Tertiary health prevention and saving

Abstract

The paper studies the interaction between saving and tertiary health prevention in a two-period framework where future decline in health is certain. The cost of investment in prevention occurs in the health attribute and future benefits include an increase in the probability of receiving treatment and a partial recovery of health if treated. In this framework, we show that the effects of changes in the returns on saving and on tertiary prevention depend on agent's correlation attitude (either correlation aversion or correlation loving). Furthermore, we study how prudence/imprudence in health and cross-prudence/imprudence in wealth determine the impact of a background health risk on optimal choices. Finally, we analyze the effect of high-order preferences in case of high-degree risk changes.

Keywords: Tertiary prevention, Saving, Health, Correlation attitude, Prudence.

JEL Classification: D81, I12, E21.

1 Introduction

In cases of chronic illness leading to declining health, an individual can choose to implement some activities which require a physical or mental effort or cause a short-run pain (usually called "side effects") but are important ways of alleviating future adverse health effects caused by the established disease. Several examples can be easily provided. Some kinds of surgery, such as coronary angioplasty, although painful in the short-run, reduces the probability of future heart attacks in patients with chronic cardiovascular diseases. Taking insulin or other drugs, although potentially causing hypoglycemia problems, reduces the probability of worsening in case of diabetes. Physical exercise involved in gardening projects for organic vegetables, intellectual activities such as playing board games, and regular interaction with community organizations, although energy consuming and even detrimental to current health, are expected to improve physical coordination and thus future health for a patient with Alzheimer's disease.

Similar behaviour, although involving different activities, is relevant for other chronic diseases, such as obstructive pulmonary disease or chronic musculoskeletal pain.[1] In all these cases, both the cost and benefit of health investment are mainly nonpecuniary, whereas the usual approach in prevention literature in decision theory is to put them in the same dimension of financial wealth.

The kind of health investment described above corresponds to behaviour usually called "tertiary prevention" in medical and health care literature (e.g. Leavell and Clark, 1958 and U.S. Preventive Services Task Force, 1996), which consists of activities or treatments, involving an established chronic disease and made in the attempt to reduce its future negative health effects. For this reason, and because of its formalization described below, we identify the health investment studied in this work as a kind of investment in tertiary health prevention.

Given these premises, the present paper examines a two-choice problem of saving and tertiary prevention in a two-period "prevention" framework under a two-argument utility

[1] Examples for the chronic diseases described above and for some others can be found, for instance, in Prince et al. (2014). On side effects see also Waters et al. (2009).

of wealth and health.[2] Making investment to lessen future loss in health, which is certain, lowers the current health level of a decision maker (DM) via side effects. As a return, the DM will increase the probability of receiving treatment and is expected to improve future health level if being treated.

Our work is related to Eeckhoudt et al. (2007) who studied tertiary prevention as the effect of an investment involving a reduction in current health level and an increase in future health level. However, the analysis in the present paper is different in several respects. First, we assume that the positive effect of health investment is not certain and that it occurs with a probability increasing in DM's effort. In this respect, our model considers a form for health investment which is a combination of the kind of investment described by Eeckhoudt et al. (2007) and standard two-period prevention, which involves an effort made in the present in order to reduce the probability that a future bad event occurs.[3] Second, our paper analyzes tertiary prevention decision together with saving decision.[4] This allows us to examine the interaction between these decisions involving several aspects of DM preferences, and to study cross-effects (such as cross-effects of returns) and joint effects of background risk and Nth-order risk changes.

This work also differs significantly from recent studies of the single choice of self-insurance-cum-protection including Lee (1998) and Hofmann and Peter (2014b), and models studying saving and financial prevention, such as Menegatti and Rebessi (2011) and Hofmann and Peter (2014a), who employ univariate utilities of wealth to investigate the activity of prevention. Indeed the introduction of a non-financial dimension of utility in the present paper implies considering the role of certain aspects of DM preferences (such as correlation aversion/loving and cross-prudence/cross-imprudence) not analyzed in these works. Lastly the health investment studied in this work is different from the kind of health investment studied

[2]In the context of health savings accounts (HSAs), Steinorth (2011) investigates a two-choice problem of saving and prevention using unidimensional utilities of wealth.

[3]This kind of prevention was first studied in a two-period framework by Menegatti (2009) with reference to a financial risk. Several subsequent papers used this approach in studying prevention (e.g. Eeckhoudt et al., 2012; Menegatti, 2014; Li and Wang, 2014).

[4]Notice that these two choices are related since they determine two different kinds of investment. Saving determines an investment requiring a cost in terms of wealth in the present and generating an increment in wealth in the future. Tertiary prevention can instead be seen as a kind of investment requiring a cost in terms of health in the present determining a possible increment in future health status and a contemporaneous increase in the probability that this increment occurs.

by Denuit et al. (2011), which concerned a single choice of financial investment determining a sure and known future health improvement.

Given the framework described above, we develop our analysis along the following lines. First, we examine optimal choices of saving and tertiary prevention and we study how these choices are affected by changes in the returns on each kind of investment, analyzing which aspects of DM preferences are relevant in direct and cross effects. We then study the impact of a background risk in health under different assumptions on prudence and cross-prudence. A generalization to the case of Nth-order risk changes in the background risk is finally provided.

The paper proceeds as follows. Section 2 presents the model and derives optimal levels of saving and tertiary prevention. Section 3 studies changes in returns. Section 4 introduces the background risk. Section 5 analyzes Nth-degree risk changes. Section 6 concludes.

2 A model with tertiary prevention and saving

We assume that a DM has a two-attribute utility $u[W, H]$ from wealth and health. We introduce the usual assumptions of non-satiation and risk aversion with respect to both arguments of the utility function, such that $u_1 > 0$, $u_{11} < 0$, $u_2 > 0$, $u_{22} < 0$.[5] Since there is no consensus in empirical and theoretical literature, we do not introduce any assumption on the sign of u_{12}.[6] This means that the DM can be either averse to ($u_{12} < 0$) or loving ($u_{12} > 0$) the correlation between the wealth level and the health condition. We will show later that the sign of u_{12} is relevant for some of our results. In order to simplify the notation we assume that the utility function is the same in the two periods. Notice that all our results also hold when utility is different in the two periods.

At date $t = 0$, the DM is given a certain wealth w that can be allocated between consumption and saving s. The amount saved in the financial market yields a constant gross return $R \geq 1$. We assume for simplicity that second-period income is null. Notice that all

[5]We use the following notation for derivatives: $\frac{\partial u}{\partial W} = u_1$, $\frac{\partial u}{\partial H} = u_2$, $\frac{\partial^2 u}{\partial W^2} = u_{11}$, $\frac{\partial^2 u}{\partial H^2} = u_{22}$, etc.

[6]For instance, Evans and Viscusi (1991), Lillard and Weiss (1997), and Edwards (2008) find the evidence of correlation aversion, while correlation loving receives support from studies by Viscusi and Evans (1990), Sloan et al. (1998), Carthy et al. (1999), and recently Finkelstein et al. (2013). See Finkelstein et al. (2009) for a detailed review.

our results hold also if this income is positive.

The DM also faces declining future health due to a chronic illness or an established disease. Because of this, the DM's health status falls from the initial level of h to the level of $h-d$ in the second period. This reduction in health can be counteracted by the DM by means of an investment in tertiary prevention e. Due to side effect of medical treatment, the investment lowers current health to a new level of $h-e$. The same investment may have a positive effect on health status in the future. The size of this effect and the probability that its treatment occurs are increasing in the level of e. This is formalized assuming that there are two states of nature at date $t=1$: bad state and good state. In the bad state, which occurs with a probability $p(e)$, the health level falls to $h-d$, where $0 \leq d < h$ stands for the reduction of health status. In the good state, which occurs with a probability $1-p(e)$, the investment takes effect by a constant productivity $\theta > 0$ such that health condition is improved to a level of $(h-d+\theta e)$. It is assumed that $p^{'}(e) < 0$ and, as usual in prevention literature, $p^{''}(e) > 0$.

In this study, given the framework described above, future loss in health becomes inevitable and therefore health specific investment in prevention is made to increase simultaneously the probability of receiving medical treatment and the size of its effect. As explained in Section 1, these elements, together with the contemporaneous presence of saving, are novel to the literature and determine a clear difference from previous models of health investment and tertiary prevention. The use of a two-argument utility function also distinguishes our work from the studies of self-insurance-cum-protection.

Without loss of generality, let the DM's subjective discount factor be one. The DM's optimization problem is thus written as

$$\begin{aligned} \max_{s,e} U(s,e) &= u[w-s, h-e] \\ &\quad + p(e)u[sR, h-d] + (1-p(e))u[sR, h-d+\theta e] \qquad (1) \end{aligned}$$

For notational simplicity, let $u[w-s,h-e] = u^1$, $u[sR,h-d] = u^{2B}$, and $u[sR,h-d+\theta e] = u^{2G}$, where "$u$ superscript" stands for a time and state (if any) dependent utility. The first-order conditions are[7]

$$U_s = -u_1^1 + p(e)u_1^{2B}R + (1-p(e))u_1^{2G}R = 0 \tag{2}$$

$$U_e = -u_2^1 + p'(e)[u^{2B} - u^{2G}] + \theta \cdot (1-p(e))u_2^{2G} = 0 \tag{3}$$

for saving and for health investment, respectively. Notice that $u^{2B} - u^{2G} < 0$ since $u_2 > 0$. The joint optimum (s^*, e^*) is obtained when both (2) and (3) are satisfied.

The sufficient second-order conditions are

$$U_{ss} = u_{11}^1 + p(e)u_{11}^{2B}R^2 + (1-p(e))u_{11}^{2G}R^2 < 0 \tag{4}$$

$$U_{ee} = u_{22}^1 + p''(e)[u^{2B} - u^{2G}] - 2\theta p'(e)u_2^{2G} + \theta^2(1-p(e))u_{22}^{2G} < 0 \tag{5}$$

$$H = U_{ss}U_{ee} - (U_{se})^2 > 0 \tag{6}$$

where

$$U_{se} = U_{es} = u_{12}^1 + p'(e)[u_1^{2B} - u_1^{2G}]R + \theta(1-p(e))u_{12}^{2G}R. \tag{7}$$

Condition (4) is satisfied given the concavity assumption on u. With reference to conditions (5) and (6), we follow previous literature on two-choice problems, such as Dionnne and Eeckhoudt (1984) and Menegatti and Rebessi (2011) in explicitly assuming that they hold in the whole domain of the utility function.

Also notice that the sign of U_{se} depends on the sign of u_{12}. Indeed we have[8]

$$U_{se} \begin{array}{l} < 0 \text{ when } u_{12} < 0 \\ > 0 \text{ when } u_{12} > 0 \end{array} \tag{8}$$

[7]We use the following notation for derivatives: $\frac{\partial U}{\partial s} = U_s$, $\frac{\partial U}{\partial e} = U_e$, $\frac{\partial^2 U}{\partial s \partial e} = U_{se}$, $\frac{\partial^2 U}{\partial s^2} = U_{ss}$ $\frac{\partial^2 U}{\partial e^2} = U_{ee}$, etc.

[8]Note that $p'(e)[u_1^{2B} - u_1^{2G}]$ is negative when $u_{12} < 0$ and positive when $u_{12} > 0$.

This implies that when $u_{12} < 0$ the optimal value of saving in (2) is a decreasing function of tertiary prevention and that the optimal value of tertiary prevention in (3) is a decreasing function of saving. When instead $u_{12} > 0$ the two functions are increasing.
These two functions can be represented by two curves (*ss* curve and *ee* curve) in a Cartesian diagram. Moreover condition (6) implies that the slope of the *ss* curve is steeper than the slope of the *ee* curve (see Appendix A for the proof). The *ss* curve and the *ee* curve in the two cases $u_{12} > 0$ and $u_{12} < 0$ are depicted in Figure 1.

INSERT FIGURE 1 HERE

3 The effects of changes in returns

Given the equilibrium studied in Section 2, we now examine what happens when there are changes in the returns on saving R and in the returns on investment in tertiary prevention θ. These second returns are a measure of the efficiency of tertiary prevention in counteracting the disease worsening. First, we study how each kind of investment, either financial investment (i.e. saving) or health investment, reacts when its own returns change. This means analyzing how saving changes when R changes and how tertiary prevention changes when θ changes. We thus determine the signs of ds^*/dR and of $de^*/d\theta$. We obtain:

Proposition 1. *a)* $ds^*/dR > 0$ *if* $[-u_{11}^{2i} \cdot (sR)/u_1^{2i}] < 1$ *where* $i = B, G$
b) $de^*/d\theta > 0$ *if* $[-u_{22}^{2G} \cdot (\theta e)/u_2^{2G}] < 1$

Proof. By totally differentiating (2) and (3) we obtain, after some algebra, that $ds^*/dR = -\frac{1}{H}\left[U_{ee}U_{sR} - U_{se}U_{eR}\right]$ and that $de^*/d\theta = -\frac{1}{H}\left[U_{ss}U_{e\theta} - U_{es}U_{s\theta}\right]$.

Note now that

$$U_{eR} = p'(e)[u_1^{2B} - u_1^{2G}] \cdot s + \theta(1 - p(e))u_{21}^{2G} \cdot s \quad \begin{array}{l} < 0 \text{ when } u_{12} < 0 \\ > 0 \text{ when } u_{12} > 0 \end{array} \tag{9}$$

and

$$U_{sR} = p(e)[u_1^{2B} + u_{11}^{2B} sR] + (1 - p(e))[u_1^{2G} + u_{11}^{2G} sR] > 0 \tag{10}$$

By (8), (9) and by second-order conditions, we know that $U_{se}U_{eR}$ is positive and that U_{ee} is negative. This implies that a sufficient condition ensuring that $ds^*/dR > 0$ is $U_{sR} > 0$. By (10) we get that this occurs if $[-u_{11}^{2i} \cdot (sR)/u_1^{2i}] < 1$.
Also note that

$$U_{s\theta} = e(1-p(e))u_{12}^{2G}R \begin{array}{l} < 0 \text{ when } u_{12} < 0 \\ > 0 \text{ when } u_{12} > 0 \end{array} \tag{11}$$

and

$$U_{e\theta} = -ep'(e)u_2^{2G} + (1-p(e))[u_2^{2G} + \theta e \cdot u_{22}^{2G}] > 0 \tag{12}$$

By (8), (11) and by second-order conditions, we know that $U_{se}U_{s\theta}$ is positive and that U_{ss} is negative. This implies that a sufficient condition ensuring that $de^*/d\theta > 0$ is $U_{e\theta} > 0$. By (12) we get that this occurs if $[-u_{22}^{2G} \cdot (\theta e)/u_2^{2G}] < 1$. □

Proposition 1 shows sufficient conditions for saving and tertiary prevention to increase when R and θ increase. The interpretation of these conditions is the same as similar conditions in usual saving/investment models. These models in fact show that, when the return on a type of investment increases, we have two different counteracting effects: the "substitution effect", which pushes the agent to increase investment level, and the "income effect", which pushes the agent to reduce investment level. The two conditions in Proposition 1 are those ensuring that the substitution effect prevails in the case of saving and in the case of investment in tertiary prevention respectively. In order to confirm these conclusions we show in Appendix B the parallel of these conditions for a simple standard two-period saving problem.

From now on we focus on the case where the substitution effect is larger than the income effect which implies that an increase in returns raises investment level. We thus focus on the case where $[-u_{11}^{2i} \cdot (sR)/u_1^{2i}] < 1$ and $[-u_{22}^{2G} \cdot (\theta e)/u_2^{2G}] < 1$ which implies that a larger R increases optimal saving ($ds^*/dR > 0$) and a larger θ increases optimal health investment ($de^*/d\theta > 0$).

Under these assumptions, we now study a second comparative statics effect, analyzing how the level of each choice variable (either s or e) changes when the return of the *other variable* changes. We thus study the "cross-effects" of the returns $ds^*/d\theta$ and de^*/dR. We obtain that:

Proposition 2. *a) If $u_{12} < 0$, an increase in R reduces e^* ($de^*/dR < 0$) and an increase in θ reduces s^* ($ds^*/d\theta < 0$).*
b) If $u_{12} > 0$, an increase in R raises e^* ($de^*/dR > 0$) and an increase in θ raises s^* ($ds^*/d\theta > 0$).

Proof. By totally differentiating (2) and (3) we obtain, after some algebra, that

$$de^*/dR = -\frac{1}{H}\left[U_{ss}U_{eR} - U_{es}U_{sR}\right] \tag{13}$$

and

$$ds^*/d\theta = -\frac{1}{H}\left[U_{ee}U_{s\theta} - U_{se}U_{e\theta}\right] \tag{14}$$

Note that, given (8) and (9), the sign of U_{es} and of U_{eR} is the same and is negative when $u_{12} < 0$ and positive when $u_{12} > 0$. Considering this conclusion together with (13) and recalling that $U_{ss} < 0$ by second-order conditions and that $U_{sR} > 0$ under our assumptions, we get that de^*/dR is negative when $u_{12} < 0$ and positive when $u_{12} > 0$.
Note now that, given (8) and (11), the sign of U_{se} and of $U_{s\theta}$ is the same and is negative when $u_{12} < 0$ and positive when $u_{12} > 0$. Considering this conclusion together with (14) and recalling that $U_{ee} < 0$ by second-order conditions and $U_{e\theta} > 0$ under our assumptions, we get that $ds^*/d\theta$ is negative when $u_{12} < 0$ and positive when $u_{12} > 0$. □

Proposition 2 shows that the cross-effects of the returns depend on the sign of u_{12}. The interpretation of this sign has been widely analyzed in the literature, starting from Richard (1975), and has proved to be relevant in many decision problems involving the optimal level of insurance (Rey and Rochet, 2004), the optimal level of saving (without health investment) when health status changes over time (Eeckhoudt et al., 2007) and the optimal choices of prevention and cure (Menegatti, 2014). As recalled in Section 2, the case where $u_{12} < 0$ is called "correlation aversion" while the case where $u_{12} > 0$ is "correlation loving". As emphasized by Eeckhoudt et al. (2007), a DM is correlation averse if "*she always prefers a 50-50 gamble of a loss in wealth or a loss in health over another 50-50 gamble offering a loss in neither dimension or a loss in both*" (see Eeckhoudt et al., 2007, p. 119). Furthermore the sign of u_{12} indicates how marginal utility of wealth depends on the level of health status and *vice versa*. In particular, if $u_{12} < 0$ then the increase in utility due to an additional unit of wealth (health) is larger when the DM is less healthy (wealthy) while if $u_{12} > 0$ then the same increase in utility is larger when the DM is healthier (wealthier).

This reasoning helps us in understanding the effects in Proposition 2. Indeed, when the DM is correlation averse, an increase in the return on saving R reduces marginal utility of health u_2. Since $u_{22} < 0$, this pushes the DM to reduce tertiary prevention in order to increase u_2. Similarly, an increase in the return on the investment in tertiary prevention θ reduces marginal utility of wealth u_1. Since $u_{11} < 0$, this pushes the DM to reduce saving in order to increase u_1. The opposite occurs when $u_{12} > 0$.[9]

4 A background risk on the health dimension

In this section we study the effect on health investment and saving of the introduction of a background risk. Suppose that in the second period the DM faces a zero-mean background risk $\widetilde{\epsilon}$ on the health dimension. Notice that, coherently with our framework studying chronic diseases, we can intepret the backgroud risk also as a random change in the disease level, which can produce an unexpected worsening or an unexepcted improvement in the health status. For this reason, we also assume that the range of possible realizations of $\widetilde{\epsilon}$ is bounded above by d.

Given these assumptions, the DM's objective function is:

$$\begin{aligned} \max_{s,e} \overline{U}(s,e) &= u(w-s,h-e) \\ &+p(e)Eu(sR,h-d+\widetilde{\epsilon})+(1-p(e))Eu(sR,h-d+\theta e+\widetilde{\epsilon}) \end{aligned} \quad (15)$$

[9]Notice that, since the results in Proposition 2 show the effect on one instrument when the return of the other changes under the assumption that marginal cost of each investment is constant, Proposition 2 also suggests that, under the assumption of correlation aversion (correlation loving), tertiary prevention and saving can be seen as substitutes (complements).

Let us evaluate $d\overline{U}/ds$ at the optimal levels of saving for Eq. (2). This gives

$$d\overline{U}/ds\mid_s = -u_1(w-s,h-e)$$
$$+p(e)Eu_1(sR,h-d+\widetilde{\epsilon})R+(1-p(e))Eu_1(sR,h-d+\theta e+\widetilde{\epsilon})R \tag{16}$$

It follows in a straightforward manner from (16) that with the introduction of background health risk, the ss curve depicted in Figure 1 will necessarily shift to the right if and only if

$$p(e)[Eu_1(sR,h-d+\widetilde{\epsilon})-u_1(sR,h-d)]R$$
$$+(1-p(e))E[u_1(sR,h-d+\theta e+\widetilde{\epsilon})-u_1(sR,h-d+\theta e)]R>0 \tag{17}$$

which is guaranteed if u_1 is convex in the second attribute, i.e. if $u_{122}>0$. Eeckhoudt et al. (2007) called this feature of DM preferences "cross-prudence in wealth".[10] Conversely, the ss curve will necessarily shift to the left if u_1 is concave in the second attribute, i.e. if $u_{122}<0$. Eeckhoudt et al. (2007) called this feature of DM preferences "cross-imprudence in wealth".

We now evaluate $d\overline{U}/de$ at the optimal levels of health investment for Eq. (3) and find:

$$d\overline{U}/de\mid_e = -u_2(w-s,h-e)$$
$$+p'(e)[Eu(sR,h-d+\widetilde{\epsilon})-Eu(sR,h-d+\theta e+\widetilde{\epsilon})] \tag{18}$$
$$+\theta(1-p(e))Eu_2(sR,h-d+\theta e+\widetilde{\epsilon})$$

Again, it is straightforward from (18) that with the introduction of backround health risk, the ee curve will necessarily move upward if and only if

$$p'(e)\{[Eu(sR,h-d+\widetilde{\epsilon})-Eu(sR,h-d+\theta e+\widetilde{\epsilon})]$$
$$-[u(sR,h-d)-u(sR,h-d+\theta e)]\} \tag{19}$$
$$+\theta\cdot(1-p(e))[Eu_2(sR,h-d+\theta e+\widetilde{\epsilon})-u_2(sR,h-d+\theta e)]>0$$

[10]The concept of prudence was first introduced by Kimball (1990), describing the case where $u_{111}>0$. Different kinds of prudence and cross-prudence were introduced by Eeckhoudt et al. (2007).

or

$$
\begin{aligned}
&[-p'(e)]\{[Eu(sR, h-d+\theta e+\widetilde{\epsilon}) - u(sR, h-d+\theta e)] \\
&\qquad -[Eu(sR, h-d+\widetilde{\epsilon}) - u(sR, h-d)]\} \\
&+\theta\cdot(1-p(e))[Eu_2(sR, h-d+\theta e+\widetilde{\epsilon}) - u_2(sR, h-d+\theta e)] > 0
\end{aligned} \tag{20}
$$

This is equivalent to

$$
\begin{aligned}
&[-p'(e)]\int_{h-d}^{h-d+\theta e}[Eu_2(sR,\omega+\widetilde{\epsilon}) - u_2(sR,\omega)]d\omega \\
&+\theta\cdot(1-p(e))[Eu_2(sR, h-d+\theta e+\widetilde{\epsilon}) - u_2(sR, h-d+\theta e)] > 0
\end{aligned} \tag{21}
$$

which is guaranteed if u_2 is convex in the health attribute, i.e. if $u_{222} > 0$. This feature of DM preference is called "prudence in health". Conversely, the ee curve will necessarily move downward if u_2 is concave in the health attribute, i.e. if $u_{222} < 0$. This feature of DM preference is called "imprudence in health".

According to the positive or negative slope of the curves ss and ee (depending on the sign of u_{12}) and to the different possible combinations of signs of u_{122} and u_{222}, the shift in the ss curve and in the ee curve can determine four different changes in optimal levels of e and s, caused by the presence of the background risk. Recalling that, as shown in Section 2, the slope of the ss curve is always steeper than the slope of the ee curve, the four cases are represented in Figure 2. From this figure we also get the following results.

INSERT FIGURE 2 HERE

Proposition 3. *Let(s^*, e^*) be the joint optimum for problem (1) and (s^{**}, e^{**}) be the joint optimum for problem (15). We thus have the following comparative statics results.*

a) $s^{**} < s^*$ and $e^{**} > e^*$ if the agent is correlation averse, cross-imprudent in wealth and prudent in health ($u_{12} < 0$, $u_{122} < 0$ and $u_{222} > 0$);

b) $s^{**} > s^*$ and $e^{**} > e^*$ if the agent is correlation loving, cross-prudent in wealth and prudent in health ($u_{12} > 0$, $u_{122} > 0$ and $u_{222} > 0$);

c) $s^{**} > s^*$ and $e^{**} < e^*$ if the agent is correlation averse, cross-prudent in wealth and imprudent in health ($u_{12} < 0$, $u_{122} > 0$ and $u_{222} < 0$);

d) $s^{**} < s^*$ and $e^{**} < e^*$ if the agent is correlation loving, cross-imprudent in wealth and imprudent in health ($u_{12} > 0$, $u_{122} < 0$ and $u_{222} < 0$).

The results in Proposition 3 clearly show that prudence/imprudence in health and cross-prudence/imprudence in wealth determine different effects of a background health risk on optimal levels of saving and tertiary prevention. In particular, prudence in health implies larger tertiary prevention, while imprudence in health implies lower tertiary prevention. Similarly, cross-prudence in wealth implies larger saving and cross-imprudence in wealth implies lower saving. The case where both saving and tertiary prevention increase is thus the case where the DM is both prudent in health and cross-prudent in wealth.

Eeckhoudt et al. (2007) show that, in a model with only saving and no tertiary prevention, cross-prudence in wealth is necessary and sufficient for precautionary saving when a background risk in health is introduced. The results in the present paper generalize this conclusion to a framework where saving and tertiary prevention are introduced together. Furthermore, as noted above, our results also show that also prudence/imprudence in health is relevant when tertiary prevention is considered.

An interpretation of this result can be provided by referring to the meaning of prudence in precautionary saving literature. As shown by Eeckhoudt and Schlesinger (2006) and Menegatti (2007), in a framework where utility only depends on wealth, prudence can be seen as the desire by a DM to have a larger wealth in the period where the DM bears a financial risk. This desire pushes the DM to increase saving. Similarly, in the present framework, cross-prudence in wealth can be interpreted as the desire to increase wealth in the period where the DM bears a background risk on health and prudence in health can be interpreted as the desire to raise the health status in that period. For these reasons, when the DM is contemporaneously prudent in health and cross-prudent in wealth, the DM chooses to raise both tertiary prevention and saving. Finally correlation loving is also required in order to ensure that the agent likes the cross-effect on utility due to these contemporaneous increases.

The effect of tertiary prevention described above is related to the term θe in the health argument of the utility function, which affects expected health status in the future. Tertiary prevention, however, also has an impact on future health via probability $p(e)$. However, as shown by Eeckhoudt et al. (2012), in a model where the DM faces a background risk in a two-period framework where utility only depends on wealth, prudence determines an increase in a kind of prevention effort acting on the probability of the occurrence of a given financial loss[11]. The same effect occurs in the present framework with reference to prudence in health, reinforcing the effect on θe described above.

Notice finally that, obviously, a similar reasoning applies to the other cases described in Proposition 3.

5 Nth-degree increase in background risk

We now generalize the analysis in the previous section by studying the case of an Nth-degree increase in zero-mean background risk.

We first introduce the definition of Nth-degree increase in risk given by Ekern (1980):

Definition 1. Consider two random variables $\widetilde{\delta}$ and $\widetilde{\epsilon}$ with their cumulative probability distribution functions, contained within the same open interval (a, b), being $F(z)$ and $G(z)$, respectively. Also define $F_0 = F$ and $G_0 = G$, and $F_n(z) = \int_a^z F_{n-1}(t)dt$ for $n \geq 1$ and similarly for G_n. In this case $\widetilde{\delta}$ is an Nth-degree increase in risk over $\widetilde{\epsilon}$ if $F_N(z) \geq G_N(z)$ for all z, and $F_n(b) = G_n(b)$ for $n = 1, 2, ..., N-1$.

Studying the Nth-degree increase in risk from $\widetilde{\epsilon}$ to $\widetilde{\delta}$ entails comparing the optimal choices derived in problem (15) with the optimal choices derived in the following problem

$$\begin{aligned}\max_{s,e} \widetilde{U}(s,e) &= u(w-s, h-e) + \\ &+p(e)Eu(sR, h-d+\widetilde{\delta}) + (1-p(e))Eu(sR, h-d+\theta e+\widetilde{\delta}) \qquad (22)\end{aligned}$$

A similar reasoning to that in Section 4 suggests that the ss curve associated with the solution of problem (22) is shifted to the right compared to the curve associated with the

[11] A loss whose size is not affected by prevention

solution of problem (15) if and only if

$$p(e)[Eu_1(sR, h-d+\widetilde{\delta}) - Eu_1(sR, h-d+\widetilde{\epsilon})]R$$
$$+(1-p(e))[Eu_1(sR, h-d+\theta e+\widetilde{\delta}) - Eu_1(sR, h-d+\theta e+\widetilde{\epsilon})]R > 0 \tag{23}$$

Integrating by parts (see Appendix C), it can easily be shown that Condition (23) holds if $u_{122} > 0$ for the second-degree increase in risk (or mean-preserving spread as defined by Rothschild and Stiglitz 1970), if $u_{1222} < 0$ for the third-degree increase in risk (or increase in downside risk as defined by Menezes et al. 1980), and so on for increases in risk of higher degrees. Obviously there is a shift of the *ss* curve in the opposite direction when the direction of previous inequalities is reversed.

By the same token, moving from $\widetilde{\epsilon}$ to $\widetilde{\delta}$, the *ee* curve will necessarily move upward if and only if

$$\begin{aligned} p'(e)\{[Eu(sR, h-d+\widetilde{\delta}) - Eu(sR, h-d+\theta e+\widetilde{\delta})] \\ -[Eu(sR, h-d+\widetilde{\epsilon}) - Eu(sR, h-d+\theta e+\widetilde{\epsilon})]\} \\ +\theta\cdot(1-p(e))[Eu_2(sR, h-d+\theta e+\widetilde{\delta}) - Eu_2(sR, h-d+\theta e+\widetilde{\epsilon})] > 0 \end{aligned} \tag{24}$$

which is equivalent to

$$[-p'(e)]\int_{h-d}^{h-d+\theta e}[Eu_2(sR, \omega+\widetilde{\delta}) - Eu_2(sR, \omega+\widetilde{\epsilon})]d\omega$$
$$+\theta\cdot(1-p(e))[Eu_2(sR, h-d+\theta e+\widetilde{\delta}) - Eu_2(sR, h-d+\theta e+\widetilde{\epsilon})] > 0 \tag{25}$$

Again, integrating by parts, it can easily be shown that Condition (25) holds if $u_{222} > 0$ for the second-degree increase in risk, if $u_{2222} < 0$ for the third-degree increase in risk, and so on. Again there is a movement of the *ee* curve in the opposite direction when the direction of previous inequalities is reversed.

Considering these movements of the *ss* and *ee* curves together and recalling that the positive or negative slope of the two curves depends on the sign of u_{12}, we get the following results for the cases. In case of second-degree risk change we have:

Proposition 4. *Let (s^{**}, e^{**}) be the joint optimum for problem (15) and (s^{***}, e^{***}) be the joint optimum for (22). For a second-degree increase in risk where $E(\widetilde{\delta}) = E(\widetilde{\epsilon}) = 0$, we obtain*

a) $s^{***} < s^{**}$ and $e^{***} > e^{**}$ if the agent is correlation averse, cross imprudent in wealth and prudent in health ($u_{12} < 0$, $u_{122} < 0$ and $u_{222} > 0$);

b) $s^{***} > s^{**}$ and $e^{***} > e^{**}$ if the agent is correlation loving, cross prudent in wealth and prudent in health ($u_{12} > 0$, $u_{122} > 0$ and $u_{222} > 0$);

c) $s^{***} > s^{**}$ and $e^{***} < e^{**}$ if the agent is correlation averse, cross prudent in wealth and imprudent in health ($u_{12} < 0$, $u_{122} > 0$ and $u_{222} < 0$);

d) $s^{***} < s^{**}$ and $e^{***} < e^{**}$ if the agent is correlation loving, cross imprudent in wealth and imprudent in health ($u_{12} > 0$, $u_{122} < 0$ and $u_{222} < 0$).

In case of third-degree risk we obtain:

Proposition 5. *Let (s^{**}, e^{**}) be the joint optimum for problem (15) and (s^{***}, e^{***}) be the joint optimum for (22). For a third-degree increase in risk where $E(\widetilde{\delta})^2 = E(\widetilde{\epsilon})^2$ and $E(\widetilde{\delta}) = E(\widetilde{\epsilon}) = 0$, we have*

a) $s^{***} < s^{**}$ and $e^{***} > e^{**}$ if $u_{12} < 0$, $u_{1222} > 0$ and $u_{2222} < 0$;

b) $s^{***} > s^{**}$ and $e^{***} > e^{**}$ if $u_{12} > 0$, $u_{1222} < 0$ and $u_{2222} < 0$;

c) $s^{***} > s^{**}$ and $e^{***} < e^{**}$ if $u_{12} < 0$, $u_{1222} < 0$ and $u_{2222} > 0$;

d) $s^{***} < s^{**}$ and $e^{***} < e^{**}$ if $u_{12} > 0$, $u_{1222} > 0$ and $u_{2222} > 0$.

Lastly in the general case of Nth-degree risk change we have:

Proposition 6. *Let (s^{**}, e^{**}) be the joint optimum for problem (15) and (s^{***}, e^{***}) be the joint optimum for (22). For a Nth-degree change in risk (defined as in Definition 1 above), we have*

a) $s^{***} < s^{**}$ and $e^{***} > e^{**}$ if $u_{12} < 0$, $(-1)^{N+1}\frac{\partial^{N+1}u}{\partial W \partial^N H} > 0$ and $(-1)^{N+1}\frac{\partial^{N+1}u}{\partial^{N+1}H} < 0$;

b) $s^{***} > s^{**}$ and $e^{***} > e^{**}$ if $u_{12} > 0$, $(-1)^{N+1}\frac{\partial^{N+1}u}{\partial W \partial^N H} < 0$ and $(-1)^{N+1}\frac{\partial^{N+1}u}{\partial^{N+1}H} < 0$;

c) $s^{***} > s^{**}$ and $e^{***} < e^{**}$ if $u_{12} < 0$, $(-1)^{N+1}\frac{\partial^{N+1}u}{\partial W \partial^N H} < 0$ and $(-1)^{N+1}\frac{\partial^{N+1}u}{\partial^{N+1}H} > 0$;

d) $s^{***} < s^{**}$ and $e^{***} < e^{**}$ if $u_{12} > 0$, $(-1)^{N+1}\frac{\partial^{N+1}u}{\partial W \partial^N H} > 0$ and $(-1)^{N+1}\frac{\partial^{N+1}u}{\partial^{N+1}H} > 0$.

Propositions 4, 5 and 6 show how different high-order derivatives of the utility function determine DM reaction to increase in risks of different orders.

Also note that Brockett and Golden (1987) and Caballé and Pomansky (1996) study a class of one-argument utility functions, exhibiting derivatives with alternating signs.[12] Denuit et al. (1999) examine a generalization of this class of functions to two-argument utility. Considering this class of functions in our framework and given results above we get that

Proposition 7. *If $U(W,H)$ is such that $u_{12} > 0$ and $(-1)^{j+1}\frac{\partial^{j+1}u}{\partial W \partial^j H} < 0$ for $j = 2,3,...,M$ and $(-1)^{j+1}\frac{\partial^{j+1}u}{\partial^{j+1}H} < 0$ for $j = 2,3,...,M$ then increases of risk of each order up to M implies both larger saving and larger tertiary prevention.*

6 Conclusions

In this paper we analyzed a model where saving and investment in tertiary prevention are jointly chosen by a DM exhibiting a declining health status because of chronic illness. The kind of health investment studied determines a partial health recovery, whose size depends on investment size and which occurs with a probability depending again on investment level.

Our results show that direct effects of changes in returns depend, as usual, on substitution and income effects and cross-effects are determined by the DM correlation attitude. In particular an increase in the return on saving reduces tertiary prevention and an increase in the return on tertiary prevention reduces saving when the DM is correlation averse. The opposite occurs when the DM is a correlation lover.

Moreover the introduction of a background risk on the health dimension generates a change in optimal choices of saving and tertiary prevention which depends on DM attitude toward prudence and cross-prudence. In particular, tertiary prevention increases when the DM is prudent in health and decreases when she is imprudent in health. Similarly, saving increases when the DM is cross-prudent in wealth and decreases when the DM is cross-imprudent in wealth.

Finally, we show that, when considering a Nth-degree increase in the background health risk, the change in saving and tertiary prevention depends on DM's high-order preferences. A

[12]Brockett and Golden (1987) call these functions "completely monotone". On these functions see also the recent results by Menegatti (2015).

specific case, where all high-degree increases in risk raise both saving and tertiary prevention occurs when the derivatives and the cross-derivatives of the utility function alternate in sign.

Appendix A

Totally differentiating (2) and (3) with respect to s and e, we have the slope of the ss curve as

$$de/ds = -U_{ss}/U_{se} \tag{26}$$

and the slope of the ee curve as

$$de/ds = -U_{es}/U_{ee} \tag{27}$$

To see the slope of the ss curve is steeper than the slope of the ee curve, we first show that this is the case for a correlation averse agent. Notice that $U_{se} = U_{es} < 0$ when $u_{12} < 0$. Given that (6) is satisfied and dividing both sides of (6) with $-U_{se}$, which is positive, we obtain

$$-U_{ss}U_{ee}/U_{se} > -U_{es} \tag{28}$$

Given that (5) is satisfied and dividing both sides of the above inequality with U_{ee}, which is negative, we then obtain

$$-U_{ss}/U_{se} < -U_{es}/U_{ee} < 0 \tag{29}$$

Since, in this case, both $-U_{ss}/U_{se}$ and $-U_{es}/U_{ee}$ are negative this ensures that the slope of the ss curve is steeper than the slope of the ee curve.

For the other case when an agent is correlation loving ($u_{12} > 0$), a similar reasoning gives us

$$-U_{ss}/U_{se} > -U_{es}/U_{ee} > 0 \tag{30}$$

Since, in this case, both $-U_{ss}/U_{se}$ and $-U_{es}/U_{ee}$ are positive this ensures that the slope of the ss curve is steeper than the slope of the ee curve.

Appendix B

In this appendix we show the sufficient condition ensuring that the substitution effect prevails over the income effect in a simple two-period model of consumption and saving. Assume that the DM maximizes with regard to s the total intertemporal utility, given by:

$$u(W_0 - s) + u(W_1 + Rs) \tag{31}$$

where W_0 and W_1 is wealth in the first and second periods respectively, s is saving and R is saving return. The first-order condition of this problem requires:

$$-u_1(W_0 - s^*) + Ru_1(W_1 + Rs^*) = 0 \tag{32}$$

By totally differentiating (32) we obtain that

$$\frac{ds^*}{dR} = -\frac{u_1(W_1 + s^*R) + s^*Ru_{11}(W_1 + s^*R)}{u_{11}(W_0 - s^*) + u_{11}(W_1 + s^*R)R^2} \tag{33}$$

The two addends in the numerator of this fraction, representing income and substitution effects respectively, have opposite signs. The substitution effect prevails, implying $\frac{ds^*}{dR} > 0$, when $-\frac{s^*Ru_{11}(W_1+s^*R)}{u_1(W_1+s^*R)} < 1$. It is clear that this condition is equivalent to the conditions derived in Proposition 1 for the effect of an increase in R on saving and the effect of an increase in θ on tertiary prevention.

Appendix C

In this appendix we show that condition (23) is satisfied if and only if the DM's preferences exhibit some features for an Nth-degree increase in risk, where $N = 2, 3, 4, ...$ Observe that condition (23) is the same as condition $Eu_1(sR, H+\widetilde{\delta}) - Eu_1(sR, H+\widetilde{\epsilon}) > 0$, where $H = h-d$ or $h - d + \theta e$, or equivalently,

$$\int_a^b u_1(sR, H + z)d[F(z) - G(z)] > 0 \tag{34}$$

Integrating by parts, we obtain

$$u_1(sR, H+z)[F(z)-G(z)]\mid_a^b - \int_a^b u_{12}(sR, H+z)d[F_1(z)-G_1(z)] > 0 \tag{35}$$

Note that the first part in the left-hand side of the above inequality is zero since $F(b) = G(b)$. Integrating by parts again, we obtain

$$-u_{12}(sR, H+z)[F_1(z)-G_1(z)]\mid_a^b + \int_a^b u_{122}(sR, H+z)d[F_2(z)-G_2(z)] > 0 \tag{36}$$

which is true if $u_{122} > 0$ for a second-degree increase in risk from $\widetilde{\epsilon}$ to $\widetilde{\delta}$ since $F_1(b) = G_1(b)$ and $F_2(z) \geq G_2(z)$ for all z. By the same token, it can be shown that condition (23) is satisfied if $u_{1222} < 0$ for a third-degree increase in risk, and so on for increases in risk of higher degrees.

Finally, similar reasoning holds for the sign of condition (25).

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Figure 1: The ss curve and the ee curve in the two cases where $u_{12} > 0$ and $u_{12} < 0$.

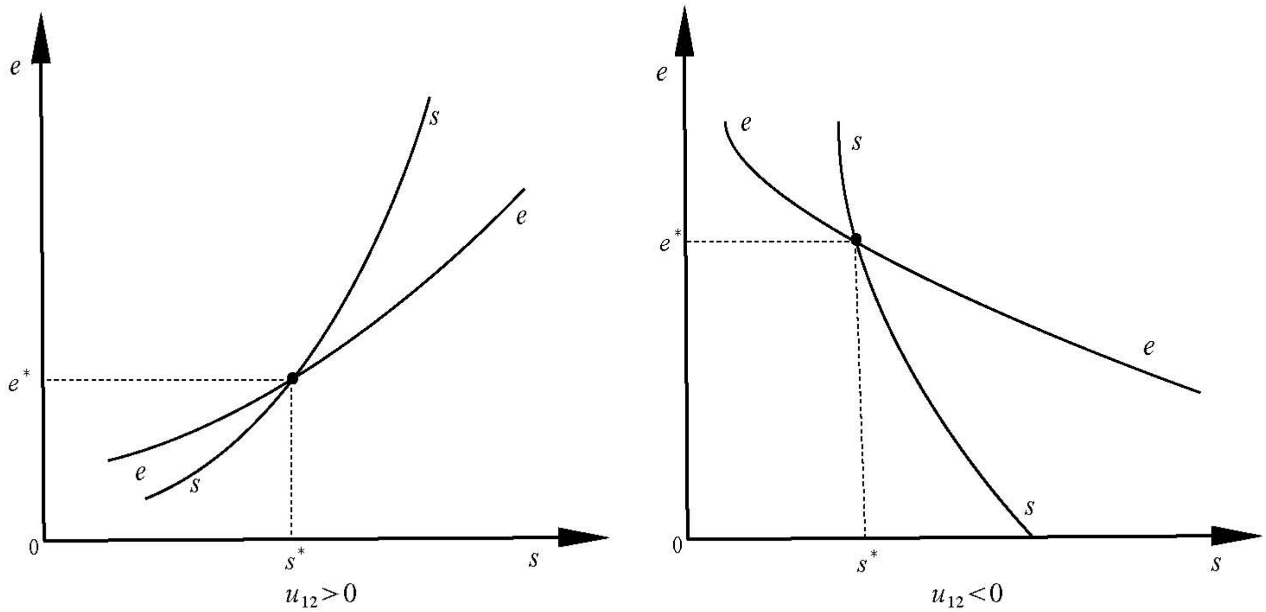

Figure 2: The effects of background risk on the optimal levels of s and e.