How Do Prudent Consumers Respond To Transitory Income Shocks? Reconciling Income Panel Data and Natural Experiments

Abstract

Estimations from panel data on income and consumption find that transitory income shocks are not followed by significant consumption changes, while results based on natural experiments obtain statistically significant and economically large responses of consumption to tax rebates. I account for these discrepancies by showing that existing panel data estimators neglect the correlation between log-consumption growth and past shocks caused by precautionary behavior. These interactions undermine the exogeneity of the instruments used to identify the shocks in panel data and generate a downward bias. I present an estimator that is robust to log-consumption growth being contingent on the consumers' history of shocks. The estimated elasticity of consumption to transitory shocks on net income shifts from 0.05 to 0.10 and becomes significant.

1 Introduction

How do consumers respond to transitory income shocks? Answering this question has widespread implications for a variety of macroeconomic questions, including the economy's response to fiscal shocks, the behavior of equilibrium asset prices, or the relation between income and consumption inequalities. Yet, two opposing views coexist. The literature interested in the impact of tax rebates, which constitute natural experiments of transitory income gain, find that the rebates have statistically significant and economically large effects on consumption. These strong responses are observed even though tax shifts are possibly anticipated and their impact blurred by simultaneous expectations of future tax increases [1] . On the other hand, a number of papers adopt a structural approach and build a life-cycle model with an income process whose parameters are estimated using panel survey data. Among these, the consensus is that transitory shocks have no impact on consumption, except possibly for the small fraction of consumers that are constrained[2]. This view is supported by both theoretical and empirical findings. Theoretically, even models that incorporate very few possibilities of insurance against income shocks conclude that transitory innovations should hardly be transmitted to consumption. A life-cycle model with quadratic utility and self-insurance—individuals can only smooth consumption by saving and borrowing a risk-free asset—predicts that consumers will almost completely smooth transitory shocks (Deaton (1992)). Also, with Constant Relative Risk-Aversion (CRRA) preferences and self-insurance, Blundell, Pistaferri and Preston (2008) and Blundell, Low and Preston (2013) obtain that the solution for the elasticity if consumption to transitory shocks can be approximated by a ratio that is close to zero. Empirically, the seminal paper of Blundell, Pistaferri and Preston (2008) estimates from panel survey data that only 5% of transitory income innovations are passed into consumption, in a framework that encompasses the standard model. Even more remarkably, Blundell, Pistaferri and Saporta-Eksten (2016) generalize the method to allow for endogenous labor supply and find that transitory wage shocks are associated with a significantly negative response of consumption[3].

The conflict between these two approaches is a problem: the study of income variations from panel survey data gives access to the typical shocks experienced by consumers and is not limited by the availability of natural experiments so it is important to dispose of such a technique, but it matters that it be reliable and consistent with other methods when comparable. In this paper, I show that the discrepancy between the results is resolved when accounting for precautionary behavior, which is implicitly ignored in the structural panel survey data estimation.

More precisely, I make three contributions. First, in a standard model with CRRA preferences and self-insurance, I derive an expression for the elasticity of consumption to transitory shocks that does not neglect the precautionary terms, and I find that this elasticity does not have to be small. The value predicted

[1]Souleles (1999) finds that, within the quarter, consumers raise significantly their expenditures in strictly non-durable by 2.6% of the tax rebate and 3% (not precisely estimated) for expenditures in nondurables; Souleles, Parker and Johnson (2006) find that it increases expenditures in strictly non-durable by 24% of the rebate and 37% of the rebate for expenditures in nondurables; Parker, Souleles, Johnson and McClelland (2013) obtain an increase of 9% of the rebate for strictly non-durables and 12% of the rebate for non-durables.

[2]A number of papers directly assume that transitory shocks are perfectly smoothed in order to improve the estimation of other aspects of their model. Examples are Blundell and Preston (1998), Heathcote, Storesletten and Violante (2007), Heathcote, Storesletten and Violante (2014) or Blundell, Pistaferri and Saporta-Eksten (2016).

[3]They interpret it as resulting from a non-separability between consumption and hours worked.

by existing approximations is almost zero because they do not account for precautionary effects. Second, I make the point these precautionary terms introduce a correlation between log-consumption growth and past shocks that biases the typical estimation method. In survey data, income shocks are identified with instruments that isolate the part of income volatility attributable to each type of shock. The instrument variables, however, depend on past shocks so that in the presence of precautionary behavior they covary with the log-consumption growth through both past and current shocks and the condition of instrument exogeneity does is not met. Variations in consumption caused by past shocks are erroneously construed as responses to the current shocks. Third, I generalize the estimation method to make it robust to the presence of a correlation between log-consumption growth and past shocks. When comparing the results before and after correction, I find that the bias caused by the correlation is significant and quantitatively important. With my corrected method, the consumption response to transitory income shocks is significantly different from zero and its magnitude is in line with results from the papers studying tax rebates.

I consider a standard life-cycle model. Finite-lived consumers face an uninsurable stochastic wage, subject to permanent and transitory shocks[4]. Consumers with isoelastic utility maximize their intertemporal utility subject to a budget constraint. Because such consumers are prudent, which to say their marginal utility is convex, the presence of risk modifies their consumption and saving decisions: they make precautionary saving, defined as the additional saving they do because of uncertainty[5] Consumers can save and borrow, but cannot default on their debt which generates a natural credit constraint: they never borrow more than the worst possible amount they expect to earn in the future so they can always repay what they owe.

In this model, precautionary behavior has three effects on log-consumption growth. First, the principle of precautionary behavior is that, when marginal utility is convex, negative shocks to consumption raises more the marginal utility of consuming an additional unit of good than positive shocks decreases it, which induces consumers to transfer consumption from the present period to future periods, at which the possibility of an unfortunate event raises expected marginal utility. As a result, these precautionary transfers increase expected consumption growth and expected log-consumption growth. Second, the response to unexpected income changes is modified too. This precautionary saving implies that prudent individuals consume less than they would in the absence of uncertainty. As a result, variations in consumption mechanically correspond to larger percentage changes of their consumption, which is to say larger changes in log-consumption growth. Finally, a given income innovation does not cause exactly the same variation in consumption than it would under perfect foresight, because the income news modifies expected precautionary saving. The direction of this effect depends on the type of shocks considered. A favorable transitory shock reduces the need for precautionary saving (it increases the income that is certain without modifying the distribution of future income), so precautionary behavior amplifies the increase

[4]This is consistent with microeconomic data on earnings: Attanasio and Davis (1996) show that consumers' revenue is not perfectly insured; the permanent-transitory structure of shocks is found to fit well with the observed dynamics of microeconomic of earnings (MaCurdy (1982), Abowd and Card (1989), Blundell and Preston (1998), Blundell, Graber and Mogstad (2015)).

[5]The alternative assumption that the impact of uncertainty can be neglected (certainty-equivalence) is unappealing as experimental evidence shows that the degree of risk associated with choices affect individual's decisions. Also, simulations show that even a small amount of uncertainty shifts substantially the predictions of consumption models, so that the effect is quantitatively important (Barsky, Mankiw and Zeldes (1986), Zeldes (1989)). The particular choice of isoelastic preferences is in accordance with experimental evidence on individual's behavior in front of risk (Eeckhoudt and Schlesinger (2006), Guiso and Paiella (2008)).

in consumption. A favorable permanent shock strengthens the need for precautionary saving (it raises expected income but increases its dispersion), so precautionary behavior mitigates the increase in consumption. These three effects drop in existing approximations of log-consumption growth, because these derivations impose that expected log-consumption growth be unaffected by past shocks, so that the first effect is neglected. It eventually implies that precautionary transfers be proportional to consumption so that the last two effects I mention are ignored too. A consequence is that empirical estimates of consumption elasticity not coinciding with the values obtained from existing approximations does not necessary indicate a failure of the model from which the approximations are derived, which is the standard model with self-insurance. The difference could come from precautionary effects.

The precautionary components of log-consumption growth are affected by past shocks, because they depend on the consumers' level of net assets, which is determined by the history of their past shocks. Therefore, when taking precautionary behavior into account, it is not possible to use instrumental variables that also depend on the realization of past shocks to identify the response of log-consumption growth to a contemporaneous shock. Otherwise, the instrument is not be exogenous and the covariance between log-consumption growth and past shocks is mistakenly attributed to the current shock.

My remedy to this lack of instrument exogeneity is to replace log-consumption growth by its innovation: when dropping the expected component, which is the part that correlates with past shocks, I eliminate the bias. This substitution restores the condition of instrument exogeneity when it is breached by a correlation with past shocks, but is innocuous in the absence of such a correlation. In effect, the unexpected component of log-consumption is the only part that is affected by current shocks, so none of the impact of an income innovation is being dropped out. Also, the corrected estimator is robust to the presence of any correlation with past shocks, whether it is generated by precautionary behavior, as in a standard model, or by other features that produce the same effect, such as borrowing constraints. The cost of this correction is that I need to make an assumption on the information set of consumers, to disentangle between the part of log-consumption growth that is expected and the part that is not. However, even if the set I build contains less information than consumers actually have, this substitution would still reduce the bias caused by the correlation of log-consumption growth with past shocks.

I implement this corrected estimator into data from the Panel Study of Income Dynamics (PSID) between 1979 and 1992 combined with imputed consumption data from the Consumer Expenditure Survey (CEX) over the same period, which is the exact same dataset as used in the seminal paper of Blundell, Pistaferri and Preston (2008). The reason I do not include later periods is that some questions regarding income and household characteristics change after 1992. The response to transitory shocks is 0.10 with my corrected estimator and is significant, while it is only 0.05 and not significant with an estimator that does not correct for the correlation of log-consumption growth with past shocks. The response to a permanent income shock is 0.61, as opposed to 0.66 without correction. When allowing for endogenous labor supply and insurance between the members of a household, the response to a transitory shock on the wage of the male earner is 0.09 and is significant, as compared to 0.04 without correction. The response to a permanent wage shock is 0.18, and 0.16 without correction. These findings suggest that there is indeed a correlation between the expected part of log-consumption and past shocks that biases the traditional estimation method. The results are robust to variations in the assumption regarding the information set available to consumers and in the persistence of the transitory component of earnings or wages.

Related Literature

This paper pertains to the literature that investigates the robustness and extends the applicability of the Blundell, Pistaferri and Preston estimator. Kaplan and Violante (2010) note that a number of biases could be altering the predictions of this estimator. In particular, they make the point that the identification strategy requires that log-consumption growth be independent from past income shocks, but they do not check analytically whether this condition is met in the model of Blundell, Pistaferri and Preston (I show it does not hold because of precautionary behavior). They also note that advanced information, mean-reverting shocks and heterogeneous income profiles could shift the estimator of Blundell, Pistaferri and Preston away from the true value of the parameters. To measure the quantitative impact of these possible biases, they implement the estimator on simulated data, and obtain that the biases are very small, except in the presence of strong borrowing constraints. It is possible that the estimator of Blundell, Pistaferri and Preston is close to the true values of the parameters when applied to these simulations, and yet substantially biased with survey data if the underlying income process differs from the one used in simulations, in particular if the income innovations are not normally distributed but skewed, which would bolster the precautionary motive. Blundell, Low and Preston (2013) extend the method of Blundell, Pistaferri and Preston (2008) to more general specifications of income dynamics. Heathcote, Storesletten and Violante (2014) explore the same consumption insurance mechanisms, but with a model that delivers closed-form solutions for consumption and hours worked, at the cost of a few additional assumptions about the economic environment. In particular, individuals smooth shocks within the family, but households are hand-to-mouth, so the bias I describe does not apply (but the quality of the estimates still rests upon the hypothesis that there is no precautionary behavior at the household level).

Section 2 exposes the baseline model and derives an approximation for log-consumption growth that does not ignore the precautionary correlation between log-consumption growth and past shocks. Section 3 presents an identification strategy that is robust to interactions between log-consumption growth and past shocks, and shows that ignoring them leads to a bias in the estimation of the consumption response to income shocks. Section 4 details the implementation of the estimators in panel data and the results: after correction, the response to transitory shocks on both earnings or wages is large and significant. Its values are in line with results from the literature on tax rebates. The overall estimation bias caused by ignoring the history dependence of log-consumption growth is significant. Section 5 concludes.

2 Model

The framework I consider is standard and encompasses the model underlying the estimation of Blundell, Pistaferri and Preston (2008). Finite-lived consumers maximize their intertemporal utility from consumption and leisure, subject to a budget constraint. They face a stochastic wage rate, shifted by permanent and transitory shocks at each period. Markets are incomplete and consumers only have a risk-free asset available to save and borrow. To clarify the presentation, I neglect the presence of the natural borrowing constraint, which prevents consumers from borrowing more than the maximum they could repay in any state of the world. The impact of this constraint on the response of consumption is presented in Appendix

B, together with the case of exogenous borrowing constraints.

2.1 Income Process

The log-wage rate of household i at period t is modeled as a permanent-transitory process, which is to say the sum of a permanent component p_t that follows a random walk, of a transitory component ε_t that follows an MA(q) process, and of a term capturing the influence of possibly time-varying individual characteristics $z_{i,t}$:

$$ln(w_{i,t}) = p_{i,t} + \varepsilon_{i,t} + \kappa_t z_{i,t} \tag{2.1}$$
$$with \begin{cases} p_{i,t} & = p_{i,t-1} + \eta_{i,t} \\ \varepsilon_{i,t} & = \mu_{i,t} + \theta_1 \mu_{i,t-1} + ... + \theta_q \mu_{i,t-q} \end{cases}$$

The shocks $\eta_{i,t}$ and $\mu_{i,t}$ are i.i.d. across households and across periods. I don't impose a log-normal distribution; in particular, the shocks can be drawn from a mixture of log-normals to match with recent evidence of skewed log-income distribution (Busch, Domeij, Guvenen and Madera (2015)). The variable $z_{i,t}$ is a vector of income characteristics, observable and known by consumers at time t. I allow their impact κ_t to vary over time and across cohorts. This specification encompasses models with fixed effects if some of the z variables are not time-varying ($z_{i,t} = z_i$), and allows for a common time/age trend if one the variable is the year or the consumers' age ($z_{i,t} = t$).

In the reminder, I drop the consumers' index i. This specification implies that for $0 \leq s \leq T - t$:

$$\Delta(ln(w_t) - \kappa_t z_t) = \eta_t + \Delta\varepsilon_t \tag{2.2}$$

The number of hours worked, denoted h_t, is a linear combination of a fixed, exogenous, number of hours $\bar{h}$ and a number of hours chosen by the worker $\hat{h}_t$: $h_t = (1-\alpha)\bar{h} + \alpha\hat{h}_t$. A model with exogenous labor supply corresponds to case where $\alpha = 0$. In that situation, income is proportional to the wage rate and can be represented as a stochastic endowment. In general, the period income of the consumers, denoted y_t is the product of the number of hours they worked, and their wage rate: $y_t = w_t h_t$.

2.2 Consumers' Problem

Consumers' intertemporal optimization problem is as follows:

$$\max_{c_t,...c_T} \quad E_t \left[\sum_{s=0}^{T-t} \beta^{t+s} e^{\delta_t z_t} \left(u(c_{t+s}) - g(h_{t+s}) \right) \right] \tag{2.3}$$

$$s.t. \quad \sum_{s=0}^{T-t} \frac{c_{t+s}}{(1+r)^s} = (1+r)a_t + \sum_{s=0}^{T-t} \frac{y_{t+s}}{(1+r)^s} \tag{2.4}$$

Time is discrete and indexed by t = 0, 1, ... T. Finite lived consumers with discount factor $\beta < 1$ and time-separable preferences derive utility from streams of consumption $\{c_s\}_{s=t}^T$, which is mitigated by a separate disutility from hours worked $\{h_s\}_{s=t}^T$. Period utility from consumption, $u(c)$ is in the Constant Relative

Risk Aversion (CRRA) class of functions or their counterparts with shifted origins[6]. Its functional form is $u(c) = \frac{c^{1-\rho} - cst}{1-\rho}$ and it is defined over $]0, +\infty[$. This implies in particular that marginal utility is decreasing (consumers are risk-averse) and convex (consumers are prudent). Period disutility from hours worked, $g(h)$, is of the form $g(h) = \rho \frac{h^{1+\sigma}}{1+\sigma 2}$. Net utility can be influenced by a vector of individual characteristics z_t whose impact is measured by coefficients δ_t. They may overlap with the characteristics that shift income. Consumers face the stochastic income endowment, y_t, bounded below by $\underline{y}_t = 0$. There are no state-contingent securities to insure idiosyncratic endowment risk, only a risk-free asset, a_t, which yields a constant gross interest rate $(1+r)$. Consumers can save and borrow but cannot default on their debt: $a_T \geq 0$. Together with the period budget constraints $a_{t+1} = (1+r)a_t + y_t - c_t$, this terminal wealth condition yields the intertemporal budget constraint (2.4). I present the more general case with borrowing constraints in Appendix B.

2.3 Consumption Allocation

Appendix A details formally the steps of the reasoning developped here. The equilibrium condition of the consumers' problem, known as the Euler equation, states that optimizing consumers equalize their expected marginal utility over time—weighted by $R_{t,t+k} = (\beta(1+r))^k e^{\delta_{t+k} z_{t+k} - \delta_t z_t}$ to capture the impact of the interest rate, the discount factor and changes in demographics :

$$u'(c_t) = E_t[u'(c_{t+1})] R_{t,t+1}$$

Following Kimball (1990b), I define the equivalent precautionary premium for consumption at $t+1$, denoted φ_t. It is the counterpart of the equivalent risk premium, applied to marginal utility instead of utility: φ_t is such that $E_t\left[u'(c_{t+1})\right] = u'(E_t[c_{t+1}] - \varphi_t)$. Under perfect foresight, defined as a situation in which income is certain and equal to its expected value, $c_{t+1} = E_t[c_{t+1}]$ and the premium φ_t is zero. In the presence of uncertainty, however, Jensen's inequality implies that the premium φ_t is strictly positive for prudent consumers, because their marginal utility is strictly convex.[7]. I combine this expression with the Euler equation and apply $u'(c)^{-1} = c^{-1/\rho}$ to each side:

$$c_t = (E_t[c_{t+1}] - \varphi_t) R_{t,t+1}^{-1/\rho}$$

The presence of φ_t indicates that prudent consumers choose, not only to equalize current consumption to future expected consumption (weighted by $R_{t,t+1}^{-1/\rho}$), but to transfer additional ressources from the current period to the next because of uncertainty. In effect, prudent consumers facing risk anticipate that, if an unfortunate event occurs in the future, their utility from consuming additional units of goods is going to be very high, while a good shock will not lower their marginal utility as much: they are willing to move consumption from the current, certain, period to future, uncertain, periods to have more resources in case a negative shock hits.

[6]This is because I impose that the function exhibits Hyperbolic Absolute Risk Aversion (HARA) with decreasing risk-aversion ($\frac{u''(c)}{-u'(c)}$ is decreasing). The Decreasing Absolute Risk Aversion subset of the HARA functions are the CRRA functions and their counterparts with shifted origins.

[7]When marginal utility $u'(c)$ is strictly convex, Jensen's inequality states that:

$$E_t\left[u'(c_{t+1})\right] > u'(E_t[c_{t+1}]) \Leftrightarrow u'(E_t[c_{t+1}] - \varphi_t) > u'(E_t[c_{t+1}]) \Leftrightarrow E_t[c_{t+1}] - \varphi_t < E_t[c_{t+1}] \Leftrightarrow 0 < \varphi_t$$

Iterating forward, I obtain that $c_t = E_t[c_{t+s}]R_{t,t+s}^{-1/\rho} - \sum_{k=1}^{s} E_t[\varphi_{t+k-1}]R_{t,t+k}^{-1/\rho}$, for any $0 < k < T-t$: because of uncertainty, consumers are willing to transfer an amount $\sum_{k=1}^{s} E_t[\varphi_{t+k-1}]R_{t,t+k}^{-1/\rho}$ from t to each future period $t+s$. Combining these expressions with the intertemporal budget constraint (2.4), consumption writes as a constant share of consumers' expected ressources, net of the sum of these expected precautionary transfers:

$$c_t = \frac{1}{l_{t,0}}\left(\underbrace{(1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s}}_{\text{total expected ressources: } W_t} - \underbrace{\sum_{k=1}^{T-t} l_{t,k} \frac{E_t[\varphi_{t+k-1}]}{(1+r)^k}}_{\text{total expected precautionary saving: } PS_t} \right)$$

The behavior of consumers facing risk can be interpreted as a permanent-income style decision, but applied to an uncertainty-adjusted measure of their total expected ressources instead of their raw total expected resources. Intuitively, in a risky environement, prudent consumers act as if they were poorer than they actually are: they mentally discard a part of their expected ressources that they reserve for the uncertain future. The term $\frac{1}{l_{t,0}} = (\sum_{s=0}^{T-t} \frac{R_{t,t+s}^{1/\rho}}{(1+r)^s})^{-1}$ measures the share of their total uncertainty-adjusted resources that consumers want to allocate to consumption at period t. It is exogenous and identical to the share obtained under perfect foresight. [8]. More generally, the term $\frac{1}{l_{t,k}} = (\sum_{s=0}^{T-t-k} \frac{R_{t+k,t+k+s}^{1/\rho}}{(1+r)^s})^{-1}$ is the share of ressources that consumers want to allocate to consumption between the beginning of period t and the beginning of period $t+k+1$. The sum, $\frac{1}{l_{t,0}} \sum_{k=1}^{T-t} l_{t,k} \frac{E_t[\varphi_{t+k-1}]}{(1+r)^k}$, corresponds to precautionary saving at period t, as it coincides with the difference between what would be consumed under perfect foresight (a share $\frac{1}{l_{t,0}}$ of total expected ressources) and what is actually consumed[9]. It is the net present value sum of the expected precautionary transfers at period t to all future periods $t+s$. Total expected precautionary saving is the sum of expected precautionary saving at all remaining periods.

2.4 Transmission of Income Shocks to Consumption

I take the difference in (weighted) consumption between two consecutive periods:

$$c_{t+1}R_{t+1}^{-\frac{1}{\rho}} - c_t = \underbrace{\varphi_t R_{t+1}^{-\frac{1}{\rho}}}_{\text{precautionary trend}} + \underbrace{\frac{1}{l_{t+1,0}} \sum_{s=0}^{T-t-1} \frac{(E_{t+1}-E_t)[y_{t+1+s}]}{(1+r)^s}}_{\text{revision of future resources}} - \underbrace{\sum_{k=1}^{T-t-1} \frac{l_{t+1+k,0}}{l_{t+1,0}} \frac{(E_{t+1}-E_t)[\varphi_{t+k}]}{(1+r)^k}}_{\text{revision of future precautionary saving}}$$

This expression clarifies the structure of the innovation to future consumption. Unexpected shifts in consumption between two periods are driven, first, by news about future income, second, by the revisions of future precautionary saving they imply. Also, precautionary behavior generates an expected transfer of consumption from period c_t to c_{t+1}, which raises expected consumption growth by an amount $\varphi_t R_{t+1}^{-\frac{1}{\rho}}$.

[8]When consumers are neither patient nor impatient ($\beta = \frac{1}{1+r}$) and individual characteristics are constant ($z_t = z$), $l_{t,0}$ tends toward one as T approaches infinity.

[9]Households consume less than they would under perfect foresight *at a given level of net assets a_t*. However, consumers that have been facing income risk for several periods might be consuming more than if they had had perfect foresight during these periods, because in the latter case they accumulate more assets than in the former ($(1+r)a_t > (1+r)a_t^{perfect foresight}$) and this additional wealth may offset the decrease in consumption caused by precautionary saving.

I take the logarithm of the above expression and I expand around the point where $(\varepsilon_{t+1},\eta_{t+1})=(0,0)$. I denote with a star the variables taken at this point. Log-consumption growth can be expressed as:

$$\Delta ln(c_{t+1}) = \underbrace{\frac{1}{\rho}ln(\beta(1+r))}_{\text{impatience}} + \underbrace{\frac{1}{\rho}\Delta(\delta_{t+1}z_{t+1})}_{\text{demographics}} + \underbrace{ln(1+\frac{\varphi_t}{R^{1/\rho}_{t,t+1}c_t} + \frac{c^*_{t+1}-E_t\left[c_{t+1}\right]}{R^{1/\rho}_{t,t+1}c_t})}_{(1)} \tag{2.5}$$

$$+\varepsilon_{t+1}\frac{\left(\frac{dW_{t+1}}{d\varepsilon_{t+1}}\right)^* \overbrace{-\left(\frac{dPS_{t+1}}{d\varepsilon_{t+1}}\right)^*}^{(3)}}{W^*_{t+1}\underbrace{-PS^*_{t+1}}_{(2)}} + \eta_{t+1}\frac{\left(\frac{dW_{t+1}}{d\eta_{t+1}}\right)^* \overbrace{-\left(\frac{dPS_{t+1}}{d\eta_{t+1}}\right)^*}^{(3)}}{W^*_{t+1}\underbrace{-PS^*_{t+1}}_{(2)}} + o(\varepsilon_{t+1},\eta_{t+1})$$

where W_{t+1} denotes total expected resources at $t+1$ and PS_{t+1} total expected precautionary saving at $t+1$. Let me first analyze this expression in the situation of perfect foresight, in which case the precautionary premium is zero so that the terms designated with numbers drop. The expected component of log-consumption growth is equal to $\frac{1}{\rho}ln(\beta(1+r))+\frac{1}{\rho}\Delta(\delta_{t+1}z_{t+1})$. It is exogenous and fully determined by the parameters of the model. The response of log-consumption to a transitory shock ε_{t+1}, which is a measure of the elasticity of consumption to transitory income, is simply the percentage change in future resources caused by a transitory shock, taken at the point $(\varepsilon_{t+1},\eta_{t+1})=(0,0)$. In the case of fixed hours worked, when income is exogenous, the percentage change in resources is equal to the ratio of expected future income over total expected resources, because a transitory gain of one unit increases total resources by $1\times y^*_{t+1}$ at the approximation point. This value is indeed very small, and under perfect foresight the impact of transitory shock on consumption should be practically imperceptible. Similarly, the response of log-consumption to a permanent shock is the percentage change in total resources it generates. In the case of fixed hours worked, this percentage change is equal to the ratio of total expected future income over total expected resources.

Precautionary behavior has three effects on the value of log-consumption growth, indicated with (1), (2) and (3) in equation (2.5). First, because of precautionary transfers between period t and $t+1$, expected log-consumption in larger: there is an additional, strictly positive term, denoted with (1), in the expression of expected log-consumption growth[10]. As the strength of the precautionary motive depends on the level of the state variables a_t and p_t, this term introduces some depency between consumers' history and their log-consumption growth. Second, prudent consumers spend a share of their uncertainty-adjusted resources, instead of a share of their total resources, and an income shock that raises resources without modifying precautionary saving generates a larger percentage change in uncertainty-adjusted resources than in total resources. In equation (2.5) this shows in the fact that the percentage change is computed with respect to adjusted resources, net of precautionary saving (denoted (2)). Third, wage shocks do not only cause changes in expected income, but also in expected precautionary saving. The sign of this effect depends on the persistence of the shock considered. Commault (2016) shows that, in the same model, a transitory shock reduces the need for precautionary saving while a permanent shock raises it (intuitively, because shocks are mutliplicative, a larger permanent income means that the magnitude of future shocks

[10] It is strictly positive because consumption is concave in transitory and permanent income or wage shocks (Carroll and Kimball (1996), Commault (2016)). Therefore, Jensen's inequality implies that $c_{t+1}(E_t[\varepsilon_{t+1}],E_t[\eta_{t+1}]) > E_t[c_{t+1}(\varepsilon_{t+1},\eta_{t+1})]$.

is increased). As a result, revisions in future precautionary saving amplifies the response to transitory shocks but mitigate the response to permanent shocks. Note that the comparison with the case of perfect foresight is made at a given level of net assets. Over time risk stimulates the accumulation of assets which would modify these conclusions.

As a result of these effects, the response of log-consumption growth to a shock does not have to coincide with the percentage change in resources caused by the shock. In the case of transitory shocks, both considering uncertainty-adjusted resources instead of total resources (2) and revising future expected precautionary transfers (3) raise the response above its perfect foresight value. In the case of permanent shocks, the impact of precautionary behavior on the response of log-consumption is undetermined because effects (2) and (3) have opposite directions. In all cases, the fact that the estimated elasticity of consumption to shocks differ from the percentage change in total resources caused by a shock cannot be used a test of whether the standard model with sef-insurance holds, because the standard model *does not* predict such a value for the elasticity. Finally, note that the comparison with perfect foresight is made at a given level of net assets. Over time risk stimulates the accumulation of assets which would modify these conclusions.

2.5 Comparison with Existing Approximations

How come that approximations derived from the same model an expression for log-consumption growth i) that is independent from past shocks and ii) in which the response of consumption to the shocks is the same as the percentage of total resources obtained under perfect foresight?

This is because the authors impose that expected log-consumption growth does not respond to past shocks. Precisely, in Blundell, Low and Preston (2013), to obtain that the difference between $t-1$ and t of the Taylor expansion of the log-total consumption (equation (30)) coincides with the innovation to log-consumption growth and a term that behaves as the variance of this innovation, one needs to assume that precautionary component of log-consumption growth is unaffected by shocks[11]. Blundell, Pistaferri and Preston (2008) use the approximation derived in Blundell, Low and Preston (2013), so they rely on this hypothesis too. In Blundell, Pistaferri and Saporta-Eksten (2016), this assumption is explicitely made[12]. With the notations presented here, it amounts to assuming that the term denoted (1) in equation (2.5) is independent from past shocks.

Yet, this assumption implies the elimination of all the other contributions of precautionary behavior to log-consumption growth. Formally, the consequence of the hypothesis that $ln\left(1+\frac{\varphi_t}{R_{t,t+1}^{1/\rho}c_t}+\frac{c_{t+1}^{*}-E_t[c_{t+1}]}{R_{t,t+1}^{1/\rho}c_t}\right)$ does not respond to past shocks is that $\frac{\varphi_t}{R_{t+1}^{1/\rho}c_t}=k_t$, with k_t an exogenous constant, and therefore that

[11] In effect, when taking the difference of equation (30) between $t-1$ and t, the term $\sum_{j=0}^{T-t}\theta_{it+j}\sum_{l=0}^{j}(E_t-E_{t-1})\mathcal{O}(E_{t+j-1}[|\varepsilon_{it+j}|^2])$. Setting it to zero, as the authors do, is equivalent to assuming that, for all j, $\mathcal{O}(E_{t+j-1}|\varepsilon_{it+j}|^2)$, which is the endogenous component of expected future log-consumption growth at $t+j-1$—it behaves like the variance of the change in marginal utility at $t+j$—, is unaffected by shocks between $t-1$ and t (past shocks)

[12] "The first component[of growth of the marginal utility of wealth e.g. of log-consumption growth], ω_t, is a function of the interest rate r, the discount factor δ, and the variance in the change of marginal utility and captures the intertemporal substitution and precautionary motives for savings. Assuming that the only source of uncertainty in this setup is the idiosyncratic wage shocks, ω_t is fixed over the cross-section." (p10)

$\varphi_t = k_t(R_{t+1}^{1/\rho} c_t)$. The Euler equation is $(1+k_t)c_t R_{t,t+1}^{1/\rho} = E_t[c_{t+1}]$. As a consequence, the optimal level of consumption is a share of total expected ressources, as in the perfect foresight case but with share coefficients different from $\frac{1}{l_{t,0}}$. The approximation of log-consumption growth around small shocks is therefore identical to what would be obtained under perfect foresight:

$$\Delta ln(c_{t+1}) = \underbrace{\frac{1}{\rho} ln(\beta(1+r))}_{\text{impatience}} + \underbrace{\frac{1}{\rho}(\Delta\delta_{t+1} z_{t+1})}_{\text{demographics}} + \varepsilon_{t+1} \frac{\left(\frac{dW_{t+1}}{d\varepsilon_{t+1}}\right)^*}{W_{t+1}^*} + \eta_{t+1} \frac{\left(\frac{dW_{t+1}}{d\eta_{t+1}}\right)^*}{W_{t+1}^*} + o(\varepsilon_{t+1}, \eta_{t+1})$$

Intuitively, by imposing that past shocks do not affect the precautionary component of expected log-consumption, they mechanically assume that current shocks do not affect the future expected precautionary component of log-income growth. Also, because the variance in the change rate of marginal utility has to be constant, changes in marginal utility have to be proprotional to marginal utility, and the precautionary premium has to be proportional to the level of consumption. Thus, consumption writes as a constant share of total expected resources, not uncertainty-adjusted resources.

The response of consumption to each shock coincide with the perfect foresight ratios, yet Blundell, Pistaferri and Preston (2008) interpret these expressions as reflecting precautionary behavior[13]. This interpretation misses the fact that the authors have thrown out precautionary behavior from their model. The form of the consumption response as a ratio over total expected ressources is not, here, a result of precautionary saving but an artifact of the logarithm.

3 Identification

The coefficients I want to estimate are as follow:

$$\phi^{\varepsilon} = \frac{cov(\Delta ln(c_t), \varepsilon_t)}{var(ln(\varepsilon_t))}$$
$$\phi^{\eta} = \frac{cov(\Delta ln(c_t), \eta_t)}{var(\eta_t)}$$

They capture how much log-consumption growth is expected to vary with respect to a given change in ε_t and η_t: they correspond to the coefficients of a linear regression of the shocks ε_t and η_t over log-consumption growth $\Delta ln(c_t)$. Because both the explanatory variable (shock to log-wage) and the dependent variable (log-consumption) are in logs, the coefficient ϕ can be interpreted as the percent change in consumption from a one percent change in wage, transitory or permanent, which is to say the elasticity of consumption to the transitory or permanent component of wage. If log-consumption growth is indeed a linear function of the shocks, those coefficients coincide exactly with the marginal effect of the shocks on log-consumption and thus with the elasticity; otherwise they represent a linear approximation of this

[13]" For individuals who are a long time from the end of their life with the value of current financial assets small relative to remaining future labor income, $\pi_t \approx 1$, and permanent shocks pass through more or less completely into consumption, whereas transitory shocks are (almost) completely insured against through saving. Precautionary saving can provide effective self-insurance against permanent shocks only if the stock of assets built up is large relative to future labor income, which is to say π_t is appreciably smaller than unity, in which case there will also be some smoothing of permanent shocks through self insurance." (page 1898) [$\pi_t = \left(\frac{E_t[\sum_{s=0}^{T-t-1} \frac{y_{t+1+s}}{(1+r)^s}]}{(1+r)a_t + E_t[\sum_{s=0}^{T-t} \frac{y_{t+s}}{(1+r)^s}]}\right)$ denotes the coefficient associated with permanent shocks]

marginal effect around small shocks.

The problem is that ε_t and η_t are not directly observed. To identify the covariance and variance that compose the coefficients, Blundell, Pistaferri and Preston rely on instruments: they regress log-consumption growth and log-income growth on instrumental variables that covary with log-income growth only through the realization of the transitory or of the permanent shock.

I do not write down the contribution of demographic variables, as they are assumed to be known in advance by consumers and do not covary with anything. To clarify the exposition, I assume in this section that ε follows an MA(0) process, but the spirit of the identification method is identical with an MA(1), which is the specification that best fit the data. A generalization of the method to any MA(q) process is detailed in the Appendix of Blundell, Pistaferri and Preston and can be applied to the identification presented here.

3.1 Transitory Shocks

An appropriate instrument to identify the impact of transitory shocks is future log-wage growth, $\Delta ln(w_{t+1})$. I use equations (2.2) and (2.5) to substitute for log-wage growth and log-consumption growth:

$$\begin{aligned} cov(\Delta ln(w_t), \Delta ln(w_{t+1})) &= cov(\eta_t + \varepsilon_t - \varepsilon_{t-1}, \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_t) \\ &= -var(\varepsilon_t) \\ cov(\Delta ln(c_t), \Delta ln(w_{t+1})) &= cov(\Delta ln(c_t), \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_t) \\ &= -cov(\Delta(ln(c_t), \varepsilon_t) \end{aligned}$$

An estimator of the transitory coefficient is:

$$\hat{\phi}^{\varepsilon} = \frac{cov(\Delta ln(c_t), \Delta ln(w_{t+1}))}{cov(\Delta ln(w_t), \Delta ln(w_{t+1}))}$$

This amounts to instrumenting the impact current log-wage growth by future log-wage growth. The reason why future log-wage growth is a good instrument here is because the current realization of the transitory shock is the only component of current log-wage growth that introduces a variation in both current log-wage growth and future log-wage growth: when a transitory shock hits, it increases current log-wage growth, but reduces it by the same amount at the next period, as the wage goes back to its initial value. On the contrary, permanent shocks last for all remaining periods, therefore they do not cause any variation in future log-wage growth; past transitory shocks affect current wage growth but not future wage growth so their impact is also eliminated by the instrumentation.

To this point, the only assumption needed regarding $\Delta ln(c_t)$ is that it is independent from future shocks but no absence of correlation with past shocks is required. Alone, this estimator is unbiased, even in the presence of precautionary effects. Yet, because the response to transitory shocks is estimated jointly with the permanent coefficient, its measure can be altered if a correlation with past shocks distorts the latter.

3.2 Permanent Shocks

Instrumenting by the sum of past, current and future log-wage growth eliminates the variations in current log-wage growth and log-consumption growth that are caused by contemporaneous and past transitory shocks:

$$\begin{aligned}
cov(\Delta ln(w_t), \Delta ln(w_{t-1}) + \Delta ln(w_t) + \Delta ln(w_{t+1})) &= cov(\eta_t + \varepsilon_t - \varepsilon_{t-1}, \eta_{t-1} + \eta_t + \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_{t-2}) \\
&= var(\eta_t) \\
cov(\Delta ln(c_t), \Delta ln(w_{t-1}) + \Delta ln(w_t) + \Delta ln(w_{t+1})) &= cov(\Delta ln(c_t), \eta_{t-1} + \eta_t + \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_{t-2}) \\
&= cov(\Delta ln(c_t), \eta_t) + \underbrace{cov(\Delta ln(c_t), \eta_{t-1}) - cov(\Delta ln(c_t), \varepsilon_{t-2})}_{\text{precautionary effects}}
\end{aligned}$$

In effect, contemporaneous transitory shocks increase the log-wage growth at one period and reduce it by the same amount at the next: they have no impact on the sum of current and future log-wage growth so they do not cause any variations in the instrument and their impact is selected out. This method also excludes variations caused by past transitory shocks because these raise past log-wage growth but then reduce current log-wage growth by the same amount and thus have no effect on their sum.

This instrument identifies the variance of the permanent shock, because it correlates with current log-wage growth only through η_t. When consumers have a precautionary motive, however, it covaries with log-consumption growth both through η_t and through past shocks, which influence the precautionary terms in log-consumption growth. In effect, the realizations of past shocks determine the amount of net assets that consumers have at their disposal, thus their current need for precautionary saving and the steepness of their log-consumption growth. Intuitively, the estimator erroneously captures the correlation of log-consumption with past transitory shocks (through precautionary saving) as a correlation with the current shock.

This precautionary effect can be recovered and eliminated at the cost of making an assumption on the information set of consumers at $t-1$, by building $E_{t-1}[\Delta ln(c_t)]$:

$$\begin{aligned}
cov(E_{t-1}[\Delta ln(c_t)], \Delta ln(w_{t-1}) + \Delta ln(w_t) + \Delta ln(w_{t+1})) &= cov(E_{t-1}[\Delta ln(c_t)], \eta_{t-1} + \eta_t + \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_{t-2}) \\
&= \underbrace{cov(\Delta ln(c_t), \eta_{t-1}) - cov(\widetilde{\varphi}_{t-1}, \varepsilon_{t-2})}_{\text{precautionary effects}}
\end{aligned}$$

An estimator of the coefficient associated with permanent shocks is:

$$\hat{\phi}^{\eta} = \frac{cov(\Delta ln(c_t) - E_{t-1}[\Delta ln(c_t)],\ \Delta ln(y_t) + \Delta ln(y_{t+1}))}{cov(\Delta ln(y_t), \Delta ln(y_{t-1}) + \Delta ln(y_t) + \Delta ln(y_{t+1}))}$$

Log-consumption growth is replaced by its innovation, which is independent from past shocks. The covariance between log-consumption and the permanent shock is identified with using $\Delta ln(w_t) + \Delta ln(w_{t+1})$ only as an instrument, because the modification eliminates any correlation with past variables: $\Delta ln(w_{t-1})$ is independent from $\Delta ln(c_t) - E_{t-1}[\Delta ln(c_t)]$ and this term has no impact on the covariance.

The hypothesis that I have to make on the information available to consumers at $t-1$ can be tested by looking into the impact of variations in the information set. I present such robustness checks in section 4. Also, if the information set I use contains less information than is available to consumers, replacing total log-consumption growth by its innnovation would still improve the estimation and reduce the bias caused by precautionary behavior. In the limit case when I assume that consumers have zero information, their expectation is a constant and innovation to log-consumption growth coincides with total log-consumption growth: the estimator is identical to one that ignores the correlation between log-consumption growth and past shocks.

When the coefficients ϕ are estimated independently, only the one associated with permanent innovations should be subject to lack of instrument exogeneity. Yet, Blundell, Pistaferri and Preston implement their estimator in survey data; they use more moments than required for identification and estimate the coefficients jointly. In that case, biases can affect the measure of any of the parameters that are being estimated, in particular the coefficient associated with transitory shocks, and I cannot predict their directions.

3.3 Empirical Implementation

The model provides more restrictions on the autocovariance of consumption growth, the autocovariance of wage growth and the covariance of the two than just those required for identification. Following Blundell, Pistaferri and Preston (2008), to take advantage of these additional moments and get a more precise estimation, I use a minimum distance estimator. I build a vector m that contains the empirical counterparts of $cov(\Delta w_t, \Delta w_{t+s})$ and $cov(\Delta ln(c_t) - E_{t-1}[\Delta ln(c_t)], \Delta w_{t+s})$ for $1 \leq t \leq T$ and $0 \leq s \leq q+1$—where q is the dimension of the MA(q) transitory component of log-wage.

The estimation model is:

$$m = f(\Lambda) + \Upsilon$$

where Λ is the vector of parameters I am interested in. It contains the variance of the transitory shock at each period $var(\varepsilon_t)$, the variance of the permanent shock at each period $var(\eta_t)$, the elasticities ϕ^{ε} and ϕ^{η}, and the coefficient of the income process θ_1 (in the case when the transitory income process is an MA(1) only). The vector Υ captures sampling variability. I estimate Λ by solving:

$$\min_{\Lambda} (m - f(\Lambda))' A (m - f(\Lambda))$$

A is a weighting matrix. In the case of the diagonally weighted minimum distance estimator used here, it is a diagonal matrix. The elements in the main diagonal are given by $diag(V^{-1})$, with V the variance-covariance matrix of m.

The estimator of Blundell, Pistaferri and Preston uses restrictions on the autocovariance of log-consumption growth, $cov(\Delta c_t, \Delta c_{t+s})$, that do not hold when there is a precautionary correlation between log-consumption growth and past variables. These moments generate additional estimation biases that may intensify or lessen the initial bias, depending on their direction. I do not use them in my control estimation, so that the difference I observe be entirely driven by the correlation of log-consumption growth to past shocks. I compare the results with and without these moments and find that the bias they induce is very small.

4 Data and Results

4.1 Data

For comparison purposes, I use the same dataset as Blundell, Pisteferri and Preston, which they kindly make available online. The files contain observations from the Panel Study of Income Dynamics (PSID) between 1978 and 1992[14]. The part of the sample focused on low-income families (SEO sample) is excluded. The dataset selects households followed for at least two consecutive years, composed of a married couple (with or without children) whose head is between 30 and 65 years old. This is to avoid problems associated with changes in family composition (for the youngest) and changes in income process due to retirement (for the oldest). Households facing some dramatic family composition change over the sample period are dropped: the dataset contains only those with either no change, or changes in members other than the head or the wife. This is to avoid modeling the risk associated with divorce, widowhood, or other household breaking-up factors, and focus on income risk. Finally, households with missing report on race, education, and region and some income outliers are eliminated. The final sample is composed of 12,058 observations and 1,765 households.

I use alternatively income (in the case of exogenous labor supply) and the wage rate (in the case of endogenous labor supply) as the source of uncertainty for consumers. Net income is made of the taxable family income reported by the household, from which I remove income from financial assets, and federal taxes on nonfinancial income, and which I deflate by the Consumer Price Index (CPI). I assume that federal taxes on nonfinancial income are a proportion of total federal taxes; the proportionality coefficient is given by the ratio between nonfinancial income and total income. Raw income is the taxable family income, net of financial assets and deflated by the CPI Each earner's wage rate is built as its yearly real labor income divided by its yearly number of hours of worked. Questions on income are retrospective and refer to the previous calendar year.

Unfortunately, the PSID only reports food consumption, while it is more adequate to use a broader category of non-durable consumption for the present exercise. To overcome the problem, non-durable consumption is imputed from demographics and food consumption, with the coefficients used for the imputation computed from the Consumer Expenditure Survey (CEX) over the same period. Further details are provided in the original paper by Blundell, Pistaferri and Preston (2008). Non-durable consumption is the sum of food (at home and away from home), alcohol, tobacco, non-durable services, heating fuel, public and private transport (including gasoline), personal care, clothing and footwear. In particular, this definition excludes expenditure on housing, health, and education. To obtain the real analog to nominal consumption, it is deflated by the CPI. The PSID survey questions on food expenditure ask about typical weekly spending: it has been argued that people report their food expenditures for an average week around March (the period of the survey), rather than for the previous calendar year as is the case for family income. Blundell, Pistaferri and Preston test this alternative assumption and find no significant effect.

[14]I considered including additional years after 1992, but a number of the questions used by Blundell, Pistaferri and Preston are redesigned in 1993, and the impact of these changes is difficult to measure. From 1999, the survey is remodeled again, more substantially, and is only conducted every two years.

I consider variables that are net of the deterministic effects of the period and their individual characteristics. More precisely, they are regressed on year and year-of-birth dummies, and on dummies for education, race, family size, number of children, region, employment status, residence in a large city, outside dependent, and presence of income recipients other than husband and wife, interacted with a cohort dummy.

4.2 Innovation To Log-Consumption Growth

Expected log-consumption is the fitted value of log-consumption (net of deterministic components), after additionally regressing it on the variables constituting the information set of the consumers. My baseline information set includes lagged consumption growth and income or wage rate growth as well as the lagged value of the households' house, financial assets, food consumption (at home and away from home) and food stamps. These latter variables should capture the strength of the precautionary motive.

Table 1: Predicted log-consumption growth - baseline information set

Variable	Coefficient	p-value
$ln(c_{t-1})$	-0.486	0.000
$ln(y_{t-1})$	0.111	0.000
wife income at $t-1$	-0.000	0.001
house at $t-1$	0.000	0.000
financial income at $t-1$	0.000	0.001
food at $t-1$	-0.000	0.001
food out at $t-1$	0.000	0.000
food stamps at $t-1$	-0.000	0.000
constant	0.002	0.815
Adjusted R^2	0.241	
Observations	12,058	

Table 1 shows the details of the regression of $ln(c_t)$ over the variables the baseline information set. Most importantly, the R^2 indicates that, contrary to the hypothesis that once the effect of demographics is removed log-consumption growth is independent from past shocks, a fourth of the volatility in log-consumption is still predictable with past variables. It makes a case for the necesity to account for history dependence of log-consumption growth to past shocks.

4.3 Results

4.3.1 Exogenous Labor Supply

In this first section, I consider that hours worked are exogenous, so that shocks to the wage rate can equivalently be captured as income shocks, and I measure the elasticity of consumption to underlying income shocks.

Table 2: Estimates of ϕ - Shocks on Income

		Net Income	Raw Income
Corrected BPP	Transitory shocks: ϕ^{ε}	**0.10** (0.03)	**0.10** (0.03)
	Permanent shocks: ϕ^{η}	0.61 (0.09)	0.24 (0.05)
BPP	Transitory shocks: ϕ^{ε}	**0.05** (0.04)	**0.06** (0.03)
	Permanent shocks: ϕ^{η}	0.66 (0.10)	0.31 (0.06)

The first two lines of Table 2, labelled *CorrectedBPP*, report the results obtained with my estimator, robust to the presence of a correlation with past income shocks. The bottom lines, labelled *BPP*, correspond to the estimator of Blundell, Pistaferri and Preston[15]. The table shows that, when accounting for precautionary behavior, the estimated elasticity of consumption to transitory income becomes large and significant: the estimate shifts from 0.05 with the traditional estimator to 0.10 with my corrected estimator in the case of shocks to net income; it increases from 0.06 to 0.10 in the case of shocks to raw income. The corrected figures are consistent with results obtained in the tax rebate literature which find significant responses of consumption to transitory shocks. The elasticity to permanent income is not modified much by the correction: the estimate decreases from 0.66 to 0.61 when considering shocks to net income; it decreases from 0.31 to 0.24 when considering raw income. This slight overestimation of the response to permanent shocks is consistent with the direction of the estimation bias I expose. It is not suprising that the elasticity be quite below one, because most consumers finance their consumption with both their income and the positive stock of net assets they have accumulated: a percentage increase in income cannot translate in a one-for-one percentage increase in consumption.

It is consistent with the model I present to observe that, together with a strong and significant bias, I find a significant response to transitory shocks. In effect, recall from the identification section that the bias is caused by the non-zero correlation between a transitory shock ε_{t-1} and the contribution of precautionary behavior to expected log-consumption growth $\widehat{\varphi}_t$. If this correlation is strong, then the response of consumption to transitory shocks should be too, because the components of $\widehat{\varphi}_t$ are also in $\Delta ln(c_t)$. It would have been incompatible with my theoretical findings that the total bias be strong, but the response to transitory shocks remain small and non-significant in the corrected estimation. It is reassuring that it is not the case.

Finally, the comparison of the impact of shocks to net income versus shocks to raw income indicates that taxes and transfers act provide substantial insurance in particular against permanent shocks: consumers respond a little less to news about their transitory raw income than to news about their transitory net income; and much less to news about their permanent raw income than to news about their permanent net income.

[15]These results coincide almost but not exactly with those presented in the authors' original paper. This is because I exclude the autocovariance of log-consumption growth from the moments used for estimation. In effect, in the presence of correlation with past shocks, the moments of log-consumption growth simply bias the estimation further. When I incorporate these moments, the estimates are 0.05 (0.04) for ϕ^{ε} and 0.64 (0.09) for ϕ^{η}.

4.3.2 Endogenous Labor Supply

I relax the assumption that hours worked are fixed and look into the elasticity of consumption to shocks on each earner's wage rate.

Table 3: Estimates of ϕ - Shocks on Wages

		Male Wage	Female Wage
Corrected BPP	Transitory shocks: ϕ^{ε}	**0.09** (0.02)	**0.03** (0.02)
	Permanent shocks: ϕ^{η}	0.18 (0.05)	0.08 (0.04)
BPP	Transitory shocks: ϕ^{ε}	**0.04** (0.03)	**0.02** (0.02)
	Permanent shocks: ϕ^{η}	0.16 (0.06)	0.06 (0.04)

Table 3 shows that the estimated elasticity of consumption to transitory shocks on the wage rate of the male earner is large and significant: it is estimated at 0.09 instead of 0.04 without correction. The elasticity to permanent shocks is 0.18, after correction. It is close to its value before correction of 0.16. Both values are close to the elasticity of consumption to labor income: there is not much difference between considering shocks on the wage rate or shocks on labor income. This indicates that the hypothesis of fixed hours worked is not too strong of an assumption. The elasticity of consumption to the female wage rate is much smaller and not significantly different from zero: it is estimated at 0.03 after correction. The value before correction is 0.02. The response of log-consumption to permanent shocks is equally smaller: the estimate is 0.08 and is not very different from its value before correction of 0.06. This more modest response of the households' consumption to shocks on the wage rate of female must be related to the observation that that female earners work on average less hours, so that a change in the wage rate has a smaller impact on their earnings. Also, the wage rate of females is below that of males, which implies that a percentage change in their wage rate corresponds to a smaller gain.

4.4 Robustness Checks

Table 4: Estimates of ϕ - Variations in the information set

Information set	$I_0 = \emptyset$	$I_1 = (c, y)$	$I_2 = (c, y, z^s)$	$I_3 = (c, y, z^s, z^l)$
Transitory: ϕ^{ε}	**0.05** (0.04)	**0.12** (0.03)	**0.10** (0.03)	**0.10** (0.03)
Permanent: ϕ^{η}	0.66 (0.10)	0.61 (0.09)	0.61 (0.09)	0.54 (0.09)

Table 4 presents the impact of varying the hypothesis on consumers' information set on the estimated elasticity of consumption to shocks on net income. I need to make an assumption on the information set to build the empirical counterpart to innovations in log-consumption growth: $\Delta ln(c_t) - E_{t-1}[\Delta ln(c_t)]$. When the information set is empty, innovations to log-consumption growth coincide with log-consumption growth and the estimator is the BPP estimator. Results presented in Table 4 shows that my findings are fairly robust to variations in the composition of the information set: all elasticity estimates are within one standard deviation from another, except for the empty set. Even when only consumption and income are included (I_1), the elasticity to transitory income is significantly different from zero and estimated at 0.12, which is twice as large as without correction. The elasticity to permanent income decreases slightly to 0.61. The baseline information set, I_2, which contains consumption and income, and more detailed variables (denoted z^s) on consumers' financial resources—net assets, the house's value, the female wage, and the hours worked by the male earner—and the structure of their consumption—consumption of total food and of food away from home and the value of food stamps received. The elasticity to transitory income decreases a little to 0.10. The elasticity to permanent income is unchanged. Finally, I add a set of cross products variables with the idea that the impact of predictors of consumption is not linear so higher order terms would help fit more precisely future consumption growth (denoted z^l). I find that the estimated elasticity of consumption to transitory shocks is unaffected. The elasticity to permanent shocks decreases at 0.54. This suggests that the baseline set is a good representation of the information consumers use to predict their future expected consumption. I present here the results obtained when considering net income, but I find very similar outcomes with raw income.

Table 5: Estimates of ϕ - Variations in the persistence of transitory shocks

Transitory Process	MA(0)	MA(1)	MA(2)
Transitory: ϕ^{ε}	**0.11** (0.04)	**0.10** (0.03)	**0.10** (0.03)
Permanent: ϕ^{η}	0.44 (0.05)	0.61 (0.09)	0.73 (0.13)
First lag: θ_1	0 (*n.a.*)	0.11 (0.02)	0.20 (0.03)
Second lag: θ_2	0 (*n.a.*)	0 (*n.a.*)	0.04 (0.03)

In Table 5 I test the robustness of the results across different assumptions on the persistence of transitory shocks (on net income). My baseline assumption is that the transitory component of log-income has MA(1) serial correlation, because some second-autocovariance are significant. Yet the evidence of serial correlation is mixed—only some second order coefficients are significant and it is worth investigating the impact of this hypothesis. The estimated coefficients of the transitory process confirm that an MA(1) fits better: θ_1 is found to be significantly different from zero while θ_2 is not. The response of log-consumption growth to a transitory shock is very robust to variations in their persistence. The estimates obtained for the three specifications are very close. The estimate is 0.11 with an MA(0), and 0.10 with an MA(1) and an MA(2). The estimated response to a permanent shock varies more across specifications. The estimate is 0.44 with an MA(0), 0.61 with an MA(1) and 0.73 with an MA(2).

5 Conclusion

In a standard consumption model with income risk, precautionary behavior raises log-consumption growth: because it is possible that a disastrous outcome materializes in the future and because additional consumption would be very valuable if this happens, consumers facing uncertainty choose to transfer more resources to the future. The size of these precautionary transfers depends on consumers' stock of net assets, which is determined by the realizations of past income shocks. There is therefore a direct correlation between past shocks and log-consumption growth through the precautionary motive.

This effect compromises the estimation of the consumption response to permanent income innovations. In effect, the technique used to identify permanent shocks is to instrument the impact income growth on consumption growth by the sum of current and future income growth, to make sure that the change is permanent and that the current increase in income does not translate into a decrease at the next periods. In the presence of a correlation between current consumption growth and past shock, this method can erroneously capture the response of-consumption growth to past transitory shocks as a response to the current permanent shock, which causes a bias in the measure of the elasticity of consumption to shocks. A solution is to replace log-consumption growth by its innovation, which is by construction independent from past shocks. This transformation is innocuous if consumers have no precautionary motive. The technique, however, requires assumptions on the amount of information consumers have at their disposal, to build an empirical counterpart to the unexpected component log-consumption growth. Yet, in the case I assume less information than consumers actually have, I still reduce part of the bias: in the limit case when I make the hypothesis that consumers have zero information, the innovation to log-consumption growth coincides with total log-consumption growth and the corrected estimator is identical to the biased estimator.

My correction generates large and significant changes in the estimates of consumption elasticity to transitory income shocks and transitory wage shocks. The estimate raises from 0.05 to 0.10 and becomes significant in the case of an income shock. It raises from 0.04 to 0.09 in the case of a shock on the wage rate of the male earner. This indicates that neglecting the precautionary component of consumption growth produces quantitatively important biases. The larger response to transitory shocks is consistent with evidence regarding the large impact of transitory tax rebates. These results are robust to a number of variations in the estimation procedure.

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Appendix A Consumption Allocation - Without A Borrowing Constraint

A.1 Consumption Level

The Euler equation is:

$$u'(c_t) = E_t[u'(c_{t+1})]R_{t,t+1}$$

Following Kimball (1990b), I define φ_t the equivalent precautionary premium for consumption at $t+1$. It is the variable φ_t such that

$$E_t\left[u'(c_{t+1})\right] = u'(E_t[c_{t+1}] - \varphi_t)$$

From Jensen's inequality, the premium φ_t is strictly positive for strictly prudent consumers (marginal utility is strictly convex) and zero for certainty-equivalent consumers (marginal utility is linear)[16]. Combining this expression with the Euler equation and applying $u'(c)^{-1} = c^{-1/\rho}$ yields:

$$\begin{aligned} u'(c_t) &= u'(E_t[c_{t+1}] - \varphi_t)R_{t,t+1} \\ c_t &= (E_t[c_{t+1}] - \varphi_t)R_{t,t+1}^{-1/\rho} \end{aligned} \tag{A.1}$$

This is true at any period t, thus it is true at $t+1$:

$$\begin{aligned} c_{t+1} &= (E_{t+1}[c_{t+2}] - \varphi_{t+1})R_{t+1,t+2}^{-1/\rho} \\ E_t[c_{t+1}] &= (E_t[c_{t+2}] - E_t[\varphi_{t+1}])R_{t+1,t+2}^{-1/\rho} \end{aligned} \tag{A.2}$$

Plugging (A.1) in (A.2) yields:

$$\begin{aligned} c_t &= E_t[c_{t+2}](R_{t,t+1}R_{t+1,t+2})^{-1/\rho} - E_t[\varphi_{t+1}](R_{t,t+1}R_{t+1,t+2})^{-1/\rho} - \varphi_t R_{t,t+1}^{-1/\rho} \\ c_t &= E_t[c_{t+2}]R_{t,t+2}^{-1/\rho} - E_t[\varphi_{t+1}]R_{t,t+2}^{-1/\rho} - \varphi_t R_{t,t+1}^{-1/\rho} \end{aligned}$$

Iterating forward, I obtain that for any $0 < s < T-t$:

$$c_t = E_t[c_{t+s}]R_{t,t+s}^{-1/\rho} - \sum_{k=1}^{s} E_t[\varphi_{t+k-1}]R_{t,t+k}^{-1/\rho}$$

Therefore, I can express future expected consumption as a function of current consumption and the precautionary premiums:

$$E_t[c_{t+s}] = c_t R_{t,t+s}^{1/\rho} + \sum_{k=1}^{s} E_t[\varphi_{t+k-1}]R_{t+k,t+s}^{1/\rho}$$

[16] When marginal utility is strictly convex, Jensen's inequality implies:

$$E_t\left[u'(c_{t+1})\right] > u'(E_t[c_{t+1}]) \;\Leftrightarrow\; u'(E_t[c_{t+1}] - \varphi_t) > u'(E_t[c_{t+1}]) \;\Leftrightarrow\; E_t[c_{t+1}] - \varphi_t < E_t[c_{t+1}] \;\Leftrightarrow\; 0 < \varphi_t$$

I combine these expressions with the intertemporal budget constraint (2.4):

$$
\begin{aligned}
\sum_{s=0}^{T-t} \frac{E_t[c_{t+s}]}{(1+r)^s} &= (1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} \\
\sum_{s=0}^{T-t} \frac{c_t R_{t,t+s}^{1/\rho}}{(1+r)^s} + \sum_{s=1}^{T-t} \sum_{k=1}^{s} \frac{E_t[\varphi_{t+k-1}] R_{t+k,t+s}^{1/\rho}}{(1+r)^s} &= (1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} \\
\sum_{s=0}^{T-t} \frac{c_t R_{t,t+s}^{1/\rho}}{(1+r)^s} + \sum_{k=1}^{T-t} \sum_{s=k}^{T-t} \frac{E_t[\varphi_{t+k-1}] R_{t+k,t+s}^{1/\rho}}{(1+r)^s} &= (1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} \\
c_t \sum_{s=0}^{T-t} \frac{R_{t,t+s}^{1/\rho}}{(1+r)^s} + \sum_{k=1}^{T-t} \frac{E_t[\varphi_{t+k-1}]}{(1+r)^k} \sum_{s=k}^{T-t} \frac{R_{t+k,t+s}^{1/\rho}}{(1+r)^{s-k}} &= (1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} \\
l_{t,0} c_t + \sum_{k=1}^{T-t} \frac{E_t[\varphi_{t+k-1}]}{(1+r)^k} \sum_{s=0}^{T-t-k} \frac{R_{t+k,t+k+s}^{1/\rho}}{(1+r)^s} &= (1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} \\
l_{t,0} c_t + \sum_{k=1}^{T-t} l_{t,k} \frac{E_t[\varphi_{t+k-1}]}{(1+r)^k} &= (1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s}
\end{aligned}
$$

with $l_{T-t,k} = \sum_{s=0}^{T-t-k} \frac{R_{t+k,t+k+s}^{1/\rho}}{(1+r)^s}$. Therefore, an expression for consumption is:

$$
c_t = \frac{1}{l_{t,0}} \left(\underbrace{(1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s}}_{\text{total expected ressources}} - \underbrace{\sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t+s-1}]}{(1+r)^s}}_{\text{total expected precautionary saving}} \right) \tag{A.3}
$$

Intuitively, instead of consuming a given share $l_{t,0}$ of their ressources (assets plus total future expected income), prudent consumers put aside an expected precautionary amount $\sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t+s-1}]}{(1+r)^s}$ and consume a share of the remaining part of their ressources only.

A.2 Consumption Growth

I consider equation (A.3), taken at period $t+1$:

$$
c_{t+1} = \frac{1}{l_{t+1,0}} \left((1+r)a_{t+1} + \sum_{s=0}^{T-t-1} \frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s} - \sum_{s=1}^{T-t-1} l_{t+1,s} \frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s} \right)
$$

I use the period budget constraint $a_{t+1} = (1+r)a_t + y_t - c_t$ to substitute for a_{t+1} in this expression:

$$
c_{t+1} = \frac{1}{l_{t+1,0}} \left((1+r)^2 a_t + (1+r)y_t - (1+r)c_t + \sum_{s=0}^{T-t-1} \frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s} - \sum_{s=1}^{T-t-1} l_{t+1,s} \frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s} \right)
$$

I add $\frac{(1+r)}{l_{t+1,0}}\left(\sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s}-\sum_{s=1}^{T-t}l_{t,s}\frac{E_t[\varphi_{t-1+s}]}{(1+r)^s}\right)-\frac{(1+r)}{l_{t+1,0}}\left(\sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s}-\sum_{s=1}^{T-t}l_{t,s}\frac{E_t[\varphi_{t-1+s}]}{(1+r)^s}\right)=0$ on the right-hand side.

$$\begin{aligned}
c_{t+1}=&\frac{1}{l_{t+1,0}}\left((1+r)^2a_t+(1+r)y_t-(1+r)c_t+\sum_{s=0}^{T-t-1}\frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s}-\sum_{s=1}^{T-t-1}l_{t+1,s}\frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s}\right)\\
&+\frac{(1+r)}{l_{t+1,0}}\left(\sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s}-\sum_{s=1}^{T-t}l_{t,s}\frac{E_t[\varphi_{t-1+s}]}{(1+r)^s}\right)-\frac{(1+r)}{l_{t+1,0}}\left(\sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s}-\sum_{s=1}^{T-t}l_{t,s}\frac{E_t[\varphi_{t-1+s}]}{(1+r)^s}\right)\\
c_{t+1}=&\frac{1}{l_{t+1,0}}\left((1+r)\left((1+r)a_t+\sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s}-\sum_{s=1}^{T-t}l_{t,s}\frac{E_t[\varphi_{t-1+s}]}{(1+r)^s}\right)+(1+r)y_t-(1+r)c_t\right.\\
&+\left.\sum_{s=0}^{T-t-1}\frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s}-\sum_{s=1}^{T-t-1}l_{t+1,s}\frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s}-(1+r)\sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s}-(1+r)\sum_{s=1}^{T-t}l_{t,s}\frac{E_t[\varphi_{t-1+s}]}{(1+r)^s}\right)
\end{aligned}$$

From equation (A.3), I can replace the expression $\left((1+r)a_t+y_t+\sum_{s=1}^{T-t}\frac{E_t[y_{t+s}-l_{t,s}\varphi_{t+s-1}]}{(1+r)^s}\right)$ by $l_{t,0}c_t$.

$$\begin{aligned}
c_{t+1}=&\frac{1}{l_{t+1,0}}\left((1+r)l_{t,0}c_t-(1+r)c_t+(1+r)y_t+\sum_{s=0}^{T-t-1}\frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s}-(1+r)y_t-(1+r)\sum_{s=1}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s}\right)\\
&-\frac{1}{l_{t+1,0}}\left(\sum_{s=1}^{T-t-1}l_{t+1,s}\frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s}-(1+r)\sum_{s=1}^{T-t}l_{t,s}\frac{E_t[\varphi_{t-1+s}]}{(1+r)^s}\right)
\end{aligned}$$

By construction, $l_{t,s}=l_{t+1,s-1}$.

$$\begin{aligned}
c_{t+1}=&\frac{1}{l_{t+1,0}}\left((1+r)(l_{t,0}-1)c_t+\sum_{s=0}^{T-t-1}\frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s}-\sum_{s=1}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^{s-1}}-\sum_{s=1}^{T-t-1}l_{t+1,s}\frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s}+\sum_{s=1}^{T-t}l_{t+1,s-1}\frac{E_t[\varphi_{t-1+s}]}{(1+r)^{s-1}}\right)\\
c_{t+1}=&\frac{1}{l_{t+1,0}}\left((1+r)(l_{t,0}-1)c_t+l_{t+1,0}\varphi_t+\sum_{s=0}^{T-t-1}\frac{(E_{t+1}-E_t)[y_{t+1+s}]}{(1+r)^s}-\sum_{s=1}^{T-t-1}l_{t+1,s}\frac{(E_{t+1}-E_t)[\varphi_{t+s}]}{(1+r)^s}\right)
\end{aligned}$$

Because $l_{t,0}=1+(R_{t,t+1}^{1/\rho}/(1+r))l_{t+1,0}$, I obtain:

$$c_{t+1}=R_{t,t+1}^{1/\rho}c_t+\underbrace{\varphi_t}_{\text{precaution}}+\underbrace{\frac{1}{l_{t+1,0}}(E_{t+1}-E_t)\left[\sum_{s=0}^{T-t-1}\frac{y_{t+1+s}}{(1+r)^s}-\sum_{s=1}^{T-t-1}\frac{l_{t+1,s}\varphi_{t+s}}{(1+r)^s}\right]}_{\text{consumption innovation }=\,c_{t+1}-E_t[c_{t+1}]}\tag{A.4}$$

A.3 Log-Consumption Growth

I divide each side of equation (A.4) by $R_{t,t+1}^{1/\rho}c_t$:

$$\frac{c_{t+1}}{R_{t,t+1}^{1/\rho}c_t}=1+\frac{\varphi_t}{R_{t,t+1}^{1/\rho}c_t}+\frac{1}{R_{t,t+1}^{1/\rho}c_tl_{t+1,0}}(E_{t+1}-E_t)\left[\sum_{s=0}^{T-t-1}\frac{y_{t+1+s}}{(1+r)^s}-\sum_{s=1}^{T-t-1}\frac{l_{t+1,s}\varphi_{t+s}}{(1+r)^s}\right]$$

I take the logarithm:

$$\Delta ln(c_{t+1}) - \underbrace{\frac{1}{\rho} ln(R_{t,t+1})}_{\text{impatience and dem.}} = ln\left(1 + \underbrace{\frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t}}_{\text{precaution}} + \underbrace{\frac{1}{R_{t,t+1}^{1/\rho} c_t l_{t+1,0}} (E_{t+1} - E_t)\left[\sum_{s=0}^{T-t-1} \frac{y_{t+1+s}}{(1+r)^s} - \sum_{s=1}^{T-t-1} \frac{l_{t+1,s}\varphi_{t+s}}{(1+r)^s}\right]}_{\text{consumption innovation} = \frac{c_{t+1} - E_t[c_{t+1}]}{R_{t,t+1}^{1/\rho} c_t}}\right) \tag{A.5}$$

Expected future income, $E_{t+1}\left[\sum_{s=0}^{T-t-1} \frac{y_{t+1+s}}{(1+r)^s}\right]$, can be expressed as a function of ε_{t+1} and η_{t+1}[17]:

$$E_{t+1}\left[\sum_{s=0}^{T-t-1} \frac{y_{t+1+s}}{(1+r)^s}\right] = E_t[y_{t+1}] ln(e^{\varepsilon_{t+1}}) ln(e^{\eta_{t+1}}) + \left(\sum_{s=1}^{T-t-1} \frac{E_t[y_{t+1+s}]}{(1+r)^s}\right) ln(e^{\eta_{t+1}})$$

The derivatives of expected income with respect to the shocks write:

$$\sum_{s=0}^{T-t-1} \frac{\left(\frac{dE_{t+1}[y_{t+1+s}]}{d\varepsilon_{t+1}}\right)}{(1+r)^s} = E_t[y_{t+1}] e^{\varepsilon_{t+1}} e^{\eta_{t+1}}$$

$$\sum_{s=0}^{T-t-1} \frac{\left(\frac{dE_{t+1}[y_{t+1+s}]}{d\eta_{t+1}}\right)}{(1+r)^s} = E_t[y_{t+1}] e^{\varepsilon_{t+1}} e^{\eta_{t+1}} + \left(\sum_{s=1}^{T-t-1} \frac{E_t[y_{t+1+s}]}{(1+r)^s}\right) e^{\eta_{t+1}}$$

I denote with a star the variables taken at the point where $(\varepsilon_{t+1}, \eta_{t+1}) = (0,0)$. I expand equation (A.5) around this point to the first order.

$$\begin{aligned}
\Delta ln(c_{t+1}) = & \frac{1}{\rho} ln(R_{t,t+1}) + ln\left(1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{c_{t+1}^* - E_t[c_{t+1}]}{R_{t,t+1}^{1/\rho} c_t}\right) \\
& + \varepsilon_{t+1} \frac{1}{R_{t,t+1}^{1/\rho} c_t l_{t+1,0}} \left(\sum_{s=0}^{T-t-1} \frac{\left(\frac{dE_{t+1}[y_{t+1+s}]}{d\varepsilon_{t+1}}\right)^*}{(1+r)^s} - \sum_{s=0}^{T-t-1} \frac{l_{t+1,s}\left(\frac{dE_{t+1}[\varphi_{t+s}]}{d\varepsilon_{t+1}}\right)^*}{(1+r)^s}\right) \frac{1}{1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{c_{t+1}^* - E_t[c_{t+1}]}{R_{t,t+1}^{1/\rho} c_t}} \\
& + \eta_{t+1} \frac{1}{R_{t,t+1}^{1/\rho} c_t l_{t+1,0}} \left(\sum_{s=0}^{T-t-1} \frac{\left(\frac{dE_{t+1}[y_{t+1+s}]}{d\eta_{t+1}}\right)^*}{(1+r)^s} - \sum_{s=0}^{T-t-1} \frac{l_{t+1,s}\left(\frac{dE_{t+1}[\varphi_{t+s}]}{d\eta_{t+1}}\right)^*}{(1+r)^s}\right) \frac{1}{1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{c_{t+1}^* - E_t[c_{t+1}]}{R_{t,t+1}^{1/\rho} c_t}} \\
& + o(\varepsilon_{t+1} - ln(E_t[e^{\varepsilon_{t+1}}]), \eta_{t+1} - ln(E_t[e^{\eta_{t+1}}]))
\end{aligned}$$

I substitute for the expression of the derivatives of expected income, which are are equal to $E_t[y_{t+1}]$ and

[17] From equation (2.1)-(2.3), I have $y_{t+s} = e^{p_{t+s}} e^{\varepsilon_{t+s}} e^{\kappa_{t+s} z_{t+s}} = e^{p_t} e^{\eta_{t+1}} \times ... \times e^{\eta_{t+s}} e^{\varepsilon_{t+s}} e^{\kappa_{t+s} z_{t+s}}$. As the shocks η are independent from each other, independent from ε, and independent from the initial value of permanent income, the expected value of their product is the product of their expected value. They are drawn from exogenous distribution so that their expected value at $t+1$ is equal to their expected value at t. Also, shocks are normalized so that $E_t[e^{\varepsilon_{t+1}}] = E_t[e^{\eta_{t+1}}] = 1$.

$$\text{for } s=1: \; E_{t+1}[y_{t+1}] = e^{p_{t+1}} e^{\varepsilon_{t+1}} e^{\kappa_{t+1} z_{t+1}} = e^{p_t} e^{\eta_{t+1}} e^{\varepsilon_{t+1}} e^{\kappa_{t+1} z_{t+1}} = \frac{e^{\varepsilon_{t+1}}}{E_t[e^{\varepsilon_{t+1}}]} \frac{e^{\eta_{t+1}}}{E_t[e^{\eta_{t+1}}]} E_t[y_{t+1}] = E_t[y_{t+1}] e^{\varepsilon_{t+1}} e^{\eta_{t+1}}$$

$$\text{for } s>1: \; E_{t+1}[y_{t+s}] = p_{t+1} E_t[e^{\eta_{t+2}}] \times ... \times E_t[e^{\eta_{t+s}}] E_t[e^{\varepsilon_{t+s}}] e^{\kappa_{t+s} z_{t+s}} = \frac{e^{\eta_{t+1}}}{E_t[e^{\eta_{t+1}}]} E_t[y_{t+s}] = E_t[y_{t+s}] e^{\eta_{t+1}}$$

$\sum_{s=0}^{T-t-1}\frac{E_t[y_{t+1+s}]}{(1+r)^s}$ at the point $(\varepsilon_{t+1},\eta_{t+1})=(ln(E_t[e^{\varepsilon_{t+1}}]),ln(E_t[e^{\eta_{t+1}}])$, and I rearrange the coefficients associated with these derivatives:

$$\begin{aligned}\Delta ln(c_{t+1}) &= \frac{1}{\rho}ln(R_{t,t+1})+ln\left(1+\frac{\varphi_t}{R_{t,t+1}^{1/\rho}c_t}+\frac{c_{t+1}^*-E_t\left[c_{t+1}\right]}{R_{t,t+1}^{1/\rho}c_t}\right)\\ &+\varepsilon_{t+1}\left(E_t[y_{t+1}]-\sum_{s=0}^{T-t-1}\frac{l_{t+1,s}\left(\frac{dE_{t+1}[\varphi_{t+s}]}{dln(\varepsilon_{t+1})}\right)^*}{(1+r)^s}\right)\frac{1}{l_{t+1,0}(R_{t,t+1}^{1/\rho}c_t+\varphi_t+c_{t+1}^*-E_t\left[c_{t+1}\right])}\\ &+\eta_{t+1}\left(\sum_{s=0}^{T-t-1}\frac{E_t[y_{t+1+s}]}{(1+r)^s}-\sum_{s=0}^{T-t-1}\frac{l_{t+1,s}\left(\frac{dE_{t+1}[\varphi_{t+s}]}{dln(\eta_{t+1})}\right)^*}{(1+r)^s}\right)\frac{1}{l_{t+1,0}(R_{t,t+1}^{1/\rho}c_t+\varphi_t+c_{t+1}^*-E_t\left[c_{t+1}\right])}\\ &+o(\varepsilon_{t+1},\eta_{t+1})\end{aligned}$$

By construction, $R_{t,t+1}^{1/\rho}c_t+\varphi_t=E_t[c_{t+1}]$. Therefore,

$$\begin{aligned}l_{t+1,0}(R_{t,t+1}^{1/\rho}c_t+\varphi_t+c_{t+1}^*-E_t\left[c_{t+1}\right]) &= l_{t+1,0}(E_t[c_{t+1}]+c_{t+1}^*-E_t\left[c_{t+1}\right])\\ &= l_{t+1,0}\frac{1}{l_{t+1,0}}\left((1+r)a_{t+1}+\sum_{s=0}^{T-t-1}\frac{E_{t+1}[y_{t+1+s}]^*}{(1+r)^s}-\sum_{s=1}^{T-t-1}l_{t+1,s}\frac{E_t[\varphi_{t+s}]^*}{(1+r)^s}\right)\\ &= \left((1+r)a_{t+1}+\sum_{s=0}^{T-t-1}\frac{E_{t+1}[y_{t+1+s}]^*}{(1+r)^s}-\sum_{s=1}^{T-t-1}l_{t+1,s}\frac{E_{t+1}[\varphi_{t+s}]^*}{(1+r)^s}\right)\end{aligned}$$

I plugg this in the expression for log-consumption growth and obtain:

$$\begin{aligned}\Delta ln(c_{t+1}) &= \frac{1}{\rho}ln(R_{t,t+1})+ln\left(1+\frac{\varphi_t}{R_{t,t+1}^{1/\rho}c_t}+\frac{c_{t+1}^*-E_t\left[c_{t+1}\right]}{R_{t,t+1}^{1/\rho}c_t}\right)\\ &+\varepsilon_{t+1}\frac{\left(E_t[y_{t+1}]-\sum_{s=0}^{T-t-1}\frac{l_{t+1,s}\left(\frac{dE_{t+1}[\varphi_{t+s}]}{d\varepsilon_{t+1}}\right)^*}{(1+r)^s}\right)}{(1+r)a_{t+1}+\sum_{s=0}^{T-t-1}\frac{E_{t+1}[y_{t+1+s}]^*}{(1+r)^s}-\sum_{s=1}^{T-t-1}l_{t+1,s}\frac{E_{t+1}[\varphi_{t+s}]^*}{(1+r)^s}}\\ &+\eta_{t+1}\frac{\left(\sum_{s=0}^{T-t-1}\frac{E_t[y_{t+1+s}]}{(1+r)^s}-\sum_{s=0}^{T-t-1}\frac{l_{t+1,s}\left(\frac{dE_{t+1}[\varphi_{t+s}]}{d\eta_{t+1}}\right)^*}{(1+r)^s}\right)}{(1+r)a_{t+1}+\sum_{s=0}^{T-t-1}\frac{E_{t+1}[y_{t+1+s}]^*}{(1+r)^s}-\sum_{s=1}^{T-t-1}l_{t+1,s}\frac{E_{t+1}[\varphi_{t+s}]^*}{(1+r)^s}}\quad+o(\varepsilon_{t+1},\eta_{t+1})\end{aligned}$$

Appendix B Borrowing Constraints

In this section, I take into account the impact the natural borrowing constraint, which imposes that consumers cannot be indebted above the worst possible realization of their total future expected income, because they cannot die in debt. I also introduce the possibility of an exogenously imposed borrowing limit. The consumer's problem is:

$$
\begin{aligned}
&\max_{c_t,\dots c_T} \quad E_t\left[\sum_{s=0}^{T-t}\beta^{t+s}u(c_{t+s})e^{\delta_t z_t}\right] \\
&s.t. \quad \begin{cases} \sum_{s=0}^{T-t}\frac{c_{t+s}}{(1+r)^s} = (1+r)a_t + \sum_{s=0}^{T-t}\frac{y_{t+s}}{(1+r)^s} \\ a_{t+1} > \max\left(-\sum_{s=0}^{T-t-1}\frac{\underline{y}_{t+1+s}}{(1+r)^{s+1}}, -L_{t+1}\right) \end{cases}
\end{aligned} \tag{3.1}
$$

The term $\underline{y}_{t+s}$ denotes the worst possible realization of y_{t+s}. The natural borrowing constraint $a_{t=1} \geq -\sum_{s=0}^{T-t-1}\frac{\underline{y}_{t+1+s}}{(1+r)^{s+1}}$ emerges from the combination of the no default condition and the requirement that consumption be positive. The exogenous borrowing constraint $a_{t+1} > L_{t+1}$ can reflect frictions on the lending market. I assume that the borrowing limit L is exogenous, predictable and perfectly anticipated by consumers. The baseline model corresponds to the base where $L_t = \infty\ \forall t$

The Euler equation becomes:

$$u'(c_t) = E_t[u'(c_{t+1})]R_{t,t+1} + \lambda_t$$

where $\lambda_t \geq 0$ denotes the multiplier on constraint (3.1). In effect, when the constraint is binding, consumers cannot borrow as much as they want and they are forced to transfer consumption from period t, to period $t+1$: their consumption at t is smaller and their marginal utility higher than it would in the absence of a borrowing limit.

Consumption at t is:

$$c_t = \frac{1}{l_{t,0}}\left(\underbrace{(1+r)a_t + \sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s}}_{\text{total expected ressources}} - \underbrace{\sum_{k=1}^{T-t} l_{t,k}\frac{E_t[\varphi_{t+k-1}]}{(1+r)^k}}_{\text{total expected precautionary saving}} - \underbrace{\frac{1}{\rho}\sum_{k=1}^{T-t} l_{t,k}\frac{E_t[\tilde{\lambda}_{t+k-1}]}{(1+r)^k}}_{\text{total expected constrained saving}}\right)$$

with $\tilde{\lambda}_{t+k-1} = \lambda_{t+k-1}\left(E_{t+k-1}[c_{t+k}] - \varphi_{t+k-1}\right)^{1+\rho} R^{-1}_{t+k-1,t+k}$ measures the impact of the borrowing constraint at $t+k+1$ on consumption growth between $t+k+1$ and $t+k$. This term is either zero or positive: everything else equal, the forced saving generates an increase in consumption growth. Borrowing constraint have an effect that is similar to precautionary saving: consumers take out from their expected resources the amount they expect to be constrained to save and consume a share of the remainder.

Log-consumption growth at t is:

$$\Delta ln(c_{t+1}) = \frac{1}{\rho} ln(R_{t,t+1}) + ln\left(1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{\lambda_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{c_{t+1}^{*} - E_t\left[c_{t+1}\right]}{R_{t,t+1}^{1/\rho} c_t}\right)$$
$$+ \varepsilon_{t+1} \frac{\left(E_t\left[y_{t+1}\right] - \left(\frac{dPS_{t+1}}{d\varepsilon_{t+1}}\right)^{*} - \left(\frac{dCS_{t+1}}{d\varepsilon_{t+1}}\right)^{*}\right)}{(1+r)a_{t+1} + \sum_{s=0}^{T-t-1} \frac{E_{t+1}\left[y_{t+1+s}\right]^{*}}{(1+r)^s} - PS_{t+1}^{*} - CS_{t+1}^{*}}$$
$$+ \eta_{t+1} \frac{\left(\sum_{s=0}^{T-t-1} \frac{E_t\left[y_{t+1+s}\right]}{(1+r)^s} - \left(\frac{dPS_{t+1}}{d\eta_{t+1}}\right)^{*} - \left(\frac{dCS_{t+1}}{d\eta_{t+1}}\right)^{*}\right)}{(1+r)a_{t+1} + \sum_{s=0}^{T-t-1} \frac{E_{t+1}\left[y_{t+1+s}\right]^{*}}{(1+r)^s} - PS_{t+1}^{*} - CS_{t+1}^{*}} + o(\varepsilon_{t+1}, \eta_{t+1})$$

The additional terms with respect to the perfect foreisght case are colored. Borrowing constraints have the same type of effects has precautionary behavior. Both have to be considered jointly because the presence of a borrowing constraint modifies precautionary saving and vice-versa (in particular forced saving can serve as a precautionary buffer so there is less need for additional precautionary saving).