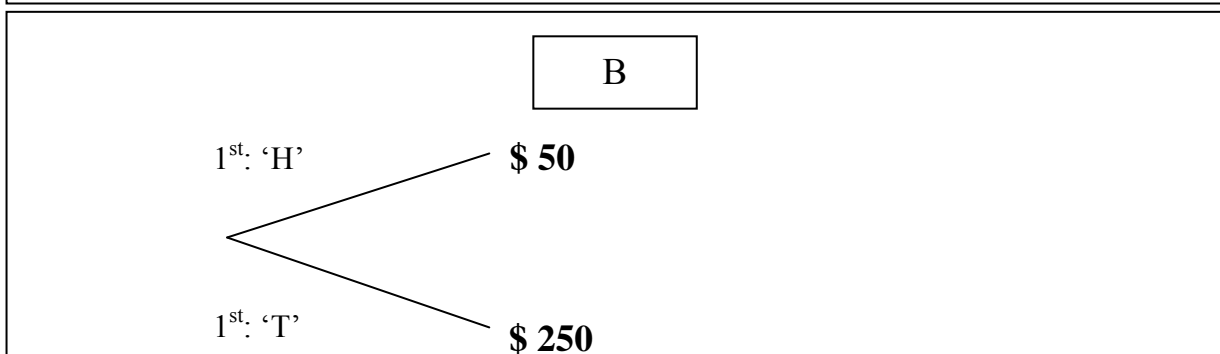
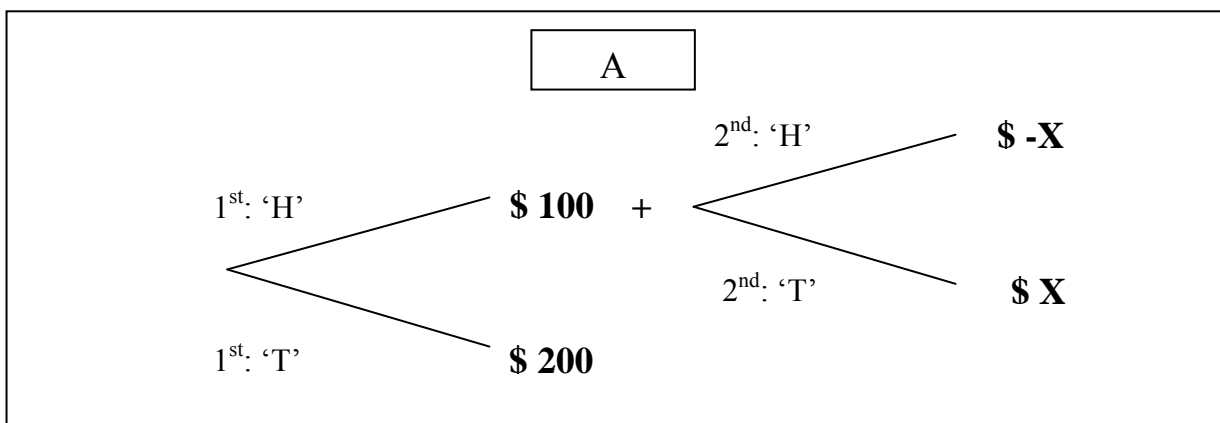
- Option A:

- If the first coin toss outcome is 'H', then a second coin toss is required:
 - If the second coin toss outcome is 'H', then the variable payoff will be **\$100-X** (*payoff U*);
 - If the second coin toss outcome is 'T', then the variable payoff will be **\$100+X** (*payoff D*).
- If the first coin toss outcome is 'T', then the variable payoff will be **\$200**.

- Option B:

- If the first coin toss outcome is 'H', then the variable payoff will be **\$50**;
- If the first coin toss outcome is 'T', then the variable payoff will be **\$250**.

The following graphs and tables show the variable payoff distribution:



Option A		Option B	
Probability	Variable Payoff	Probability	Variable Payoff
25%	$\$100-X (U)$	50%	\$50
25%	$\$100+X (D)$		
50%	\$200	50%	\$250

For example:

1. When $X=100$, if the first coin toss is 'H', the second coin toss is required. If the second toss is also 'H', then:

- *payoff of Option A = endowment + variable payoff*
 $= \$150 + (100 - 100) = \$150.$

- *payoff of Option B = endowment + variable payoff*
 $= \$150 + 50 = \$200.$

2. When $X=200$, if the first coin toss is 'T', then:

- *payoff of Option A = endowment + variable payoff*
 $= \$150 + 200 = \$350.$

- *payoff of Option B = endowment + variable payoff*
 $= \$150 + 250 = \$400.$

III. Third Stage

You receive an endowment of \$400 plus an uncertain variable payoff. The variable payoff is determined by the coin toss outcomes. Tossing the coin one or two times may be needed. The payoffs of Options A and B are explained as follows:

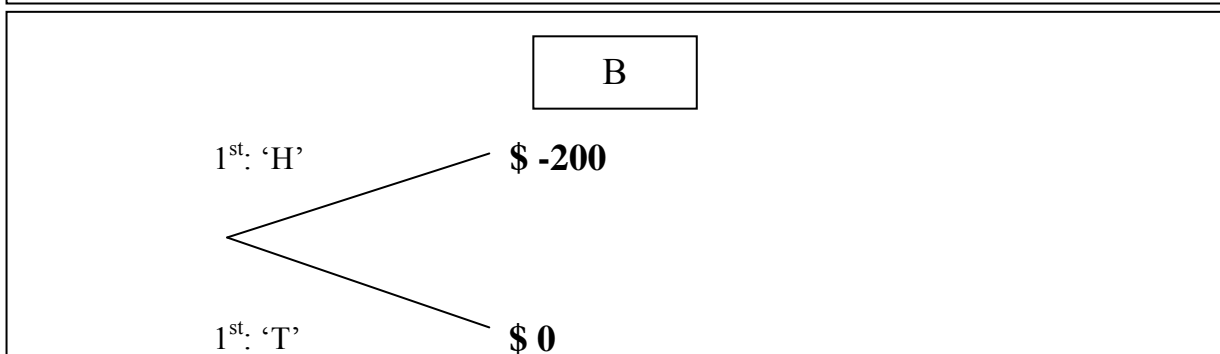
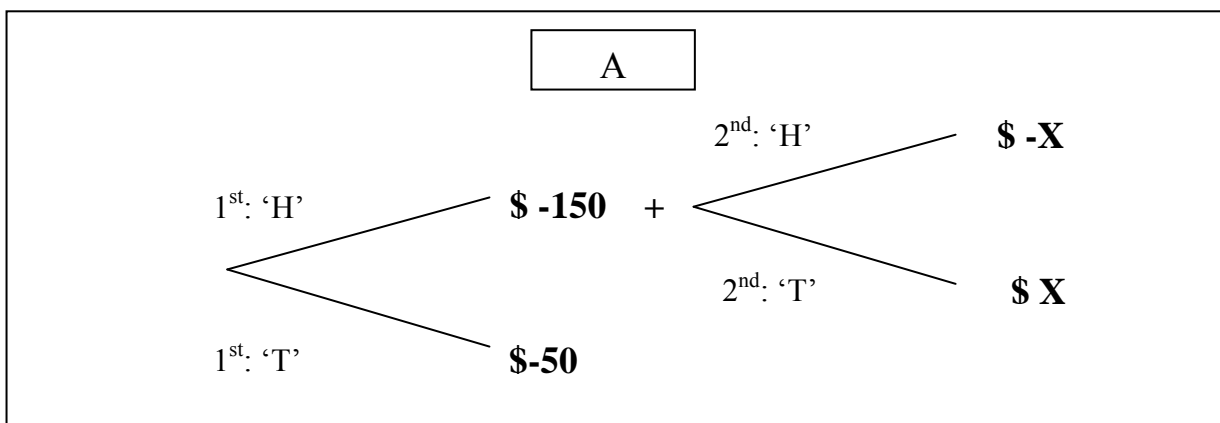
- Option A:

1. If the first coin toss outcome is 'H', then the second coin toss is required:
 - I. If the second coin toss outcome is 'H', then the variable payoff will be **\$-150-X** (payoff U);
 - II. If the second coin toss outcome is 'T', then the variable payoff will be **\$-150+X** (payoff D).
2. If the first coin toss outcome is 'T', then the variable payoff will be **\$-50**.

- Option B:

1. If the first coin toss outcome is 'H', then the variable payoff will be **\$-200**;
2. If the first coin toss outcome is 'T', then the variable payoff will be **\$0**.

The following graphs and tables show the variable payoff distribution:



Option A		Option B	
Probability	Variable Payoff	Probability	Variable Payoff
25%	$\$-150-X (U)$	50%	\$-200
25%	$\$-150+X (D)$		
50%	\$-50	50%	\$0

For example:

1. When $X=100$, if the first coin toss is 'H', the second coin toss is required. If the second toss is also 'H', then:

- *payoff of Option A = endowment + variable payoff*
 $= \$400 + (-150 - 100) = \$150.$
- *payoff of Option B = endowment + variable payoff*
 $= \$400 - 200 = \$200.$

2. When $X=200$, if the first coin toss is 'T', then:

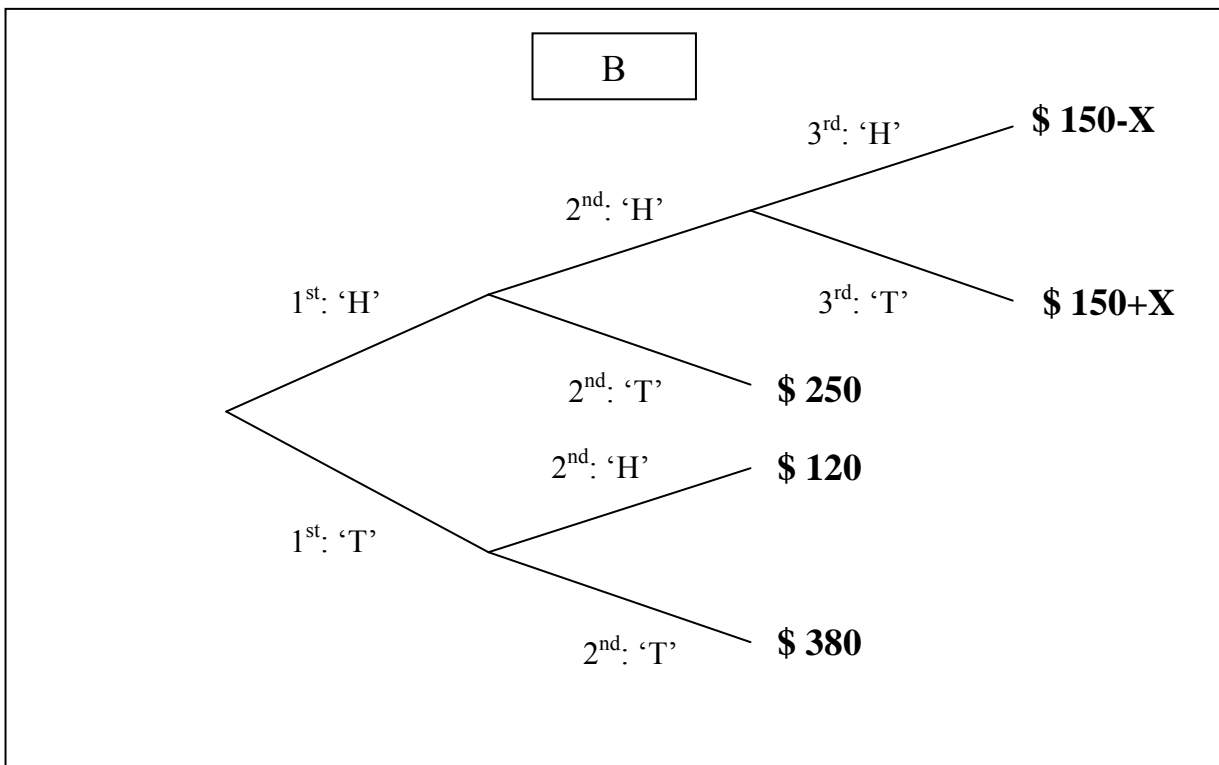
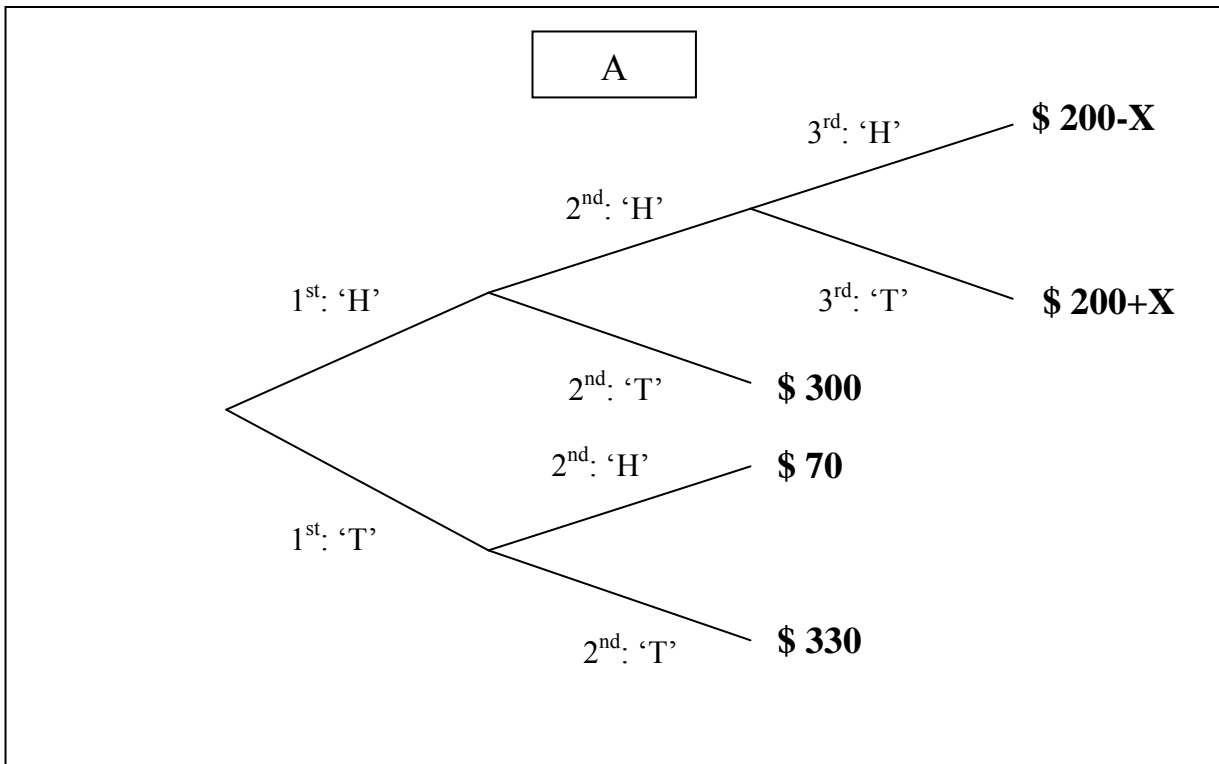
- *payoff of Option A = endowment + variable payoff*
 $= \$400 - 50 = \$350.$
- *payoff of Option B = endowment + variable payoff*
 $= \$400 + 0 = \$400.$

IV. Fourth Stage

You receive an endowment of \$100 plus an uncertain variable payoff. The variable payoff is determined by the coin toss outcomes. Tossing the coin two or three times may be needed. The payoffs of Options A and B are explained as follows:

- Option A:
 1. If the first coin toss is 'H', and:
 - I. If the second coin toss is 'H', then the second coin toss is required:
 - i. If the third coin toss is 'H', then the variable payoff will be **\$200-X**
(*payoff U1*);
 - ii. If the third coin toss is 'T', then the variable payoff will be **\$200+X**
(*payoff D1*).
 - II. If the second coin toss is 'T', then the variable payoff will be **\$300**.
 2. If the first coin toss is 'T', and:
 - I. If the second coin toss is 'T', then the variable payoff will be **\$70**;
 - II. If the second coin toss is 'H', then the variable payoff will be **\$330**.
- Option B:
 1. If the first coin toss is 'H', and:
 - I. If the second coin toss is 'H', then the second coin toss is required:
 - i. If the third coin toss is 'H', then the variable payoff will be **\$150-X**
(*payoff U2*);
 - ii. If the third coin toss is 'T', then the variable payoff will be **\$150+X**
(*payoff D2*).
 - II. If the second coin toss is 'T', then the variable payoff will be **\$250**.
 2. If the first coin toss is 'T', and:
 - I. If the second coin toss is 'T', then the variable payoff will be **\$120**;
 - II. If the second coin toss is 'H', then the variable payoff will be **\$380**.

The following graphs and tables show the variable payoff distribution:



Option A		Option B	
Probability	Variable Payoff	Probability	Variable Payoff
12.5%	\$200-X (<i>U1</i>)	12.5%	\$150-X (<i>U2</i>)
12.5%	\$200+X (<i>D1</i>)	12.5%	\$150+X (<i>D2</i>)
25%	\$300	25%	\$250
25%	\$70	25%	\$120
25%	\$330	25%	\$380

For example:

1. When $X=100$, if the first coin toss is 'H', and the second coin toss is also 'H',

then the third coin toss is required. If the third toss is 'T', then:

- *payoff of Option A = endowment + variable payoff*
 $= \$100 + (200 + 100) = \$400.$
- *payoff of Option B = endowment + variable payoff*
 $= \$100 + (150 + 100) = \$350.$

2. When $X=200$, if the first coin toss is 'T' and the second toss is 'H', then:

- *payoff of Option A = endowment + variable payoff*
 $= \$100 + 70 = \$170.$
- *payoff of Option B = endowment + variable payoff*
 $= \$100 + 120 = \$220.$

Appendix B Experiment Decision

Please fill in background information:

- Gender:
- Citizenship (country):
- Department or major:
- Graduating year (Class of):
- Age:
- Monthly average expense (please check the box):
 - TWD \$0~\$10,000
 - TWD \$10,000~\$20,000
 - Above TWD \$20,000

I. First Stage

In the following table, when $X=$ __, according to the corresponding payoffs, please decide whether you prefer Option A or Option B (circle A or B):

	When $X=$ __	You prefer A or B? (circle A or B)	
Decision 1	150	A	B
Decision 2	250	A	B
Decision 3	300	A	B
Decision 4	400	A	B
Decision 5	500	A	B
Decision 6	600	A	B
Decision 7	700	A	B
Decision 8	800	A	B
Decision 9	1000	A	B
Decision 10	1100	A	B

II. Second Stage

In the following table, when $X = \underline{\quad}$, according to the corresponding payoffs, please decide whether you prefer Option A or Option B (circle A or B):

Values of *Payoff U* and *Payoff D* corresponding to different X are also shown in the table.

	When $X = \underline{\quad}$	<i>Payoff U</i>	<i>Payoff D</i>	You prefer A or B? (circle A or B)	
				A	B
Decision 11	25	75	125	A	B
Decision 12	50	50	150	A	B
Decision 13	100	0	200	A	B
Decision 14	125	-25	225	A	B
Decision 15	150	-50	250	A	B
Decision 16	160	-60	260	A	B
Decision 17	170	-70	270	A	B
Decision 18	180	-80	280	A	B
Decision 19	190	-90	290	A	B
Decision 20	200	-100	300	A	B

III. Third Stage

In the following table, when $X = \underline{\quad}$, according to the corresponding payoffs, please decide whether you prefer Option A or Option B (circle A or B):

Values of *Payoff U* and *Payoff D* corresponding to different X are also shown in the table.

	When $X = \underline{\quad}$	<i>Payoff U</i>	<i>Payoff D</i>	You prefer A or B? (circle A or B)	
Decision 21	25	-175	-125	A	B
Decision 22	50	-200	-100	A	B
Decision 23	100	-250	-50	A	B
Decision 24	125	-275	-25	A	B
Decision 25	150	-300	0	A	B
Decision 26	160	-310	10	A	B
Decision 27	170	-320	20	A	B
Decision 28	180	-330	30	A	B
Decision 29	190	-340	40	A	B
Decision 30	200	-350	50	A	B

IV. Fourth Stage

In the following table, when $X = \underline{\quad}$, according to the corresponding payoffs, please decide whether you prefer Option A or Option B (circle A or B):

Values of *Payoff* $U1, D1, U2, D2$ corresponding to different X are also shown in the table.

	When $X = \underline{\quad}$	<i>Payoff</i>		<i>Payoff</i>		You prefer A or B? (circle A or B)	
		$U1$	$D1$	$U1$	$D1$	A	B
Decision 31	50	150	250	100	200	A	B
Decision 32	100	100	300	50	250	A	B
Decision 33	150	50	350	0	300	A	B
Decision 34	175	25	375	-25	325	A	B
Decision 35	200	0	400	-50	350	A	B
Decision 36	210	-10	410	-60	360	A	B
Decision 37	220	-20	420	-70	370	A	B
Decision 38	230	-30	430	-80	380	A	B
Decision 39	240	-40	440	-90	390	A	B
Decision 40	250	-50	450	-100	400	A	B

Thank you very much for your participation! Please fill in the following questionnaire:

1. Do you think the experiment, in general, is easy to understand? (please check the box which applies)

very easy	easy	normal	hard	very hard

2. Do you think the decisions, in general, are easy to make?

very easy	easy	normal	hard	very hard

3. Is there any stage where it is too hard to make a decision? (can tick multiple boxes)

First Stage	Second Stage	Third Stage	Fourth Stage	None

4. Do you think the experiment, in general, is interesting?

Very interesting	interesting	normal	boring	very boring

5. If there is any feedback you would like to give, please write it down here: