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Ambiguity Aversion in Competitive Insurance Markets: Adverse and Advantageous Selection

Abstract:

We analyze an extension of the Rothschild-Stiglitz model where loss probabilities are ambiguous and consumers are ambiguity averse to determine whether there are adverse or advantageous selection equilibria. We show that non-increasing absolute ambiguity aversion is sufficient for adverse selection. The effect of ambiguity on the critical proportion of high risks required for the RS equilibrium to exist can be decomposed into a deductible effect and an ambiguity effect. When single-crossing does not hold, then in a competitive insurance market with actuarially fair prices, advantageous selection cannot occur in equilibrium.

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1. Introduction

Self-selection based on policyholders' private information plays a central role in the performance of insurance markets. Standard models of asymmetric information (e.g., Rothschild and Stiglitz, 1976, Arnott and Stiglitz, 1988) predict a positive correlation between coverage and ex post risk. Chiappori et al. (CJSS, 2006) show that this positive correlation is a robust prediction of the economic theory of asymmetric information. However, the empirical evidence on the correlation between coverage and risk in insurance markets is mixed.[1] Hemenway (1990, 1992) reports that individuals who engage in risky behavior (e.g., riding a motorcycle without a helmet) are less likely to have insurance. Chiappori and Salanié (2000) report no correlation among beginning French drivers, while Cohen (2005) reports a positive correlation for experienced Israeli drivers. Finkelstein and Poterba (2002, 2004) and He (2009) provide evidence of adverse selection in annuity markets and life insurance markets, respectively. Cawley and Phillipson (1999) report that mortality is lower for individuals with life insurance. Hendel and Lizzeri (2003) also report evidence of advantageous selection in the life insurance market. Cardon and Hendel (2001) find no evidence of asymmetric information in health insurance, while Bundorf, Herring and Pauly's (2010), Handel's (2013) and Bajari et. al.'s (2014) findings are consistent with adverse selection in employer provided health insurance.[2] Cutler, Finkelstein and McGarry (2008) report positive correlations for annuities and health insurance, negative correlations for life and Medigap insurance and no correlation for long term care insurance.

[1] See Cohen and Siegelman (2010) and Chiappori and Salanié (2013) for reviews of the literature on empirical analyses of asymmetric information.
[2] Cutler and Zeckhauser (2000), in a review of the earlier literature on health insurance, report that the vast majority of studies find evidence of adverse selection.

Recent theoretical research has attempted to explain this empirical evidence. Hemenway (1990), De Meza and Webb (2001) and De Donder and Hicks (2009) propose preference-based explanations. They argue that more risk averse individuals are more likely to take actions to reduce their risk exposure and also more likely to purchase insurance or to purchase more coverage. Netzer and Schuer (2010) assume people differ with respect to productivity or patience in addition to risk. Individuals who are more productive or more patient accumulate more wealth, which reduces their marginal willingness to pay for insurance. They show that this implies there is no necessary correlation between coverage and risk. Huang, Liu and Tzeng (2010) analyze a model in which individuals may be overconfident. Individuals who underestimate their risk chose low effort, and individuals who do not underestimate their risk chose high effort. In equilibrium, insurers offer high coverage policies to attract the rational (low risk) types and low coverage contracts to attract the overconfident (high risk) types. Spinnewijn (2013) assumes that individuals have different perceptions of both their basic risk and their ability to affect the risk. Depending on the correlation of individuals' beliefs about the basic risk and the ability to control the risk, there may be a positive or negative correlation between coverage and ex post risk. All of these models posit a second dimension of private information which, given the appropriate correlation with risk type, leads to a violation of the single-crossing condition and allows for the possibility of negative correlation between coverage and risk.

Virtually all of the theoretical analyses of adverse and advantageous selection assume that the relevant loss distributions are known or can be learned with certainty. However, even with the best available information there may be "uncertainty about probability created by missing information that is relevant and could be known" (Camerer and Weber 1992, p. 330). There are several reasons why decision-makers might experience uncertainty about their true accident probability. Anderson (2002) describes the ambiguity surrounding environmental risks, even for risk-neutral corporations. Knowledge of family health history may suggest an individual has above or below average propensity to develop a disease, but not the exact risk of becoming symptomatic. For new products, engineering studies will provide, at best, estimates of the risk of injury subject to some estimation error.

In this paper we develop a model of risk-based self-selection in which otherwise identical individuals differ only with respect to their loss probabilities. We assume that the loss probabilities are not completely knowable, so that there is ambiguity. We also assume that individuals are ambiguity averse. We analyze the existence and characteristics of equilibria in competitive insurance markets with adverse selection when there is ambiguity regarding the loss probabilities and individuals are ambiguity averse.

Insurance markets with adverse selection and ambiguity have also been examined by Koufpoulos and Kazhan (2012) and by Huang, Snow and Tzeng (2012*a*, *b*). Koufopoulos and Kazhan assume that policyholders have maxmin expected utility preference (e.g., Gilboa and Schmeidler, 1989). They also allow both ambiguity and ambiguity aversion to vary between high and low risks. Finally, they employ an idiosyncratic assumption about insurers' commitment to offer policies. That is, insurers may offer some policies that they can later withdraw from the market and other policies that they cannot withdraw. This commitment assumption implies that equilibrium always exists and is unique. All of the equilibria in their model are second-best efficient. In their model there can be a pooling equilibrium at full coverage for both types. There is also a separating equilibrium in which high risks fully insure and low risk over-insure.

Huang, Snow and Tzeng (HST) analyze a model in which individuals have both a general and specific risk of loss. The general risk is common to all policyholder and is ambiguous. The specific risk is private information and is not ambiguous. HST assume that individuals have "smooth" ambiguity aversion (Klibanoff, Marrinaci and Mukerji 2005, Nielsen 2010). HST focus on the case where insurance policies have a loading. They show that there can be a pooling equilibrium with partial coverage, although this requires "very special parameter values" (p. 3). Their model has separating equilibria with both adverse and advantageous selection. HST (2012*b*) extend the analysis to allow ambiguity averse insurers; insurers require an ambiguity premium which creates an endogenous premium loading.

Our work differs from the above mentioned papers in several dimensions. Both Koufopoulos and Kazhan and HST add features to the basic Rothschild-Stiglitz framework. Thus, it is difficult to determine if their results are due to ambiguity and ambiguity aversion or to the special features of their models. We modify the canonical Rothschild-Stigliz (1976) framework less extensively, because our objective is to keep as close to the original framework as possible. First, we assume that the loss probabilities are ambiguous, and second, we assume that policyholders are averse to this ambiguity. We employ the "smooth" model of ambiguity aversion (Klibanoff, Marrinaci and Mukerji 2005, Nielsen 2010); this is becoming the standard model of ambiguity aversion. To isolate the effect of ambiguity on competitive insurance markets, we abstain from further variations. In particular, we assume individuals face only an ambiguous general risk which may be either high or low. We also assume that the degree of ambiguity and of ambiguity aversion is the same for all individuals. Snow (2009) argues forcefully that competition in insurance markets implies that firms earn zero expected profits in equilibrium. Consequently, we focus on zero expected profit equilibria.

We show that individual's indifference curves are conventionally downward sloping and convex. We show that, when ambiguity is present, ambiguity averse individuals are more averse to lotteries over wealth than ambiguity neutral individuals. Further, increases in ambiguity and in ambiguity aversion increase aversion to lotteries over wealth.[3] We show that Mossin's (1968) theorem holds, that is, individuals fully insure at actuarially fair prices. We show that ambiguity aversion implies the low risks' indifference curves are steeper (in state space) than the high risks' indifference curves at full insurance. This need not be true at points where there is less than full insurance. That is, ambiguity aversion implies that the single-crossing condition may or may not hold. We show that non-increasing absolute ambiguity aversion is sufficient for single-crossing to hold; increasing absolute ambiguity aversion is necessary for single-crossing to fail. The possibility that the single-crossing condition may not hold creates the possibility that there are equilibria with advantageous selection.

Even when single-crossing holds, ambiguity aversion has implications for the existence of equilibrium. We show that ambiguity aversion increases the critical proportion of high risks below which the Rothschild-Stiglitz equilibrium does not exist. The argument is similar to Crocker and Snow's (2008) analysis of background risk in markets with adverse selection. Both ambiguity aversion and background risk make individuals act "as if" they are more risk averse, which shifts the equilibrium contract. The high risks' increased risk aversion relaxes the self-selection constraint, so that low risks obtain more coverage. The low risks increased risk aversion makes it easier to attract them to a defecting contract. This second effect dominates and the critical proportion of high risks increases.

[3] This increased aversion to lotteries over wealth underlies the finding that ambiguity aversion increases the willingness to pay for insurance (Alary, Gollier and Treich, 2013, Bajtelsmit, Coats and Thistle, 2015, Snow, 2011).

If single crossing holds, then advantageous selection equilibria cannot arise. We focus on the case where single crossing does not hold, and equilibria with advantageous selection may potentially exist. However, we show that advantageous selection equilibria do not exist if prices are actuarially fair. For the Rothschild-Stiglitz (RS) equilibrium, this follows from the results in CJSS. For the Wilson (1977)-Myazaki (1977)-Spence (1978, WMS) equilibrium, we show that the low risks must subsidize the high risks. The incentives created by the subsidization of high risks by low risks leads to adverse selection in equilibrium.

The structure of the paper is as follows. The second section introduces the standard model and enriches it with ambiguity and ambiguity aversion and provides basic results. Section 3 analyzes the model under the assumption that the single-crossing condition holds, focusing on the effect on the existence of the RS equilibrium. Section 4 analyzes the model under the assumption that single-crossing does not hold, focusing on the existence and characterization of the equilibrium. Section 5 provides brief concluding remarks.

2. The Standard Model without and with Ambiguity

The standard model of competitive insurance markets under asymmetric information is developed along the following lines. First let us assume that there is no ambiguity or ambiguity aversion. We assume that applicants for insurance are endowed with initial wealth W. Risk preferences are characterized by the vNM utility function $u(\cdot)$. The individuals in our model incur a loss l with probability π^H if they are high risk, or with probability π^L if they are low risk, $0 < \pi^L < \pi^H < 1$. The proportion of high risks in the population is given by λ. We denote by W_N wealth in the no-loss state and by W_L wealth in the loss state. All of these parameters of the model are common knowledge, but whether a given individual is high or low risk is private information. Consequently, expected utility is given by $(1-\pi^t)u(W_N)+\pi^t u(W_L)$ for a type t individual, $t \in \{H, L\}$.

An insurance policy consists of a premium, p, paid by the insured in both states of the world and an indemnity, q, paid to the insured if a loss occurs. Then $W_N = W - p$ and $W_L = W - p - l + q$. We can also identify an insurance policy or contract as $C = (W_N, W_L)$, specifying wealth in the no loss state and the loss state.

Now let us introduce ambiguity and ambiguity aversion into the model. We assume that the probability of loss is subject to uncertainty. It is given by $\tilde{\pi} = \pi + \tilde{e}$, where $\tilde{e}$ is a random variable with distribution F on the support $[\underline{e}, \overline{e}]$. We also assume that beliefs about the loss probability are unbiased, i.e. $E\{\tilde{\pi}\} = \pi$. Since policyholders know the prices of the policies, they can infer the average probabilities of loss from the contracts offered on the market (Ligon and Thistle, 1996).[4]

To model ambiguity aversion, there are several avenues. The most popular one is the model of smooth ambiguity preferences developed by Klibanoff, Marinacci, and Mukerji (2005) and Neilson (2010). In this model individuals form a Φ-weighted average of expected utilities implied by the different probabilistic scenarios using second-order beliefs F. Naturally, Φ' is positive because higher expected utility is desirable for the individual. The curvature of Φ captures ambiguity attitude: If Φ'' is negative (zero, positive), the decision-maker is ambiguity-averse (-neutral, -loving). In the case of ambiguity neutrality, our model collapses to the standard (subjective) EUT-case in which indifference curves are decreasing, convex, and satisfy the single-crossing property.

[4] To ensure proper beliefs throughout the analysis we assume $\underline{e} > -\pi^L$ and $\overline{e} < 1 - \pi^H$.

Let us proceed to the case with ambiguity aversion, i.e. the case where $\Phi'' < 0$. We denote by $\widetilde{U} = \tilde{\pi}u(W_L) + (1-\tilde{\pi})u(W_N)$ expected utility under uncertain beliefs about the true probability of loss and by

$$V(W_N, W_L) = \mathrm{E}\{\Phi(\widetilde{U})\} = \mathrm{E}\{\Phi(\tilde{\pi}u(W_L) + (1-\tilde{\pi})u(W_N))\} \tag{3.1}$$

the objective function of an ambiguity-averse decision-maker. First, observe that the indifference curves are downward sloping. For a constant value of V, an increase in wealth in one state of the world must be offset by a decrease in wealth in the other state of the world. The slope of the indifference curve is given by

$$M(\pi) = \left.\frac{\mathrm{d}W_L}{d\mathrm{W_N}}\right]_{V=\text{const}} = -\frac{\mathrm{E}\{\Phi'(\widetilde{U})(1-\pi-\tilde{e})\}u'(W_N)}{\mathrm{E}\{\Phi'(\widetilde{U})(\pi+\tilde{e})\}u'(W_L)} < 0. \tag{3.2}$$

Convexity of the indifference curves in the state space follows from the concavity of u and Φ, which imply that V is an increasing and concave function of (W_N, W_L).[5] Now choose two points (W_N', W_L') and (W_N'', W_L'') on the same indifference curve, i.e. $V(W_N', W_L') = V(W_N'', W_L'')$. Then,

$$V(tW_N' + (1-t)W_N'', tW_L' + (1-t)W_L'') > tV(W_N', W_L') + (1-t)V(W_N'', W_L'')$$

for all $0 < t < 1$. The point $(tW_N' + (1-t)W_N'', tW_L' + (1-t)W_L'')$ lies on a higher indifference curve and it follows that the indifference curves are convex.

Observe that at full insurance $(W_N = W_L)$, the slope of the indifference curve is given by $-(1-\pi)/\pi$. As a result the indifference curve is tangent to the fair price line at full insurance. This implies that individuals fully insure when prices are actuarially fair. Alternatively, write the insurance premium as $p = (1+\gamma)\pi q$, and substitute it into V to obtain

$$V = \mathrm{E}\{\Phi(\tilde{\pi}u(W - (1+\gamma)\pi q - l + q) + (1-\tilde{\pi})u(W - (1+\gamma)\pi q))\}.$$

[5] Technically, it is straightforward to verify that $\mathrm{d}V/\mathrm{d}W_N > 0$, $\mathrm{d}V/\mathrm{d}W_L > 0$, $\mathrm{d}^2V/\mathrm{d}W_N^2 < 0$, $\mathrm{d}^2V/\mathrm{d}W_L^2 < 0$, $\mathrm{d}^2V/\mathrm{d}W_N\mathrm{d}W_L < 0$, and that $\mathrm{d}^2V/\mathrm{d}W_N^2 \cdot \mathrm{d}^2V/\mathrm{d}W_L^2 - \mathrm{d}^2V/\mathrm{d}W_N\mathrm{d}W_L > 0$.

Differentiating with respect to q yields the following first order expression,

$$\mathrm{d}V/\mathrm{d}q = \mathrm{E}\left\{\Phi'\left(\widetilde{U}\right)[(1-(1+\gamma)\pi)\tilde{\pi}u'(W_L)-(1+\gamma)\pi(1-\tilde{\pi})u'(W_N)]\right\}.$$

For actuarially fairly priced insurance, $\gamma = 0$, this expression is equal to zero at $W_N = W_L$. For $\gamma > 0$, the derivative is negative evaluated at $W_N = W_L$. Thus, for individuals who are ambiguity averse, Mossin's (1968) theorem holds: individuals buy full insurance at fair prices and less than full insurance at unfair prices. This result is consistent with Alary et al. (2013).

We now turn to the effects of ambiguity and ambiguity aversion on the shape of the indifference curves. Following Pratt (1964), individual 1 is more risk averse than individual 2 if 1 dislikes every lottery that 2 dislikes. We use the concavity of Φ to show that ambiguity aversion implies the individual is more risk averse to lotteries over wealth. We have

$$V(W_N, W_L) = \mathrm{E}\left\{\Phi\left(\tilde{\pi}u(W_L)+(1-\tilde{\pi})u(W_N)\right)\right\} \leq \Phi(E\{\tilde{\pi}u(W_L)+(1-\tilde{\pi})u(W_N)\}) =$$

$$= \Phi\left(\pi u(W_L)+(1-\pi)u(W_N)\right) = \Phi\left(U(W_N, W_L)\right)$$

Since $\Phi(U)$ and U rank lotteries over wealth the same, the ambiguity averse individual dislikes every lottery the ambiguity neutral individual dislikes. Similarly, if Φ^1 is an increasing concave transformation of Φ^2, then individual 1 is more risk averse than individual 2 towards lotteries over wealth.[6] Now suppose person 1 has beliefs G about $\tilde{\pi}$ and person 2 has beliefs F about $\tilde{\pi}$, where G is obtained from F by a mean preserving spread. Then,

$$V_G(W_N, W_L) = \mathrm{E}_G\left\{\Phi\left(\tilde{\pi}u(W_L)+(1-\tilde{\pi})u(W_N)\right)\right\}$$

$$\leq \mathrm{E}_F\left\{\Phi\left(\tilde{\pi}u(W_L)+(1-\tilde{\pi})u(W_N)\right)\right\} = V_F(W_N, W_L).$$

As a result person 1 dislikes all lotteries that person 2 dislikes, so that 1 is more risk averse than 2. Another way of looking at this is by answering the question how ambiguity affects the slope of

[6] It is well known that an increase in risk aversion decreases the demand for risky assets (Pratt, 1964). An increase in ambiguity aversion, however, need not decease the demand for risky assets (Gollier, 2011).

the indifference curves at less than full insurance ($W_N > W_L$). It is straightforward to show that the indifference curves are flatter in the presence of ambiguity than in its absence, i.e. their slopes are less negative at a given state-contingent wealth pair.

Before turning to the equilibrium analysis, we investigate whether single-crossing holds in the presence of ambiguity. The single crossing property holds if the low risk indifference curves are always steeper than the high risk indifference curves, $M(\pi^L) < M(\pi^H)$. First, observe that at full insurance we have $M(\pi^L) = -(1-\pi^L)/\pi^L < -(1-\pi^H)/\pi^H = M(\pi^H)$. In general, $M(\pi^L) < M(\pi^H)$ is equivalent to

$$(\pi^H - \pi^L)\ \mathrm{E}\{\Phi'(\tilde{U}_H)\}\mathrm{E}\{\Phi'(\tilde{U}_L)\} + \mathrm{E}\{\Phi'(\tilde{U}_H)\tilde{e}\}\mathrm{E}\{\Phi'(\tilde{U}_L)\}$$
$$-\mathrm{E}\{\Phi'(\tilde{U}_H)\}\mathrm{E}\{\Phi'(\tilde{U}_L)\tilde{e}\} > 0 \qquad (3.3)$$

The first term is positive. Since $\mathrm{E}\{\Phi'(\tilde{U}_t)\tilde{e}\}, t \in \{H, L\}$, is positive, whether the inequality in (3.3) holds depends on the relative magnitudes of the second and third term. Also notice that the ambiguity experienced by the high risks contributes to the single crossing property, i.e. makes it more likely to be satisfied, whereas the opposite is true for the ambiguity experienced by the low risks. This shows that ambiguity has differential implications depending on which risk type experiences it despite the fact that both high and low risks face the same degree of ambiguity. The following proposition states the sufficient condition for the single crossing property to be globally satisfied.

Proposition 1: Non-increasing ambiguity aversion is sufficient for the single crossing property to hold. Increasing ambiguity aversion is necessary but not sufficient for the single crossing property to fail.

Proof: See appendix.

This condition is simple and intuitive. Our analysis above shows that it is the ambiguity experienced by the low risks that might lead to the failure of the single crossing property. For a given state-contingent wealth profile, low risks have higher expected utility than high risks because they are less likely to be in the low-wealth state. Under non-increasing ambiguity aversion, higher levels of expected utility imply that a given level of ambiguity is less painful than at lower levels of expected utility (or at least not more painful). Therefore, although high risks and low risks experience the same level of ambiguity, low risks are affected less as soon as ambiguity aversion is non-increasing. This guarantees single crossing. [7]

Conversely, a necessary condition when single crossing does not hold, is that absolute ambiguity aversion is increasing. If single crossing fails, then the low risk indifference curve crosses the high risk indifference curves from above at full insurance and then crosses the high risk indifference curve again from below.

3. The Rothschild and Stiglitz Equilibrium under Ambiguity Aversion

Let us assume that single crossing is satisfied. According to Rothschild and Stiglitz (1976) an equilibrium in a competitive insurance market is a set of contracts such that, when customers choose contracts to maximize their objective function, (i) no contract makes negative expected profits in equilibrium; and (ii) there is no contract outside the equilibrium set that, if offered, will generate a non-negative profit. This draws on the Cournot-Nash equilibrium concept. Rothschild and Stiglitz show that equilibrium existence depends on a threshold level λ^{RS} for the proportion of high risks in the market. As long as $\lambda \geq \lambda^{RS}$, the equilibrium is self-separating with full insurance for high risks at their actuarially fair rate and partial insurance for low risks at their actuarially fair rate. The partial insurance contract for low risks is determined via the incentive compatibility constraint on high risks. If, however, $\lambda < \lambda^{RS}$, equilibrium fails to exist at all. The purpose

[7] In the Appendix we derive the sufficient conditions for Proposition 1 to hold when ambiguity takes the multiplicative form $\tilde{\pi} = (1 + \tilde{e})\pi$.

of this section is to revisit this result under ambiguity with ambiguity-averse agents and to see how the equilibrium, if it exists, is affected by the presence of ambiguity.

As established in Section 2, ambiguity raises the aversion towards lotteries over wealth. As a result, indifference curves of low and high risks are more convex than in the absence of ambiguity. This generates two sets of effects, see also Crocker and Snow (2008).[8] First of all, the incentive compatibility constraint on high risks is not satisfied when evaluated at the level of coverage that low risks would obtain in the absence of ambiguity. Due to the fact that ambiguity increases the aversion towards risks over wealth, high risks need to be compensated for giving up their respective full-coverage contract by more than in the absence of ambiguity. This increases the level of coverage available to low risks and lowers the threshold λ^{RS}. However, at a given level of coverage on the fair insurance line for the low risks, the low-risk indifference curve is more convex which has a positive effect on the critical threshold λ^{RS}. As a result, the net effect is a priori indeterminate.

Let us formalize these ideas. We denote by C^{H0} and C^{L0} the policy for high and low risks, respectively, in the absence of ambiguity, i.e. when ambiguity is zero. From Rothschild and Stiglitz (1976) we know that C^{H0} provides full coverage at the fair price for high risks so that terminal wealth in both states of the world is given by $W - \pi^H l$. The level of coverage, q^0, available to low risks is determined such that high risks are indifferent between their policy and the low-risk policy:

$$u(W - \pi^H l) = \pi^H u(W - \pi^L q^0 - l + q^0) + (1 - \pi^H) u(W - \pi^L q^0).$$

[8] Crocker and Snow (2008) study the introduction of background risk when preferences are risk vulnerable. This increases the aversion towards endogenous risks over wealth. They do not address ambiguity.

Notice that this condition is not affected if we apply Φ on both sides due to strict monotonicity. We want to determine the effect of ambiguity on the incentive compatibility constraint and as such on the level of coverage available to low risks. First, note that

$$\Phi\big(u(W-\pi^H l)\big) = \Phi\big(\pi^H u(W-\pi^L q^0 - l + q^0) + (1-\pi^H)u(W-\pi^L q^0)\big)$$
$$> \mathrm{E}\big\{\Phi\big(\tilde{\pi}^H u(W-\pi^L q^0 - l + q^0) + (1-\tilde{\pi}^H)u(W-\pi^L q^0)\big)\big\},$$

due to ambiguity aversion. Let us therefore evaluate how the right hand side depends on the level of coverage under ambiguity aversion. We denote by $V(q)$ the expected welfare of high risks when purchasing indemnity q on the low-risk fair-odds line. We can rearrange the first-order condition as follows:

$$\frac{dV}{dq} = \big(\pi^H(1-\pi^L)u'(W_L) - \pi^L(1-\pi^H)u'(W_N)\big)\mathrm{E}\big\{\Phi'\big(\widetilde{U}\big)\big\}$$
$$+ \big((1-\pi^L)u'(W_L) + \pi^L u'(W_N)\big)\mathrm{E}\big\{\Phi'\big(\widetilde{U}\big)\tilde{e}\big\}.$$

The first term is positive because higher coverage at the low-risk rate increases expected utility of high risks. The second one is also positive because higher coverage reduces the difference between utility in the loss state and in the no-loss state, which in a sense reduces the level of ambiguity present. To see this, remember that we can rewrite $\widetilde{U} = U_\pi - \tilde{e}\Delta u$, and as we increase the level of coverage, Δu becomes smaller so that $\widetilde{U}$ is less volatile. As a result, the equilibrium policy for low risks entails more coverage in the presence of ambiguity. Ceteris paribus, this increases the welfare of low risks so that the critical threshold λ^{RS} for equilibrium existence is lower. In the words of Crocker and Snow (2008), this is a deductible effect.

Ambiguity also affects the indifference curves of low risks. Let q^* be the amount of coverage available to low risks if ambiguity is present, which is determined by the respective incentive compatibility constraint:

$$\Phi\big(u(W-\pi^H l)\big) = \mathrm{E}\big\{\Phi\big(\tilde{\pi}^H u(W-\pi^L q^* - l + q^*) + (1-\tilde{\pi}^H)u(W-\pi^L q^*)\big)\big\}.$$

Let $\bar{\pi} = \lambda\pi^H + (1-\lambda)\pi^L$ be the average objective probability of loss among the population. Assume that the low-risk indifference curve without ambiguity through the low-risk contract is tangential to the pooled line of insurance, and that it touches it if coverage is given by $\bar{q}$. In other words, $\bar{q}$ maximizes the low-risk expected utility without ambiguity on the pooling line. This contract is denoted by $\bar{C}$.

Now introduce ambiguity. According to our results in section 2 the indifference curves of low risks through contract $\bar{C}$ are flatter in the presence of ambiguity than without ambiguity. As a result, contracts to the southeast of contract $\bar{C}$ along the indifference curve with ambiguity are characterized by higher levels of coverage than the corresponding contracts along the indifference curve without ambiguity. Consequently, as soon as ambiguity is present contract $\bar{C}$ offers higher expected welfare to low risks than the amount of coverage q^* on the low-risk fair-odds line. Said differently, the indifference through q^* in the presence of ambiguity is below the contract $\bar{C}$ so that profitable deviations from the RS separating contracts become possible. Hence, the critical threshold for equilibrium to exist, λ^{RS}, is lower when ambiguity is introduced, ceteris paribus. Intuitively, ambiguity averse low-risk individuals suffer from the introduction of ambiguity so that it becomes easier to cherry-pick on them. We call this the ambiguity effect.

4. Advantageous Selection?

In this section we analyze whether equilibria with advantageous selection can exist in competitive markets when firms earn zero expected profits. If the single crossing condition holds, then we know there cannot be advantageous selection equilibria. Consequently, throughout this section, we assume the single crossing condition does not hold and the indifference curves cross twice.

CJSS provide a fundamental result showing that the positive correlation property holds under very general conditions. The result uses standard assumptions about preferences and does not rely on any particular equilibrium concept. The key assumption is "non-increasing profits" (NIP), that is, expected profits are weakly decreasing in the level of coverage. Since the policy that offers more coverage must sell for a higher price, this assumption implies that price cannot rise faster than the level of overage.

> **Theorem 1.** (CJSS, 2006). Assume that preferences are monotonic and risk averse to lotteries over wealth and that agents have realistic expectations ($E\{\tilde{\pi}\} = \pi$). Assume that if policy C_1 covers more than policy C_2, then it does not have higher expected profits. Then the ex post risk is higher for the contract with the higher coverage.

This result has implications for both the RS and WMS equilibria. We consider the RS equilibrium first, and assume there is a sufficient number of high risks that the equilibrium exists. If each firm can offer one contract, then in a competitive market, each contract must break even individually. In particular, the policies offered to both high and low risks earn zero expected profits and therefore satisfy NIP. It then follows directly that advantageous selection cannot occur.

> **Proposition 3**: Assume that single crossing does not hold. Assume that each firm can offer one policy and that the RS equilibrium exists. Then in equilibrium there is a positive correlation between coverage and ex post risk.

The Proposition follows immediately from CJSS's Theorem. This result implies that the adverse selection equilibrium in which high risks obtain full coverage and the low risks obtain partial coverage is *always* the outcome in the RS model.

Now suppose that firms can offer menus of contracts. In competitive equilibrium, the menu must break even overall. Suppose there is advantageous selection, so the high risks have low coverage and the low risks have high coverage. There are two possibilities – either the high risks subsidize the low risks or the low risks subsidize the high risks. In the first case, we have $\Pi(C^H) > \Pi(C^L)$ so the NIP condition holds. CJSS implies this equilibrium cannot occur. In the second case, we have $\Pi(C^L) > \Pi(C^H)$ so the NIP condition does not hold and CJSS does not apply. If there is a WMS equilibrium with cross-subsidization, then, whether there is adverse or advantageous selection, the low risks subsidize the high risks.

Spence (1978) shows that the WMS equilibrium is a solution to the constrained maximization problem

$$\max_{W_A^L, W_N^L} V^L(W_N^L, W_L^L) \tag{5.1}$$

subject to

$$V^H(W_N^H, W_L^H) \geq V^H(W_N^L, W_L^L) \tag{5.2}$$

$$V^H(W_N^H, W_L^H) \geq V^H(W_N^{H*}, W_L^{H*}) \tag{5.3}$$

$$W - [\lambda\pi^H + (1-\lambda)\pi^L]D = \lambda[\pi^H W_L^H + (1-\pi^H)\, W_N^H] \tag{5.4}$$

$$+ (1-\lambda)\,[\pi^L W_L^L + (1-\pi^L)\, W_N^L].$$

Here (5.2) is the self-selection constraints. Equation (5.3) constrains the high risk contract to be no worse than the first best high risk contract. Equation (5.4) is the zero profit constraint. With actuarially fair prices, the first best high risk contract is (W^H*, W^H*) at the intersection of the high risk fair price line and the 45 degree full insurance line.

Proposition 4: Assume that single crossing does not hold. Assume that each firm can offer a menu of policies and that low risks subsidize high risks. Then in equilibrium there is a non-negative correlation between coverage and ex post risk.

Proof: If the proportion of high risks is above λ^{**}, the WMS equilibrium coincides with the RS equilibrium. By Proposition 3, there is an adverse selection equilibrium.

Now suppose the proportion of high risks is below λ^{**}. Assume that lumps sum transfers are used to provide the cross-subsidy; this is the case considered in Spence (1978), Crocker and Snow (1985a, b) and most of the literature. Then the marginal price of coverage for both high and low risks is actuarially fair, and full coverage is optimal for both types. If both types receive full coverage, then there is a pooling equilibrium. The break-even constraint implies the equilibrium is at the intersection of the full insurance and pooled fair price lines. The correlation between coverage and ex post risk is zero. Suppose there is a separating equilibrium. The binding self-selection constraint cannot increase the expected utility of the low risks, which implies the low risks receive less than full coverage. The correlation between coverage and ex post risk is negative.[9]

Now suppose that the tax/subsidy can be proportional or lump sum. With the subsidy, the premium faced by the high risks can be written as $p^H = (1 - s)\pi^H q^H - S$, where q^H is the level of coverage and s , $S \geq 0$, with at least one strict inequality, are the subsidies. The subsidy is paid for by a "tax" on the low risks, who face the premium $p^L = (1 + t)\pi^L q^L + T$, where t, $T \geq 0$. The level of coverage preferred by the high risks and low risk depends on the proportional tax/subsidy and not on the lump sum transfer.

[9] If $\lambda > \lambda^{**}$, so the RS equilibrium exists, then the transfers are zero. The argument in the proof implies the outcome in the RS equilibrium is adverse selection.

Now suppose $s > 0$ and $t = 0$. The high risks would prefer more than full insurance and the low risks full insurance. These choices could potentially satisfy the constraints, and, if so, there is a positive correlation between coverage and ex post risk. If not, then the constraints limit the low risks coverage and the correlation is positive. If the principle of indemnity applies and limits the high risks to full coverage, then the constraints (5.2) and (5.3) imply the low risks receive less than full coverage and the correlation is positive. If $t > 0$, the low risks prefer less than full insurance while the high risk prefer full insurance ($s = 0$) or more than full insurance ($s > 0$). Then, whether the principle of indemnity and/or the constraints limit coverage or not, there is a positive correlation between coverage and ex post risk. There is no combination of taxes and subsidies under which the low risk obtain more coverage than the high risks. Thus, the correlation between coverage and ex post risk is non-positive. ||

Whether the single-crossing condition holds or not, any subsidies in a WMS equilibrium must run from the low risks to the high risks. The pattern of subsidization from low risks to high risks implies an adverse selection equilibrium.

Propositions 3 and 4 do not imply that advantageous selection equilibria cannot occur. If there is advantageous selection, the consumers' preferences must have sufficiently strong increasing absolute ambiguity aversion so that the single crossing condition does not hold. In addition, the policies sold to the high risks must have a positive loading so that the first-best high risk policy involves less than full insurance. Both conditions are necessary for advantageous selection.

Except for the pooling equilibrium, the proof of Proposition 4 does not use the zero profit constraint, which leaves open the possibility that the outcome could result is positive profits. If firms can offer menus of contracts and there are no barriers to entry, then competition will erode profits; this is essentially the argument in Snow (2009). Consider a proposed pair of equilibrium contracts, $\{C^H, C^L\}$, which maximize $V^H(W_N^L, W_L^L)$ subject to the constraints (5.2) and (5.3) and would earn strictly positive profits. Then a new entrant can break the proposed equilibrium by offering a slightly smaller tax on the low risks, a slightly higher subsidy to the high risks and earning a smaller, but still positive profit. The process continues until profits are driven to zero.

5. Conclusions

Empirical research finds that there are both positive and negative correlations between coverage and ex post risk in insurance markets, that is, there is both adverse and advantageous selection. Theoretical research attempting to explain these empirical results assume a second dimension of unobserved heterogeneity, but assumes the underlying loss distributions are known. In this paper we assume risk is the single dimension of unobserved heterogeneity but there is ambiguity in the sense that the loss probabilities are not known with certainty. We extend the Rothschild-Stiglitz (1976) model by assuming that there is ambiguity and that consumers have KMM smooth ambiguity averse preferences. We make no other modifications to the Rothschild-Stiglitz model.

When consumers are ambiguity averse, the indifference curves are still downward sloping and convex (in state space). Increases in ambiguity and increases in ambiguity aversion make consumers more averse to lotteries over wealth. Consumers still fully insure at actuarially fair prices. However ambiguity aversion raises the possibility that the single-crossing condition may not hold. We show that decreasing or constant absolute ambiguity aversion is sufficient for the single-crossing condition to hold. Increasing absolute ambiguity aversion is necessary but not sufficient for single-crossing to fail.

When single-crossing holds, ambiguity aversion has implications for the existence of the RS equilibrium. We show that ambiguity aversion increases the critical proportion of high risks below which the Rothschild-Stiglitz equilibrium does not exist. Ambiguity aversion makes individuals act “as if” they are more risk averse, which shifts the equilibrium contract. The high risks’ increased risk aversion relaxes the self-selection constraint, so that low risks obtain more coverage. The low risks increased risk aversion makes it easier to attract them to a defecting contract. This second effect dominates and the critical proportion of high risks increases.

If single crossing holds, then advantageous selection equilibria cannot arise. When single crossing does not hold, equilibria with advantageous selection may potentially exist. However, we show that advantageous selection equilibria do not exist if prices are actuarially fair. For the Rothschild-Stiglitz (RS) equilibrium, this follows from the results in CJSS that the positive correlation between coverage and ex post risk holds in a broad range of settings. For the Wilson-Myazaki-Spence (WMS) equilibrium, this follows the incentives created by the subsidization of high risks by low risks; the low risks never obtain more coverage than the high risks. As in the standard case where single-crossing holds, there is a pooling equilibrium and separating equilibria where the high risks obtain full coverage and the low risks obtain partial coverage.

If there is an advantageous selection equilibrium due to ambiguous loss probabilities, then consumers must have sufficiently strong increasing ambiguity aversion and prices must be actuarially unfair. Non-increasing absolute ambiguity aversion and actuarially fair pricing are each sufficient conditions for adverse selection equilibria.

Appendix: Proof of Proposition 1

We will first explore how $M(\pi)$ behaves locally. Define $\Delta u = u(W_N) - u(W_L)$, which is positive. $\mathrm{d}M/\mathrm{d}\pi$ is a fraction whose sign is determined by the sign of the numerator. After some simplifications this numerator can be written as

$$u'(W_N)u'(W_L)\left\{\Delta u\left(\mathrm{E}\{\Phi'(\tilde{U})\tilde{e}\}\mathrm{E}\{\Phi''(\tilde{U})\} - \mathrm{E}\{\Phi'(\tilde{U})\}\mathrm{E}\{\Phi''(\tilde{U})\tilde{e}\}\right) + \mathrm{E}\{\Phi'(\tilde{U})\}\mathrm{E}\{\Phi'(\tilde{U})\}\right\}.$$

Note that $\tilde{U}$ and $-\Delta u\tilde{e}$ only differ by a positive constant; as such we can rewrite the expression in curly brackets as follows:

$$\mathrm{E}\{\Phi'(\tilde{U})\}\mathrm{E}\{\Phi''(\tilde{U})\tilde{U}\} - \mathrm{E}\{\Phi'(\tilde{U})\tilde{U}\}\mathrm{E}\{\Phi''(\tilde{U})\} + \mathrm{E}\{\Phi'(\tilde{U})\}\mathrm{E}\{\Phi'(\tilde{U})\}.$$

Let $\tilde{U}$ be distributed in $[\underline{u}, \overline{u}]$ according to the density $f(u)$. We can then expand the sum of the first and the second term according to

$$\int_{\underline{u}}^{\overline{u}} \Phi'(u)f(u)du \int_{\underline{u}}^{\overline{u}} \Phi''(v)vf(v)dv - \int_{\underline{u}}^{\overline{u}} \Phi'(u)uf(u)du \int_{\underline{u}}^{\overline{u}} \Phi''(v)f(v)dv =$$

$$= \int_{\underline{u}}^{\overline{u}} \int_{\underline{u}}^{\overline{u}} (v-u)\,\Phi'(u)\Phi''(v)f(u)f(v)dudv.$$

Rather than integrating over the entire square $[\underline{u}, \overline{u}] \times [\underline{u}, \overline{u}]$, we slice it up along the diagonal and integrate over one of the resulting triangles only. Due to the fact that

$$\int_{\underline{u}}^{\overline{u}} \int_{\{u>v\}} (v-u)\,\Phi'(u)\Phi''(v)f(u)f(v)dudv = \int_{\underline{u}}^{\overline{u}} \int_{\{u\leq v\}} (u-v)\,\Phi'(v)\Phi''(u)f(u)f(v)dudv,$$

we obtain that

$$\int_{\underline{u}}^{\overline{u}} \int_{\underline{u}}^{\overline{u}} (v-u)\, \Phi'(u)\Phi''(v) f(u) f(v) du dv =$$

$$= \int_{\underline{u}}^{\overline{u}} \int_{\{u \leq v\}} (v-u)\left(\Phi'(u)\Phi''(v) - \Phi'(v)\Phi''(u)\right) f(u) f(v) du dv$$

$$= \int_{\underline{u}}^{\overline{u}} \int_{\{u \leq v\}} (v-u)\, \Phi'(v)\Phi'(u)\left(A_\Phi(u) - A_\Phi(v)\right) f(u) f(v) du dv,$$

where $A_\Phi(u) = -\Phi''(u)/\Phi'(u)$is the index of absolute ambiguity aversion. Now on $\{u \leq v\}$ we have that $(v-u)$ is non-negative, and so is $\left(A_\Phi(u) - A_\Phi(v)\right)$ as long as absolute ambiguity aversion is non-increasing. This shows that non-increasing absolute ambiguity aversion is sufficient to obtain that $\mathrm{d}M/\mathrm{d}\pi$ is positive. Given that π is fixed but arbitrary, this argument shows that non-increasing absolute ambiguity aversion implies that $\mathrm{d}M/\mathrm{d}\pi$ is positive for all $0 \leq \pi \leq 1$. As such it follows that for any choice of $0 < \pi^L < \pi^H < 1$, non-increasing absolute ambiguity aversion is sufficient for the single-crossing property to be satisfied whereas increasing absolute ambiguity aversion is necessary (but not sufficient) for the single crossing property to fail.

Appendix: Single-Crossing with Multiplicative Ambiguity

In this appendix, we derive the sufficient condition for single-crossing when ambiguity takes the multiplicative form, $\tilde{\pi} = (1 + \tilde{e})\pi$. As in the text, the value function is

$$V(W_N, W_L) = \mathrm{E}\left\{\Phi\left(\widetilde{U}\right)\right\} = \mathrm{E}\left\{\Phi\left(\tilde{\pi} u(W_L) + (1-\tilde{\pi}) u(W_N)\right)\right\}.$$

The marginal rate of substitution is

$$M(\pi) = \left.\frac{dW_L}{dW_N}\right]_{V=const} = -\frac{\mathrm{E}\left\{\Phi'\left(\widetilde{U}\right)(1-\tilde{\pi})\right\} u'(W_N)}{\mathrm{E}\left\{\Phi'\left(\widetilde{U}\right)\tilde{\pi}\right\} u'(W_L)} < 0.$$

This can be rewritten as

$$M(\pi) = -\left(\frac{\mathrm{E}\{\Phi'(\widetilde{U})\}}{\mathrm{E}\{\Phi'(\widetilde{U})\tilde{\pi}\}} - 1\right) \cdot \frac{u'(W_N)}{u'(W_L)}.$$

The numerator of the derivative of the bracketed expression with respect to π is, after some simplifications, given by:

$$\pi\Delta u \begin{pmatrix} \mathrm{E}\{\Phi'(\widetilde{U})\}\mathrm{E}\{\Phi''(\widetilde{U})\tilde{e}\} + \mathrm{E}\{\Phi'(\widetilde{U})\}\mathrm{E}\{\Phi''(\widetilde{U})\tilde{e}^2\} \\ -\mathrm{E}\{\Phi'(\widetilde{U})\tilde{e}\}\mathrm{E}\{\Phi''(\widetilde{U})\} - \mathrm{E}\{\Phi'(\widetilde{U})\tilde{e}\}\mathrm{E}\{\Phi''(\widetilde{U})\tilde{e}\} \end{pmatrix}.$$

Using the fact that $\pi\tilde{e}\Delta u = U_\pi - \widetilde{U}$, the last expression can be rewritten as follows:

$$\begin{aligned} &\mathrm{E}\{\Phi'(\widetilde{U})\widetilde{U}\}\mathrm{E}\{\Phi''(\widetilde{U})\} - \mathrm{E}\{\Phi'(\widetilde{U})\}\mathrm{E}\{\Phi''(\widetilde{U})\widetilde{U}\} \\ &\qquad + \frac{U_\pi}{\pi\Delta u}\left(\mathrm{E}\{\Phi'(\widetilde{U})\widetilde{U}\}\mathrm{E}\{\Phi''(\widetilde{U})\} - \mathrm{E}\{\Phi'(\widetilde{U})\}\mathrm{E}\{\Phi''(\widetilde{U})\widetilde{U}\}\right) \\ &\qquad + \frac{1}{\pi\Delta u}\left(\mathrm{E}\{\Phi'(\widetilde{U})\}\mathrm{E}\{\Phi''(\widetilde{U})\widetilde{U}^2\} - \mathrm{E}\{\Phi'(\widetilde{U})\widetilde{U}\}\mathrm{E}\{\Phi''(\widetilde{U})\}\widetilde{U}\right). \end{aligned}$$

Under non-increasing absolute ambiguity aversion we know from the previous analysis that the expression in the first and the second line are negative. For the expression in the third line, we will employ the integral technique again:

$$\int_{\underline{u}}^{\overline{u}} \Phi'(u)f(u)du \int_{\underline{u}}^{\overline{u}} \Phi''(v)v^2 f(v)dv - \int_{\underline{u}}^{\overline{u}} \Phi'(u)uf(u)du \int_{\underline{u}}^{\overline{u}} \Phi''(v)vf(v)dv =$$

$$= \int_{\underline{u}}^{\overline{u}} \int_{\underline{u}}^{\overline{u}} v(v-u)\,\Phi'(u)\Phi''(v)f(u)f(v)dudv.$$

Furthermore,

$$\int_{\underline{u}}^{\overline{u}} \int_{\{u>v\}} v(v-u)\,\Phi'(u)\Phi''(v)f(u)f(v)dudv = \int_{\underline{u}}^{\overline{u}} \int_{\{u\leq v\}} u(u-v)\,\Phi'(v)\Phi''(u)f(u)f(v)dudv,$$

so that the previous integral becomes

$$\int_{\underline{u}}^{\overline{u}} \int_{\{u \le v\}} (v-u)\, \Phi'(u)\Phi'(v)\Phi''(u)\big(uA_{\Phi}(u) - vA_{\Phi}(v)\big) f(u)f(v)dudv.$$

On $\{u \le v\}$ we have that $(v-u)$ is non-negative and that $\big(uA_{\Phi}(u) - vA_{\Phi}(v)\big)$ is non-positive as long as relative ambiguity aversion is non-decreasing. This shows that non-increasing absolute ambiguity aversion and non-decreasing relative ambiguity aversion together are sufficient to obtain that $\mathrm{d}M/\mathrm{d}\pi$ is positive for any $0 \le \pi \le 1$.[10] Then, single crossing will hold. Conversely, a necessary condition when single crossing is not satisfied, is that either absolute ambiguity aversion is increasing or that relative ambiguity aversion is decreasing.

[10] Notice that the two conditions are neither necessary nor sufficient for one another. One example that satisfies both of them is an exponential specification with $\Phi(x) = -\exp(-\alpha x), \alpha > 0$.

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